The JINA Center for the Evolution of the Elements



Separator Ion Optics School NSCL, Michigan State University

Series of Four Lectures plus COSY Tutorials September 10-14, 2018

Georg P. Berg University of Notre Dame JINA Center for the Evolution of the Elements

The Lecture Series

An Introduction to Ion-Optics

1st Lecture: 9/10/18: Formalism and ion-optical elements

2nd Lecture: 9/12/18: Ion-optical systems and spectrometers

3rd Lecture: 9/12/18: Recoil separators for nuclear astrophysics, St. GEORGE

4rd Lecture: 9/13/18: The recoil separator SECAR for FRIB

Hands-on sessions in the afternoon: 9/10/18 – 9/14/18: COSY Infinity

Motivation

- Manipulate charged particles ($e^{+/-}$, ions, like p, d, α , HI)
- Beam line, beam delivery systems
- Magnetic & electric analysis/ separation (Experiments!)
- (Acceleration of ions, not covered in these lectures)

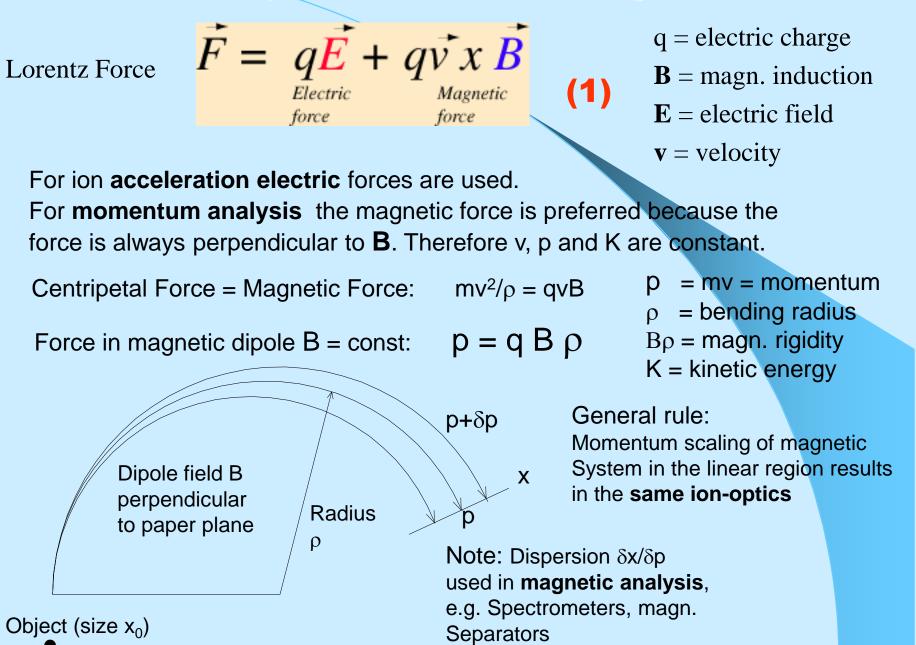
Tools

- Geometry, drawing tools, CAD drafting program (e.g. AutoCad)
- Linear Algebra (Matrix calculations), first order ion-optics (e.g. TRANSPORT)
- Higher order ion-optics code to solve the equation of motion, e.g. COSY Infinity, GIOS, RAYTRACE (historic)
- Electro-magnetic field program (solution of Maxwell's Equations), (e.g. finite element (FE) codes, 2d & 3d: POISSON, OPERA, MagNet)
- Properties of incoming charged particles and design function of electro-magnetic facility, beam, reaction products (e.g. kinematic codes, charge distributions of heavy ions, energy losses in targets (e.g. SRIM), detectors, etc, e.g. LISE++ for fragment separators)
- Many other specialized programs, e.g. for accelerator design (e.g. synchrotrons, cyclotrons) or gas-filled systems, and Monte Carlo simulations are not covered in this lecture series.

Literature

- Optics of Charged Particles, Hermann Wollnik, Academic Press, Orlando, 1987
- The Optics of Charged Particle Beams, David.C. Carey, Harwood Academic Publishers, New York 1987
- Accelerator Physics, S.Y. Lee, World Scientific Publishing, Singapore, 1999
- TRANSPORT, A Computer Program for Designing Charged Particle Beam Transport Systems, K.L. Brown, D.C. Carey, Ch. Iselin, F. Rotacker, Report CERN 80-04, Geneva, 1980
- Computer-Aided Design in Magnetics, D.A. Lowther, P. Silvester, Springer 1985

lons in static or quasi-static electro-magnetic fields



Code TRANSPORT:

(x, Θ, y, Φ, l, dp/p)
(1, 2, 3, 4, 5, 6)
Convenient "easy to use" program
for beam lines with paraxial beams

Not defined in the figure are:

dp/p = rel. momentum l = path length difference

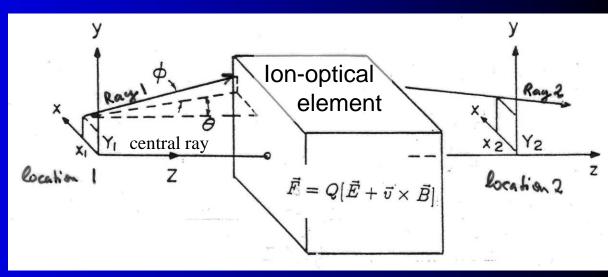
All parameters are relative to "central ray"

Code: COSY Infinity:

 $(x, a, y, b, l, \delta_K, \delta_m, \delta_z)$

Needed for complex ion-optical systems including several charge states different masses velocities (e.g. Wien Filter) higher order corrections

Defining a RAY



Not defined in the figure are:

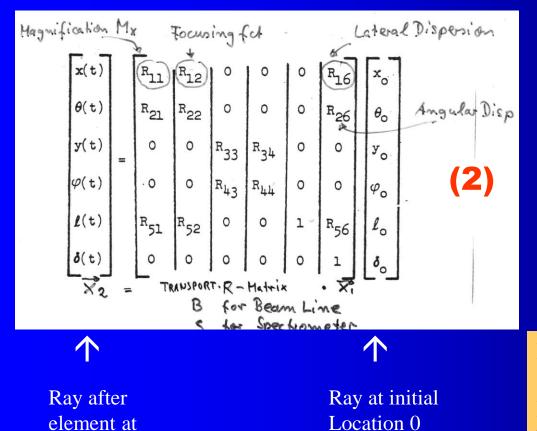
$$\begin{split} \delta_K &= dK/K = rel. \ energy \\ \delta_m &= dm/m = rel. \ mass \\ \delta_z &= dq/q = rel. \ charge \ change \\ a &= p_x/p_0 \\ b &= p_y/p_0 \\ All \ parameters \ are \ relative \\ to ``central \ ray`' \ properties \end{split}$$

Note: Notations in the Literature is not consistent! Sorry, neither will I be.

6x6 Matrix representing optic element (first order)

Transport of a ray

 \checkmark



Location t

Note: We are not building "random" optical elements. Many matrix elements = 0 because of symmetries, e.g. mid-plane symmetry

DRIFT space matrix $\vec{E} = \vec{C} = \vec{O}$ The first-order R matrix for a drift space is as follows: 1 L 0 <td

where

L = the length of the drift space.

First-order qu	adrupole mat	rix dB dr	$=0 \frac{dB}{dy} \neq 0$		6
cos k L	$\frac{1}{k_q}$ sin k_q^L	0	0	0	0
-k _q sin k _q L	cos k L q	0	0	0	0
0	0	$\cosh k_q^L$	$\frac{1}{k_q}$ sinh k_q^L	0	0
0	0	$k_q \sinh k_q L_q$	cosh k L	0	0
0	0	0	0	1	0
0	0	0	0	0	1

These elements are for a quadrupole which focuses in the horizontal (x) plane (B positive). A vertically (y-plane) focusing quadrupole (B negative) has the first two diagonal submatrices interchanged.

Definitions: L = the effective length of the quadrupole

- = the radius of the aperture
- Bo = the field at radius a
- k²_q = (B₀/a)(1/Bρ₀), where (Bρ₀) = the magnetic rigidity (momentum) of the central trajectory.

TRANSPORT matrices of a Drift and a Quadrupole

For reference of TRANSPORT code and formalism:

K.L. Brown, F. Rothacker, D.C. Carey, and Ch. Iselin, TRANSPORT: A computer program for designing charged particle beam transport systems, SLAC-91, Rev. 2, UC-28 (I/A), also: CERN 80-04 Super Proton Synchrotron Division, 18 March 1980, Geneva, Manual plus Appendices available on Webpage: ftp://ftp.psi.ch/psi/transport.beam/CERN-80-04/

David. C. Carey, The optics of Charged Particle Beams, 1987, Hardwood Academic Publ. GmbH, Chur Switzerland

Transport of a ray through a system of beam line elements

6x6 Matrix representing first optic element (usually a Drift)

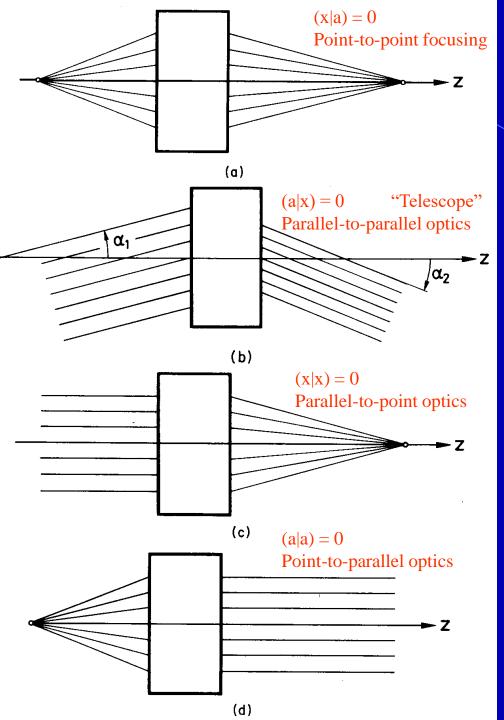
(3) $\mathbf{x}_{n} = \mathbf{R}_{n} \mathbf{R}_{n-1} \dots \mathbf{R}_{0} \mathbf{x}_{0}$ \uparrow Ray at initial Ray at final

Location n

Location 0 (e.g. a target)

Complete system is represented by one Matrix $R_{system} = R_n R_{n-1} \dots R_0$

(4)



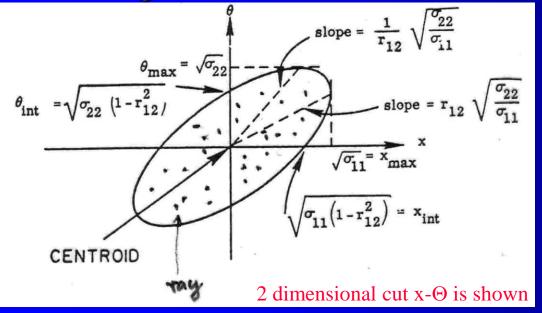
Geometrical interpretation of some TRANSPORT matrix elements

Focusing Function

$(\mathbf{x} \mathbf{a})$	Wollnik, COSY Infinity
$= dx/d\Theta$	physical meaning
$= (\mathbf{X} \Theta)$	RAYTRACE
$= R_{12}$	TRANSPORT

Achromatic system: $R_{16} = R_{26} = 0$ $(x|\delta p) = (a|\delta p)$

Defining a BEAM The 2-dimensional case (x, Θ)



Equation of Ellipse: $\sigma_{22}x^2 + 2\sigma_{21}x\Theta + \sigma_{11}\Theta^2 = \det \sigma$

Real positive definite
symmetric σ Matrix $\sigma := \begin{pmatrix} \sigma_{11} & \sigma_{21} \\ \sigma_{21} & \sigma_{22} \end{pmatrix}$ Inverse Matrix $\sigma^{-1} = 1/\epsilon^2 \begin{pmatrix} \sigma_{22} & -\sigma_{21} \\ -\sigma_{21} & \sigma_{11} \end{pmatrix}$

Warning: This is a mathematical abstraction of a beam: It is your responsibility to verify it applies to your beam Attention: Space charge effects occur when the particle density is high, so that particles repel each other (Point in $x - \Theta$ phase space)

$$\mathbf{X} = \begin{pmatrix} \mathbf{x} \\ \Theta \end{pmatrix} \qquad \mathbf{X}^{\mathrm{T}} = (\mathbf{x} \ \Theta)$$

Ellipse in Matrix notation: $X^{T} \sigma^{-1} X = 1$ (5)

Ellipse Area = π (det σ)^{1/2} is called "Phase Space Area"

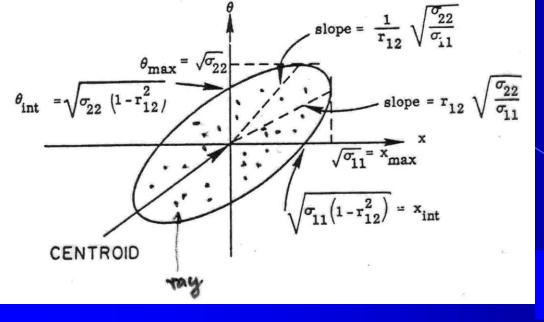
Emittance $\varepsilon = \sqrt{\det \sigma}$

$$= \sqrt{\sigma_{11}\sigma_{22}} - (\sigma_{12})^2 (6)$$

Emittance ε is constant for fixed energy & conservative forces (Liouville's Theorem)

The emittance is a precious commodity

Note: ε shrinks (increases) with acceleration (deceleration); Dissipative forces: ε increases in gases; electron, stochastic, laser cooling



Emittance ε measurement by tuning a quadrupole

The emittance ε is an important parameter of a beam. It can be measured as shown below.

Emittance: $\varepsilon = \sqrt{\sigma_{11}\sigma_{22}} - (\sigma_{12})^2$

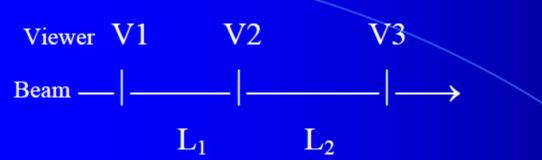
$$x_{max} = \sigma_{11} (1 + \sigma_{12} L / \sigma_{11} - L g) + (\epsilon L)^2 / \sigma_{22}$$

 $g = \frac{\partial Bz/\partial x * l}{B\rho} \qquad (Quadr. field strength) \\ l = eff. field length$

L = Distance between quadrupole and beam profile monitor

Take minimum 3 measurements of $x_{max}(g)$ and determine Emittance ε

Ref. S.Y. Lee, Accelerator Physics, p 55



L = Distances between viewers (beam profile monitors)

Emittance ε measurement by moving viewer method

The emittance ε can also be measured in a drift space as shown below.

$$(\mathbf{x}_{\max}(V2))^2 = \sigma_{11} + 2 L_1 \sigma_{12} + L_1^2 \sigma_{22}$$

 $(\mathbf{x}_{\max}(V3))^2 = \sigma_{11} + 2 (L_1 + L_2)\sigma_{12} + (L_1 + L_2)^2 \sigma_{22}$

where $\sigma_{11} = (x_{max}(V1))^2$

Emittance: $\varepsilon = \sqrt{\sigma_{11}\sigma_{22}} - (\sigma_{12})^2$

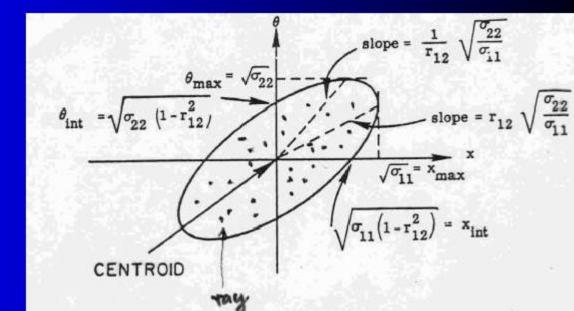
Discuss practical aspects No ellipse no ε? Phase space!

Courant-Snyder Notation

In their famous "Theory of the Alternating Synchrotron" Courant and Snyder used a Different notation of the σ Matrix Elements, that are used in the Accelerator Literature.

For you r future venture into accelerator physics here is the relationship between the σ matrix and the betatron amplitue functions α , β , γ or Courant Snyder parameters

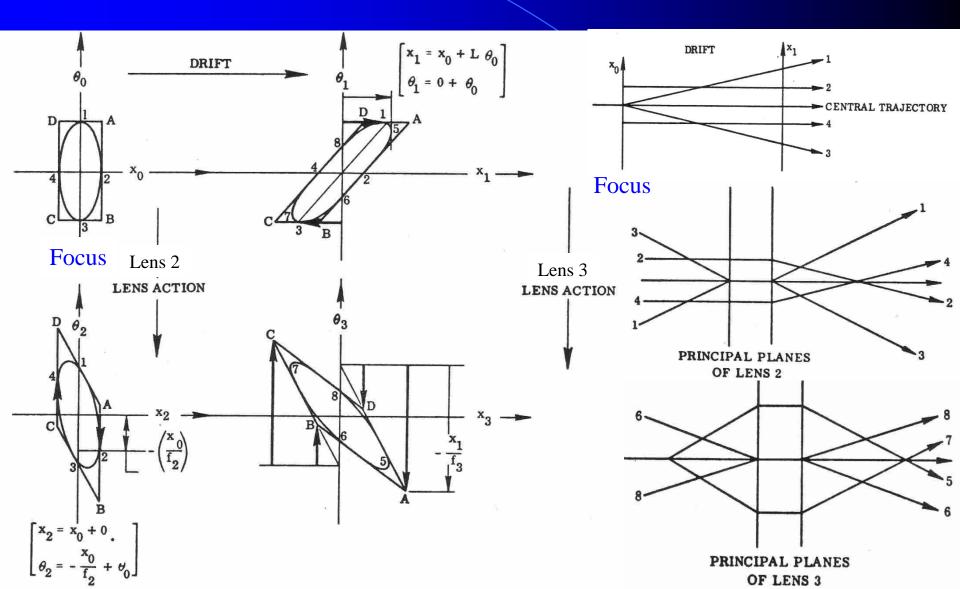
$$\sigma = \begin{pmatrix} \sigma_{11} \sigma_{21} \\ \sigma_{21} \sigma_{22} \end{pmatrix} = \epsilon \begin{pmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{pmatrix}$$

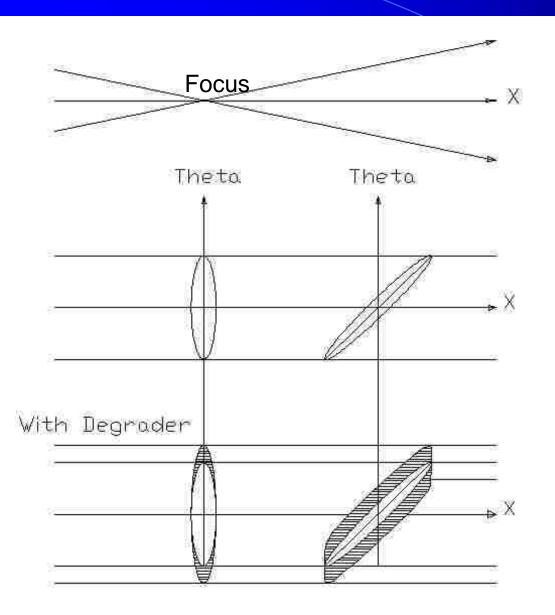


Transport of 6-dim σ Matrix

Consider the 6-dim. ray vector in TRANSPORT: $X = (x, \Theta, y, \Phi, l, dp/p)$ $X_1 = RX_0$ (7) Ray X_0 from location 0 is transported by a 6 x 6 Matrix R to location 1 by: Note: R maybe a matrix representing a complex system (3) is : $\mathbf{R} = \mathbf{R}_n \mathbf{R}_{n-1} \dots \mathbf{R}_0$ $X_0^T \sigma_0^{-1} X_0 = 1$ Ellipsoid in Matrix notation (6), generalized to e.g. 6-dim. using σ Matrix: Inserting Unity Matrix $I = RR^{-1}$ in equ. (6) it follows $X_0^{T}(R^{T}(R^{T})^{-1}) \sigma_0^{-1}(R^{-1}R) X_0 = 1$ $(\mathbf{R}\mathbf{X}_0)^{\mathrm{T}} (\mathbf{R}\sigma_0 \mathbf{R}^{\mathrm{T}})^{-1} (\mathbf{R}\mathbf{X}_0) = 1$ (8) from which we derive (9) The equation of the new ellipsoid after transformation becomes $X_1^T \sigma_1^{-1} X_1 = 1$ (10) $\sigma_1 = R\sigma_0 R^T$ where Conclusion: Knowing the TRANSPORT matrix R that transports one ray through an ion-optical system using (7) we can now also transport the phase space ellipse describing the initial beam using (10)Equivalence of Transport of ONE Ray \Leftrightarrow Ellipsoid

The transport of rays and phase ellipses in a Drift and focusing Quadrupole, Lens



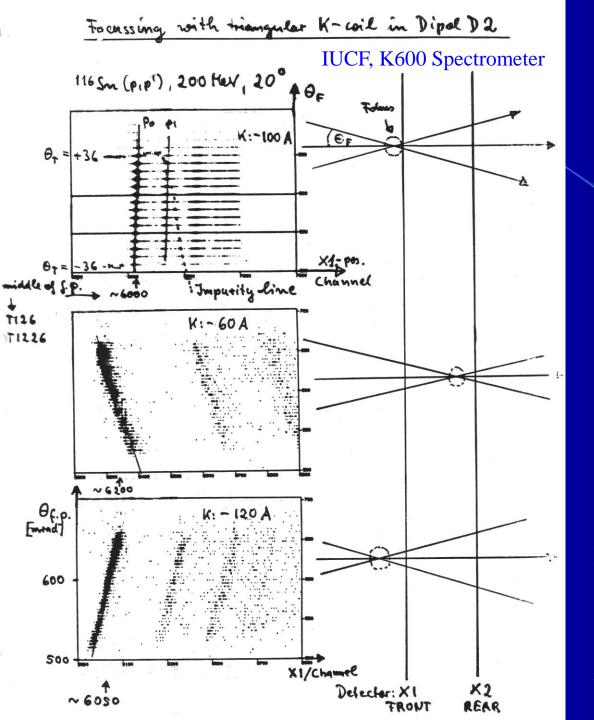


Increase of Emittance ε due to degrader

for back-of-the-envelop discussions!

A degrader / target increases the emittance ε due to multiple scattering.

The emittance growth is minimal when the degrader is positioned in a focus As can be seen from the schematic drawing of the horizontal x-Theta Phase space.



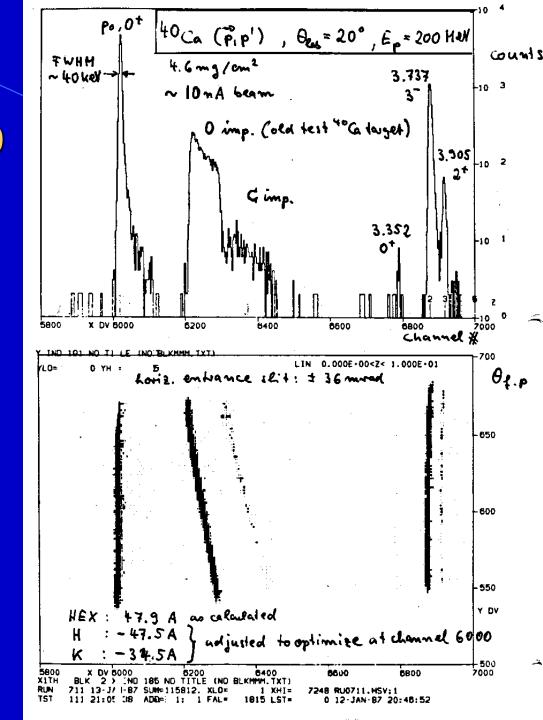
Diagnostics in focal plane of spectrometer

Typical in focal plane of Modern Spectrometers:

Two position sensitive Detectors: Horizontal: X1, X2 Vertical: Y1, Y2

Fast plastic scintillators: Particle identification Time-of-Flight

Measurement with IUCF K600 Spectrometer illustrates from top to bottom: focus near, downstream and upstream of X1 detector, respectively



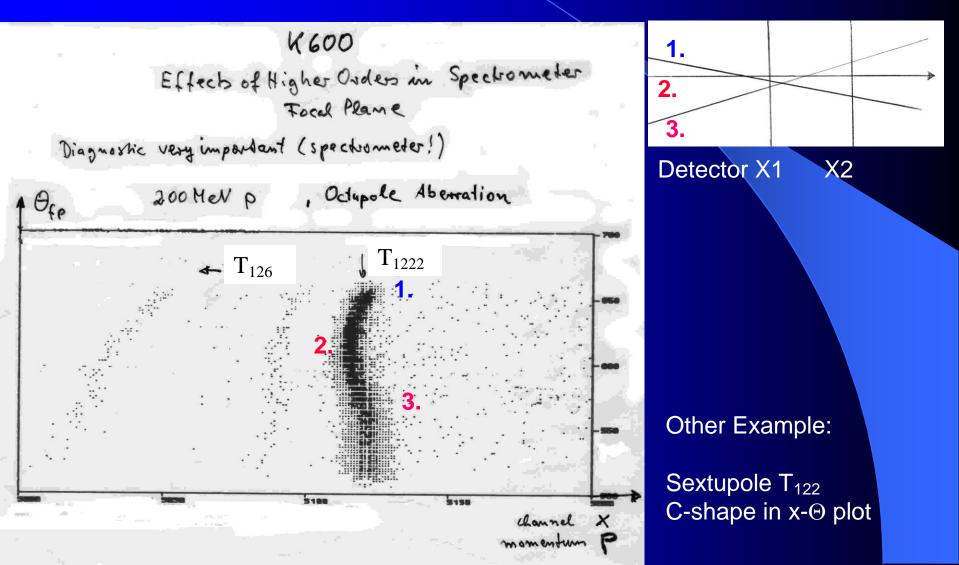
Kinematic affect in K600 spectrometer spectrum

with impurities

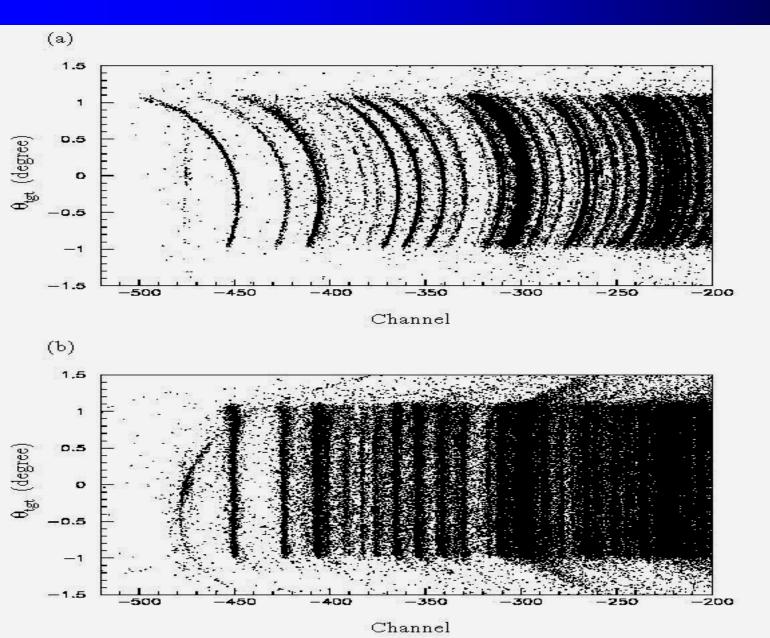
Higher order beam aberrations

Example Octupole (S-shape in x- Θ plane

3 rays in focal plane



Effect of 2nd order aberrations



Saturation of Grand Raiden at 450 MeV (³He,t) Taylor expansion in $x_1, \theta_1, y_1, \phi_1$, and δ

$$R_{11} \qquad R_{12} \qquad R_{12} \qquad R_{12} \qquad R_{12} \qquad x_{12} = (x/x)x_1 + (x/\theta)\theta_1 + (x/\delta)\delta + (x/x^2)x_1^2 \\ + (x/x\theta)x_1\theta_1 + (x/\theta^2)\theta_1^2 + (x/x\delta)x_1\delta \qquad (11) \\ + (x/\theta\delta)\theta_1\delta + (x/\delta^2)\delta^2 + (x/y^2)y_1^2 + (x/y\phi)y_1\phi_1 \\ + (x/\phi^2)\phi_1^2 + \text{higher order terms} \\ q. Transfer we fight R_{11} = (X/\kappa) = \frac{\partial \kappa_2}{\partial x_1} \qquad \text{Megnification} \\ R_{16} = (x/\delta) = \frac{\partial \kappa_2}{\partial \delta} \qquad \text{Hegnification} \\ \text{Higher orders : e.g. } (x/\theta^2) = T_{122} = \frac{\partial \kappa_2}{\partial \theta}$$

Taylor expansion

Note: Several notations are in use for 6 dim. ray vector & matrix elements.

 $T_{nmo} = (n|mo)$ TRANSPORT RAYTRACE Notation

Linear (1st order)TRANSPORT Matrix R_{nm}

Remarks:

2

- Midplane symmetry of magnets reason for many matrix element = 0
- Linear approx. for "well" designed magnets and paraxial beams
- TRANSPORT code calculates 2nd order by including Tmno elements explicitly
- TRANSPORT formalism is not suitable to calculate higher order (>2).

lagnification Mx Focusing fot					-	Lateral Dispersion				
	x(t)		R	R12	0	0	0	R_16	x	1
	$\theta(t)$		R ₂₁	R ₂₂	0	ο	0	R ₂₆	θο	Angular Disp
	y(t)	=	0	0	^R 33	^R 34	0	0	у	
	arphi(t)		· 0	0	^R 43	R ₄₄	0	0	φ_{o}	
	(t)		R ₅₁	R ₅₂	0	0	٦°	^R 56	l _o	
	ð (t)		0	0	0	0	0	1	80	
X2 = TRANSPORT. R-Hatrix · Xi										
B for Beam Line										
s for Spechampter										

$$d(m\dot{x})/dt = Q(E_x + v_y B_z - v_z B_y)$$

$$d(m\dot{y})/dt = Q(E_y + v_z B_x - v_x B_z)$$

$$d(m\dot{z})/dt = Q(E_z + v_x B_y - v_y B_x)$$



Solving the equations of Motion

Methods of solving the equation of motion:

1) Code RAYTRACE slices the system in small sections along the z-axis and integrates numerically the particle ray through the system.

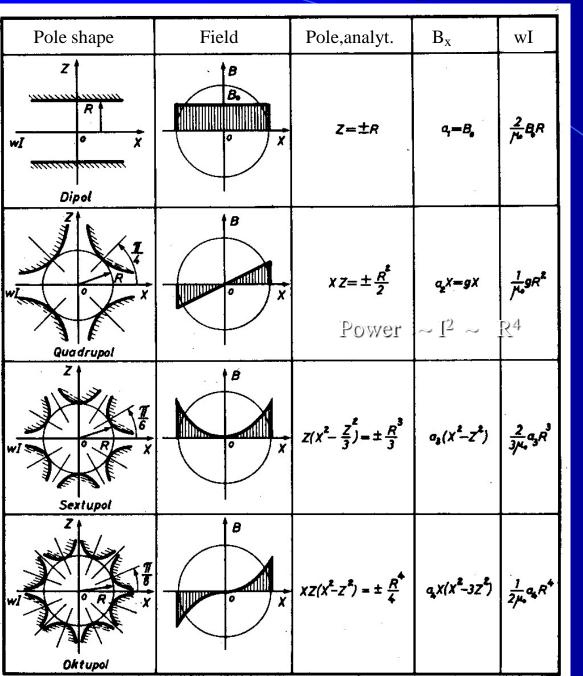
2) Determine the TRANSPORT matrix.

3) Code COSY Infinity uses Differential Algebraic techniques to arbitrary orders using matrix representation for fast calculations (See Appendix)

Ion-optics and real magnet systems

- Obviously, an ion-optical is meant to describe quantivatively a real magnet systems.
- A well-designed magnet system should have a realistic ion-optical model that allows you to plan an experiment and analyse your data. Nevertheless, there will be many limitations that need to be considered requiring e.g. calibrations.
- Also, parameters of the system may change, e.g. alignment of elements, beam parameters and alignment.
- Designing a system for a particular purpose (or many purposes) is much more that an ion-optical task. You need to include in your ion-optical concept and calculations many parameters, often before they are fully know, e.g. physics requirements, floor plan, magnet sizes and limitations, field shapes. Design of magnet system is an iterative process.
- For the magnet systems designer one important task is to extract from a ion-optical design the Specifications for the manufacturer. Are they feasible?
- Detectors and diagnostics systems need to be considered and included in the design of a magnet system.

Schematic Overview of Magnetic Elements (Iron dominated)



Iron dominated:

B field is determined by properties & shape of iron pole pieces

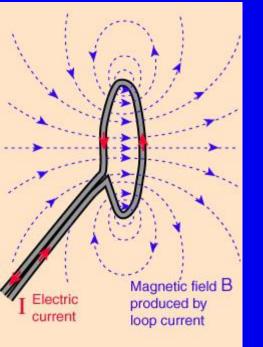
Required wI = Ampere-turns for desired magnet strength B_0 , g, a_3 , a_4 can be calculated formula in last column.

Coils are not shown in drawing in 1st columm

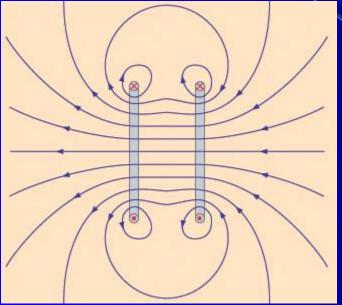
G. Schnell, Magnete, Verlag K. Thiemig, Muenchen 1973

Creation of magnetic fields using current

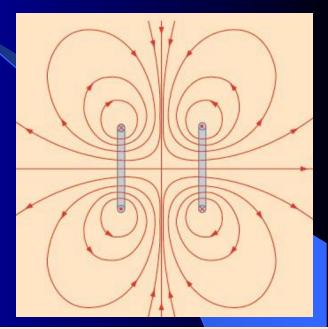
Current loop



Helmholtz coil, Dipole



Helmholtz coil, reversed current, Quadrupole

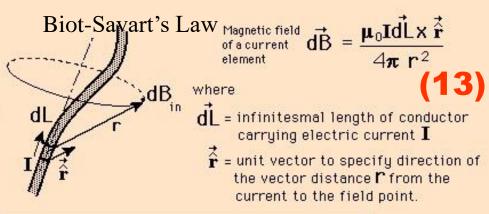


Magnetization in Ferromagnetic material:

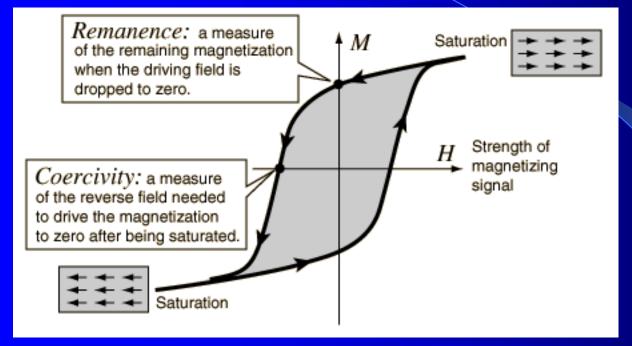
μH

В

B = magn. Induction H = magn. Field μ = magn. permeability

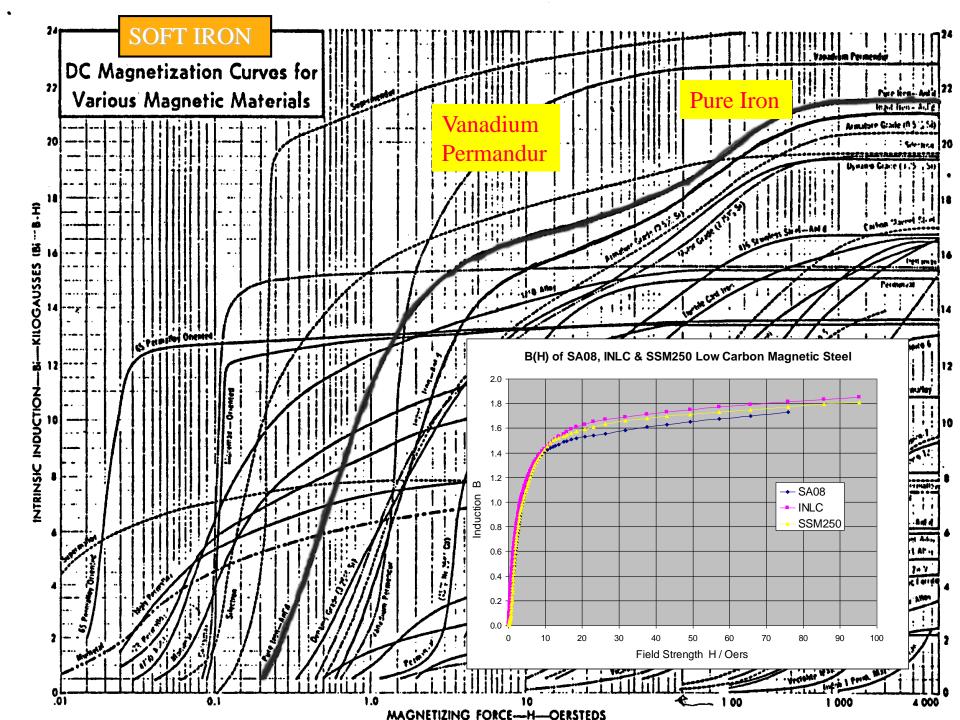


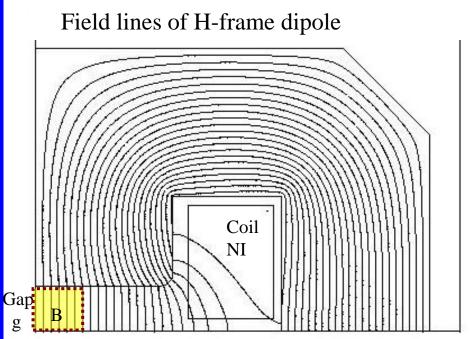
Creation of magnetic fields using permanent magnets



Magnet iron is **soft**: Remanence is very small when H is returned to 0 Permanent magnet material is **hard**: Large remaining magnetization B

Permanent magnets can be used to design dipole, quadrupole and other ionoptical elements. They need no current, but strength has to be changed by mechanical adjustment.





↑ Magnetic field B in Good-field region

defined by ion-optical requirement, e.g. dB/B < 10⁻⁴ For symmetry reasons only a quarter of the full dipole is calculated & shown

The Field calculation was performed Using the finite element (FE) code MagNet (Infolytica). Other programs are OPERA or POISSON.

Dipole magnet

From Ampere's law:

Design of an iron-dominated

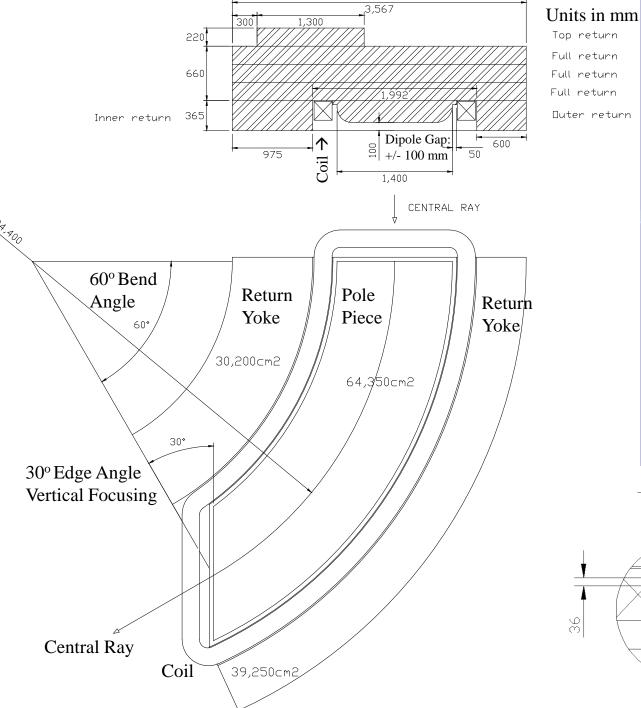
NI (Ampere turns) = ----

 $4\pi * 10^{-7}$ (m/A)

B (T) * g (m)

Units: m = meter T = Tesla A = Ampere (14

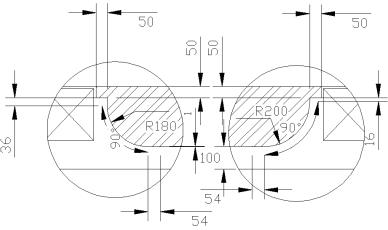
Note: FE codes solve the static of time varying Maxwells Equations numerically by "meshing" the geometry in triangles in 2d or "bricks" in 3d. This allows to precisely calculate the Fields (B,E) for any configuration of current and materials, like ferromagnetic metals



SHARAQ Dipole D2

Iron-dominated, normal conducting Dipole Magnet with constant field in Dipole Gap (Good-field region)

- Soft magnet iron, B(H)
- Hollow copper conductor for high current density < 10 A/mm²
- Iron magnetization saturates at about 1.7 T
- For B > 2 T superconducting (current dominated or hybrid) magnets are used.



OPT-9: pole INLC, return SA08

80000 90000

1.57730

1.57700

1.57670

1.57640

1.57610

.57580

.57550

1.57520

1.57490

80000

00006

70000

70000

Saturation: -3.0% (Deviation from B(I) linearity)

80cm: 2.5*E-4

40000 50000

WWW

-.10000 .00000 .10000

4.57730

1.57700

1.57670

1.57640

1.57610

1.57580

1.57550

1.57520

1.57490

0000

00000

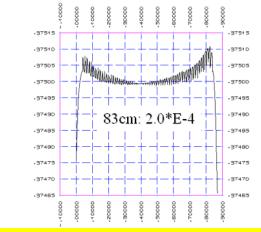
3.96m

10000 20000

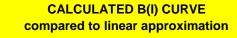
These figures show the Good-field region in the center of the magnet and the maximum field B as function of the current I in the conductor

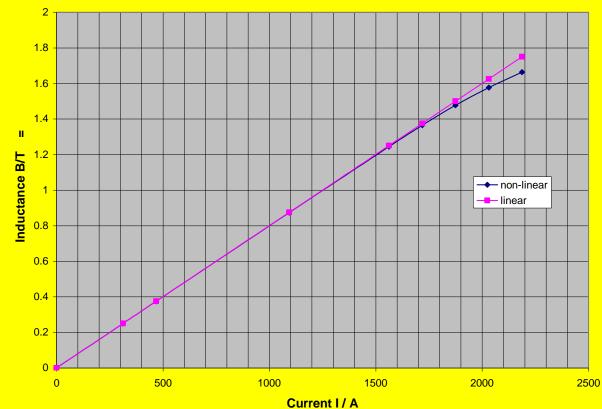
40000 50000

30000

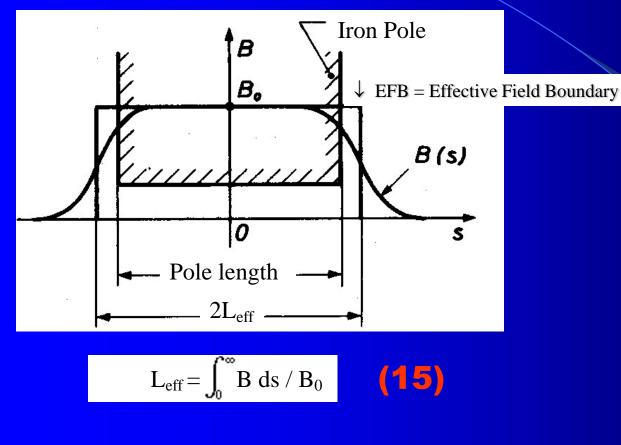


SHARAQ Dipole D2 Field calculation





Fringe field & Effective field length L_{eff}

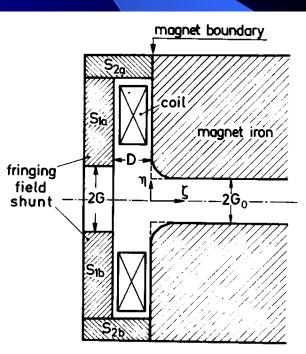


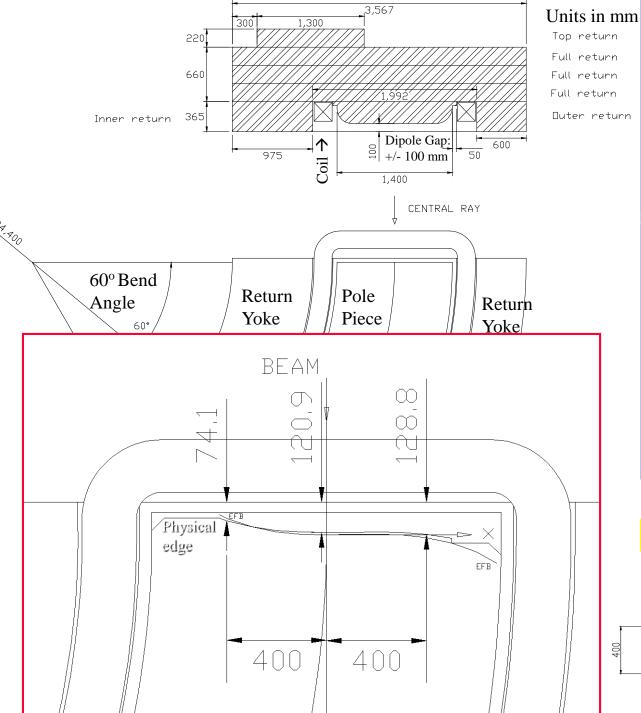
Note:

1) The fringe field is important even in 1st order ion-optical calculations.

2) Rogowski profile to make L_{eff} = Pole length.

3) The fringe field region can be modified with field clamp or shunt.

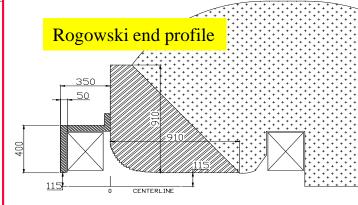




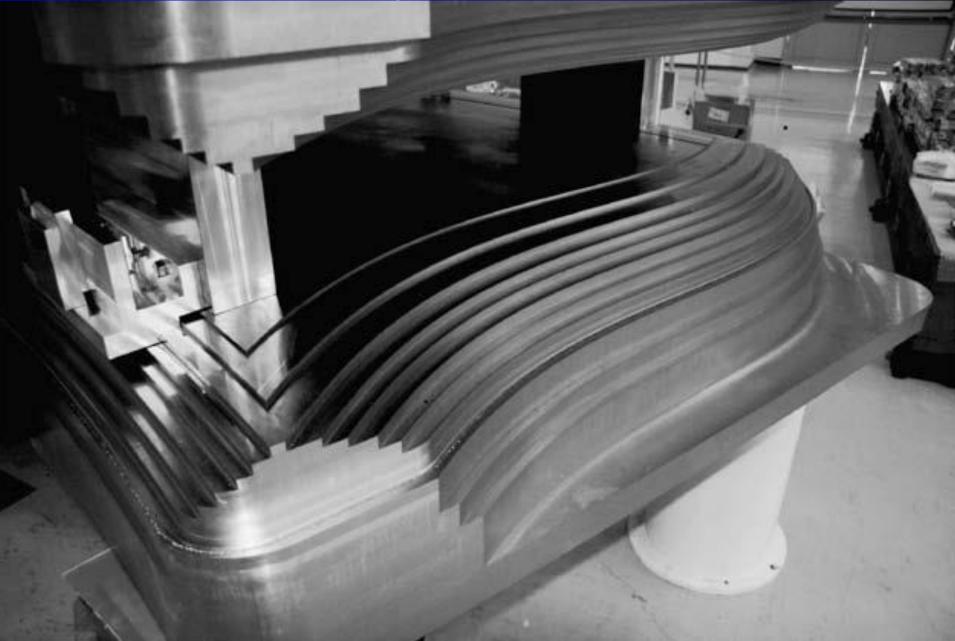
SHARAQ Dipole D2

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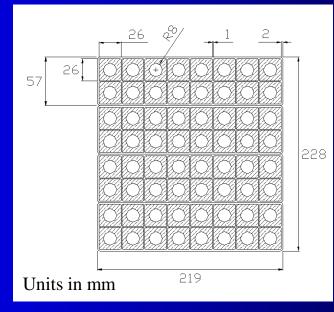
• 3rd order polynomial shaping of the SHARAQ dipole D2 for higher order corrections



SHARAQ D2 Dipole magnet

Max. Rigidity Bending Radius: Total gap Pole width Bend angle Central ray length Iron weight Weight of 2 coils Bmax	1400 60 4607.7 255 7.4 1.55	mm mm deg mm tons tons T
DC/Power DC/Current, no saturation DC, including saturation Max. allowed current	2050 2130	A A A
DC/Voltage Conductor Pancakes/coil	150 26x26/16m 4	
Turns/pancake Total turns/magnet Pressure drop	16 128 5	atm
Temp. rise Water flow, magnet Current density Resistance/magnet	150 4.3 73.2	deg C I/min A/mm2 mOhm
Inductance L/R Ramp time Dynam. Power Excess Voltage to ramp Stored Energy	618 8.6 15 480 85 1320	s s kW V

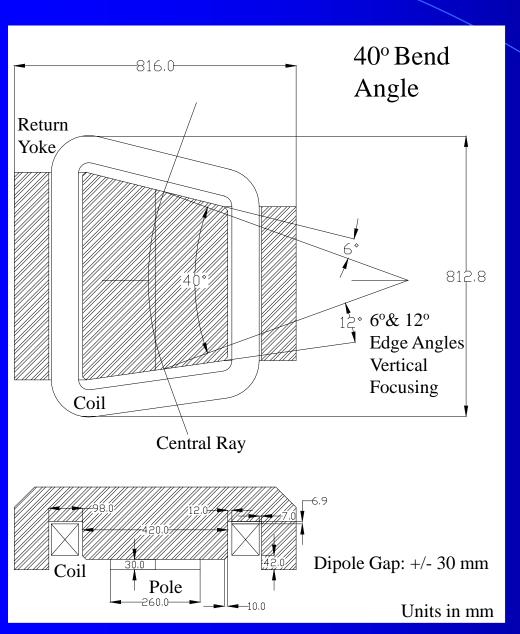
SHARAQ Dipole D2 specifications and Coil design



This figure shows the coil with hollow cupper conductor for water cooling (4.3 A/mm²)

Thumb rule: Water cooling has to dissipate the complete magnet power IR²

Dipole H-Magnet for St. George a new recoil separator at Notre Dame University for astrophysics



Iron-dominated Dipole Magnet with constant field $B_{max} = 0.6 \text{ T}$ in dipole gap (Good-field region).

- Soft magnet iron, B(H)
- Hollow copper conductor for high current density
- Iron magnetization saturates at about 1.7 T, small returns

• St. George Dipole magnet B1

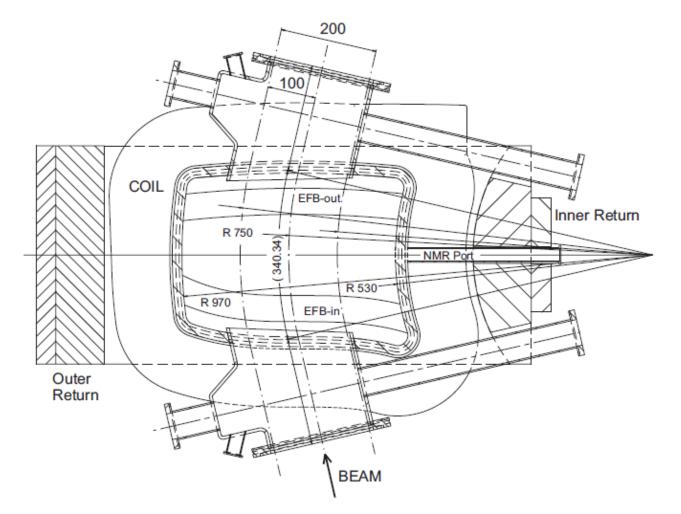
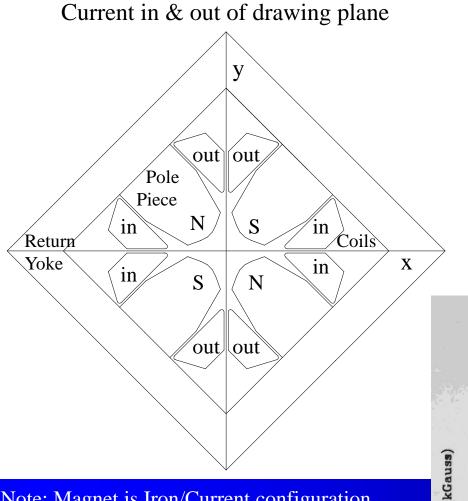
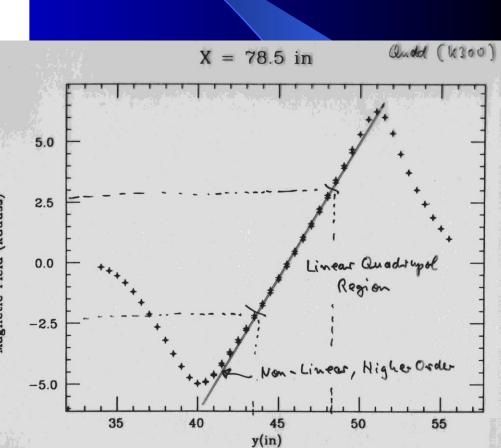


Fig. 4. View from above the midplane of dipole magnet B1 and the vacuum chamber. The physical shape of the pole piece at the entrance (EFB-in) and the exit (EFB-out) is designed to create the higher-order corrections. All dimensions are in millimeter.



Standard Quadrupole

Note: Magnet is Iron/Current configuration with field as needed in ion-optical design. 2d/3d finite elements codes solving POISSON equation are well established Magnetic Field (kGauss)



Forces on ions (quadrupole)

Quadrupole

Hexapole

Octopole

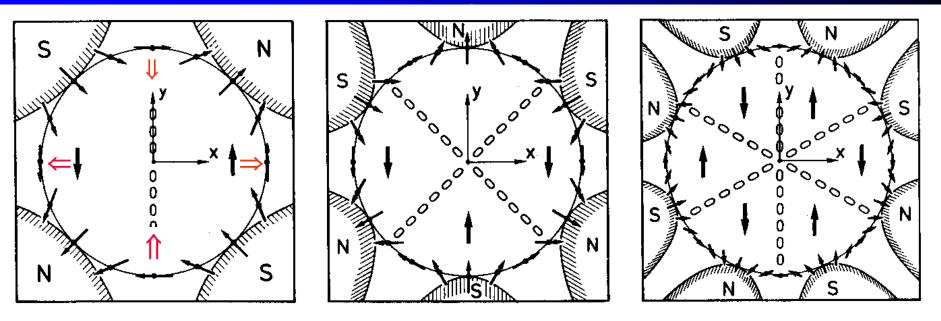


Fig. 9.15. Pole arrangements of magnetic quadrupoles, hexapoles, and octopoles are indicated. Also shown is a circle of radius r_0 along which the magnetic flux density is constant, and its direction varies as indicated. Finally, strings of zeros indicate lines along which B_y the y component of the magnetic flux density vanishes. These lines separate regions in which B_y is parallel or antiparallel to the y axis.

Horizontally defocusing quadrupole for ions along – z axis into the drawing plane. See Forces $\uparrow \leftarrow \downarrow \Rightarrow$ in direction v x B

A focusing quadrupole is obtained by a 90° rotation around the z axis

End Lecture 1

Appendix

32. COSY INFINITY and Its Applications in Nonlinear Dynamics^{*}

(Chapter of "Computational Differentiation: Techniques, Applications, and Tools", Martin Berz, Christian Bischof, George Corliss, and Andreas Griewank, eds., SIAM, 1996.)

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The code COSY INFINITY originated in the field of beam physics, where it is used as a major design and analysis tool by more than 150^{*}registered users. In this field, high-order nonlinearities correspond directly to relevant quantities called image aberrations, which traditionally have been very hard to compute and describe.

Many codes compute aberrations to second and third order [Wollnik1988a], [Brown1979a], [Dragt1985a]. A comprehensive description of the theory behind the perturbative approach to compute aberrations can be found in [Brown1982a]. With advanced and dedicated formula manipulators, it was possible to extend the work to fifth order [Berz1987c], [Berz1988a], but it seems hard to imagine that the traditional methods can be extended substantially. In contrast, the forward differentiation techniques and the differential algebraic extensions used in the code COSY routinely allow the computation of aberrations of orders around ten.

COSY also allows the computation of aberrations of systems that are otherwise particularly hard to treat, especially those occurring due to the fringe fields of particle optical elements [Hoffstätter1993a], [Hoffstätter1994b].

* In 2006 (2018) COSY Infinity had more than 1000 (>2500) registered users