# Neutron-proton pairing in nuclear density functional theory

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# Plan

- Introduction to neutron-proton pairing
- Neutron-proton pairing in nuclear density functional theory
    - isobaric analogue states and isovector pairing
    - ground state neutron-proton pair condensation in N=Z nuclei
- Neutron-proton pairing beyond nuclear DFT
    - double-beta decay nuclear matrix element and isoscalar pairing
- Summary

# Collective correlations

two levels of many-body correlation (in the terminology of mean field theory):
residual correlation and condensation

residual correlation: no effect in the mean field level
(beyond-mean-field effect)

condensation: structural change in the mean field

two main collective correlations in nuclei: pairing and multipole force

| force | residual correlation | condensation |
|---|---|---|
| pairing force | pairing vibration of normal system | superconductivity |
| quadrupole force | quadrupole surface oscillation of spherical nucleus | quadrupole deformation |

# Nucleonic pairing

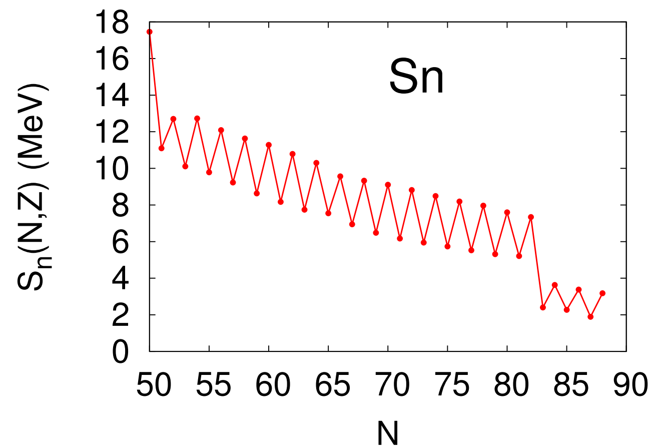
**pairing interaction**

two nucleons in a time-reversed pair of orbits get additional binding

**pairing relevant phenomena (condensation)**

- one-neutron/proton separation energy

$$S_n(N,Z) = B(N,Z) - B(N-1,Z)$$
$$S_p(N,Z) = B(N,Z) - B(N,Z-1)$$



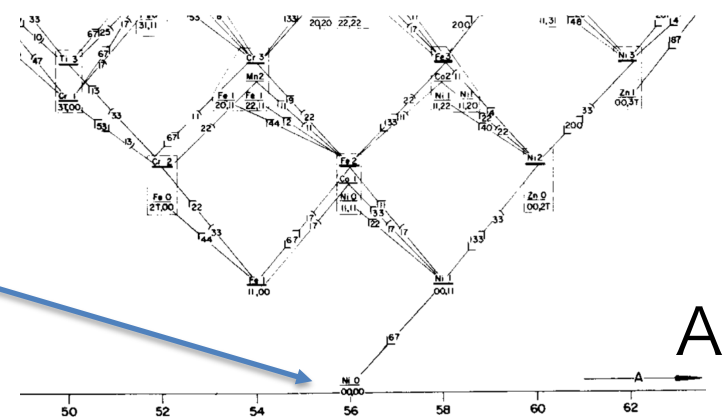

- rotational moment of inertia
- two-nucleon transfer cross section
- level densities (even and odd systems)
- two-nucleon separation energy: NH and Nazarewicz (2016)

like-particle pairing (nn and pp) condensation exists everywhere in open-shell nuclei

**pairing relevant phenomena (residual correlation)**

- pairing vibration (closed shell)

$^{56}$Ni




Bes et al., Phys. Rep. **34**, 1 (1977)

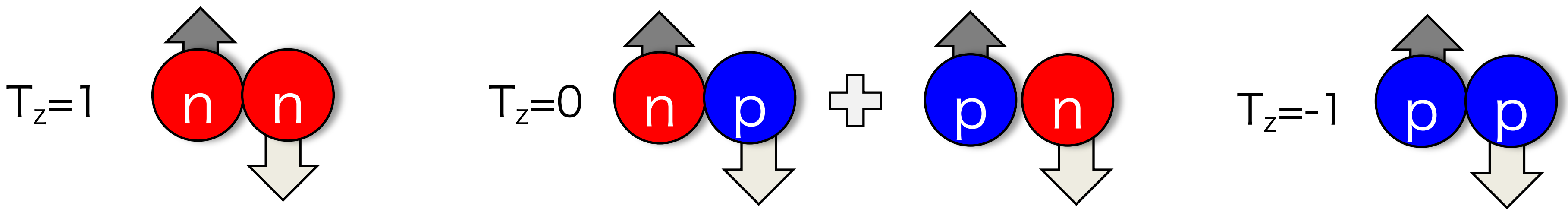
# Neutron-proton pairing

Nuclear force has (approximate) isospin symmetry
nucleon: isospin t=1/2 (neutron $t_z$=1/2, proton $t_z$=-1/2)

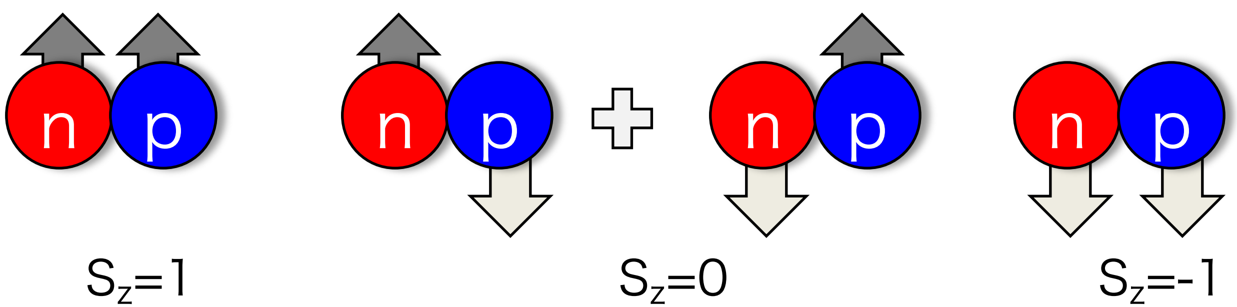
many experimental evidence for nn ($T_z$=1) and pp($T_z$=-1) pairing condensation

**Nucleon pairing** for L=0 (space-symmetric) pair

Isovector (T=1, S=0) pairings: isospin symmetric, spin antisymmetric




Isoscalar (T=0, S=1) pairings: isospin antisymmetric, spin symmetric

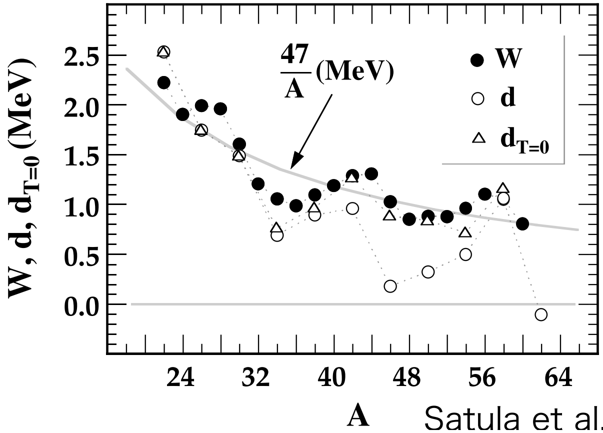



reviews: Afanasjev, Fifty years of Nuclear BCS, p.138, arXiv:1205.2134
Frauendorf and Macchiavelli, Prog. Part. Nucl. Phys. **78**, 24 (2014)

# Neutron-proton pairing

## neutron-proton pairing condensation?

possible in proton-rich region (around N=Z)-- relevant to rp process
Wigner energy




additional binding for even-even system

$$\Delta B = -E_W = -W(A)|N - Z|$$

Satula et al., Phys. Lett. B **407**, 103 (1997)

isovector or isoscalar pair condensation?


red: isoscalar condensate
blue: isovector np and isoscalar np mixture
green: like-particle pairing condensate

Woods-Saxon + np pairing int. from shell model Hamiltonian

Gezerlis, Bertsch, Luo, Phys. Rev. Lett. **106**, 252502 (2011)

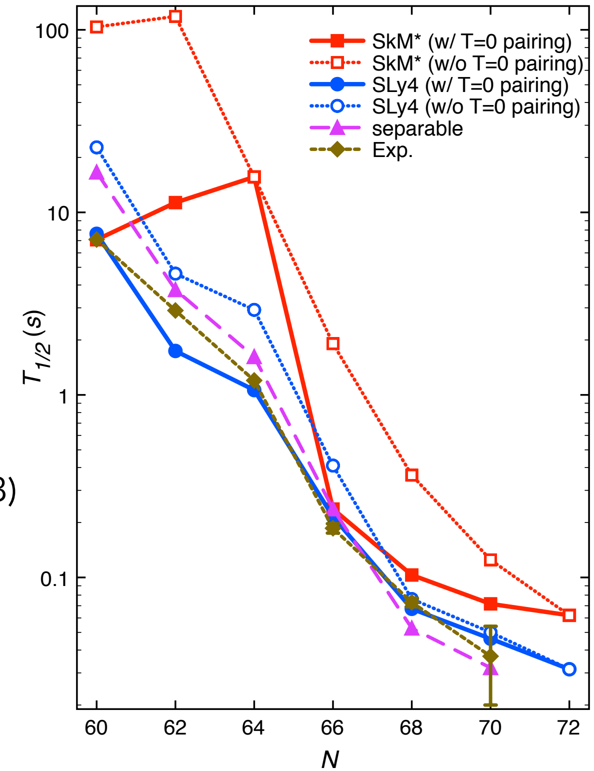
# Neutron-proton pairing

**neutron-proton pairing as residual correlation (isoscalar pairing)**

- beta-decay rate (relevant to r-process)
- double-beta decay

npQRPA calculation for beta decay

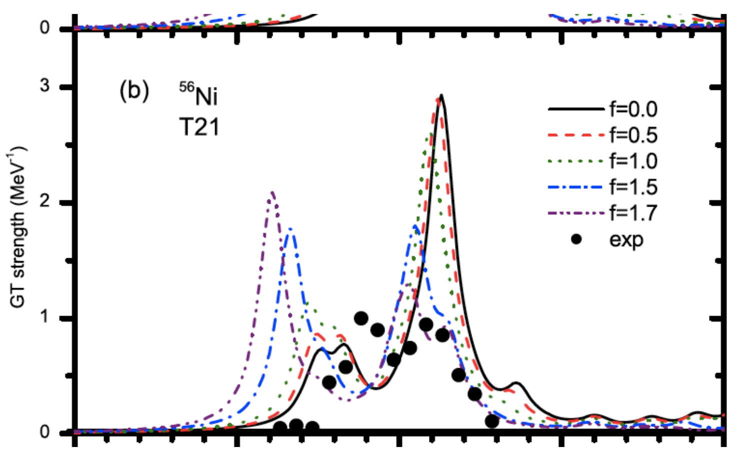
Yoshida, Prog. Theor. Exp. Phys. **2013**, 113D02 (2013)




- low-energy Gamow-Teller strength

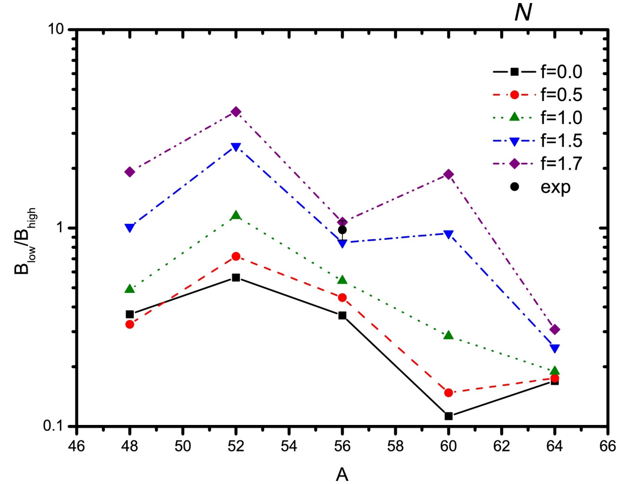
npQRPA calculation

f: relative strength of isoscalar np pairing




10 MeV

30 MeV




Bai et al. Phys. Lett. B **719**, 116 (2013)

# DFT for np pairing condensation

nuclear DFT (and beyond): microscopic and global description of finite nuclei
Most DFT calculations include np pairing only as residual correlation (npQRPA)
Most calculations for np pair condensation use schematic interactions

**DFTs with np pairing condensation**

Terasaki, Wyss, and Heenen (Skyrme, Phys. Lett. B **437**, 1(1998))
Afanasjev and Frauendorf (RHB, Phys. Rev. C **71**, 064318 (2005))

## Goal

develop a neutron-proton DFT code based on HFBTHO (Skyrme) and give a unified description for np pairing correlation and condensation

**theoretical formalism:**

"Local density approximation for proton-neutron pairing correlations: Formalism"
Perlinska, Rohozinski, Dobaczewski, Nazarewicz Phys. Rev. C **69**,014316 (2004)
(isospin-invariant EDF)

**particle-hole implementation:**

"Isospin-invariant Skyrme energy-density-functional approach with axial symmetry"
Sheikh, NH, Dobaczewski, Nakatsukasa, Nazarewicz, Sato, Phys. Rev. C **89**, 054317 (2014)

# Isospin representation of ph density

Perlinska et al., Phys. Rev. C **69**,014316 (2004)

density matrix $\hat{\rho}(\boldsymbol{r}st, \boldsymbol{r}'s't') = \langle\Psi|a^{+}_{\boldsymbol{r}'s't'} a_{\boldsymbol{r}st}|\Psi\rangle$

non-local density $\rho_k(\boldsymbol{r}, \boldsymbol{r}') = \sum_{stt'} \hat{\rho}(\boldsymbol{r}st, \boldsymbol{r}'st')\hat{\tau}^{k}_{t't}$ $\hat{\tau}^{0}_{t't} = \delta_{t't}$

k=1-3 Pauli matrix

local density $\rho_k(\boldsymbol{r}) = \rho_k(\boldsymbol{r}, \boldsymbol{r}')|_{\boldsymbol{r}=\boldsymbol{r}'}$

isoscalar $\rho_0(\boldsymbol{r}) = \rho_n(\boldsymbol{r}) + \rho_p(\boldsymbol{r})$ $\rho_n(\boldsymbol{r}) = \langle\Psi|a^{+}_{n}(\boldsymbol{r})a_n(\boldsymbol{r})|\Psi\rangle$

(conventional) isovector $\rho_3(\boldsymbol{r}) = \rho_n(\boldsymbol{r}) - \rho_p(\boldsymbol{r})$ $\rho_p(\boldsymbol{r}) = \langle\Psi|a^{+}_{p}(\boldsymbol{r})a_p(\boldsymbol{r})|\Psi\rangle$

new densities present only when neutron-proton symmetry is broken

$\rho_1(\boldsymbol{r}) = \rho_{np}(\boldsymbol{r}) + \rho_{pn}(\boldsymbol{r})$ $\rho_{pn}(\boldsymbol{r}) = \langle\Psi|a^{+}_{n}(\boldsymbol{r})a_p(\boldsymbol{r})|\Psi\rangle$

$\rho_2(\boldsymbol{r}) = -i\{\rho_{np}(\boldsymbol{r}) - \rho_{pn}(\boldsymbol{r})\}$ $\rho_{np}(\boldsymbol{r}) = \langle\Psi|a^{+}_{p}(\boldsymbol{r})a_n(\boldsymbol{r})|\Psi\rangle$

(time reversal symmetry cancels $\rho_2$)

Isospin representation allows us to have n-p mixed density

# Isospin-invariant EDF (Skyrme)

isoscalar functional

Perlinska et al., Phys. Rev. C **69**,014316 (2004)

$$\chi_0(\boldsymbol{r}) = C_0^\rho \rho_0^2(\boldsymbol{r}) \qquad \rho_0(\boldsymbol{r}) = \rho_n(\boldsymbol{r}) + \rho_p(\boldsymbol{r})$$

isovector functional (conventional)

$$\chi_1(\boldsymbol{r}) = C_1^\rho \rho_3^2(\boldsymbol{r})$$

$$\rho_3(\boldsymbol{r}) = \rho_n(\boldsymbol{r}) - \rho_p(\boldsymbol{r})$$

isovector functional (isospin-rotation invariant)

$$\chi_1(\boldsymbol{r}) = C_1^\rho \{\boxed{\rho_1^2(\boldsymbol{r}) + \rho_2^2(\boldsymbol{r})} + \rho_3^2(\boldsymbol{r})\}$$

$$= C_1^\rho \vec{\rho}(\boldsymbol{r}) \circ \vec{\rho}(\boldsymbol{r})$$ isoscalar in total

k=1,2 neutron-proton mixing terms

Isospin-invariant Skyrme functional (density bilinear, $C_t^\rho$ is density dependent)

$$\chi_0(\boldsymbol{r}) = C_0^\rho \rho_0^2 + C_0^{\Delta\rho} \rho_0 \Delta\rho_0 + C_0^\tau \rho_0 \tau_0 + C_0^{J0} J_0^2 + C_0^{J1} \boldsymbol{J}_0^2 + C_0^{J2} \underline{\mathsf{J}}_0^2 + C_0^{\nabla J} \rho_0 \nabla \cdot \boldsymbol{J}_0$$
$$+ C_0^s \boldsymbol{s}_0^2 + C_0^{\Delta s} \boldsymbol{s}_0 \cdot \Delta \boldsymbol{s}_0 + C_0^T \boldsymbol{s}_0 \cdot \boldsymbol{T}_0 + C_0^j \boldsymbol{j}_0^2 + C_0^{\nabla j} \boldsymbol{s}_0 \cdot (\nabla \times \boldsymbol{j}_0) + C_0^{\nabla s} (\nabla \cdot \boldsymbol{s}_0)^2 + C_0^F \boldsymbol{s}_0 \cdot \boldsymbol{F}_0,$$

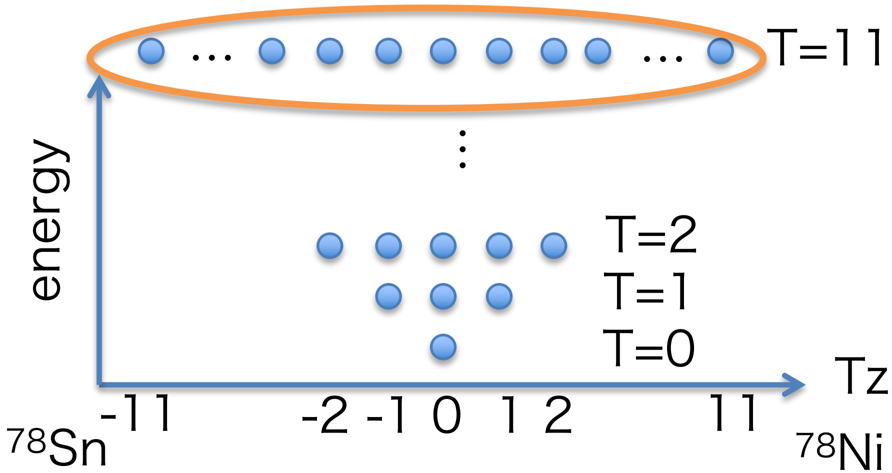
$$\chi_1(\boldsymbol{r}) = C_1^\rho \vec{\rho}^{\,2} + C_1^{\Delta\rho} \vec{\rho} \circ \Delta\vec{\rho} + C_1^\tau \vec{\rho} \circ \vec{\tau} + C_1^{J0} \vec{J}^{\,2} + C_1^{J1} \vec{\boldsymbol{J}}^{\,2} + C_1^{J2} \vec{\underline{\mathsf{J}}}^{\,2} + C_1^{\nabla J} \vec{\rho} \circ \nabla \cdot \vec{\boldsymbol{J}}$$
$$+ C_1^s \vec{\boldsymbol{s}}^{\,2} + C_1^{\Delta s} \vec{\boldsymbol{s}} \cdot \circ \Delta \vec{\boldsymbol{s}} + C_1^T \vec{\boldsymbol{s}} \cdot \circ \vec{\boldsymbol{T}} + C_1^j \vec{\boldsymbol{j}}^{\,2} + C_1^{\nabla j} \vec{\boldsymbol{s}} \cdot \circ (\nabla \times \vec{\boldsymbol{j}}) + C_1^{\nabla s} (\nabla \cdot \vec{\boldsymbol{s}})^2 + C_1^F \vec{\boldsymbol{s}} \cdot \circ \vec{\boldsymbol{F}},$$

- Isovector part is extended to include np-mixing
- No new coupling constants in ph channel
- Coulomb functional breaks isospin invariance (not shown here)

# Implementation to axial DFT solver HFBTHO

Sheikh et al. Phys. Rev. C **89**, 054317 (2014)

**Isobaric analogue states (IASs)**




ground state with N neutrons and Z protons has isospin $T=T_z=(N-Z)/2$

$|^{78}\mathrm{Ni(g.s)}\rangle = |T=11, Tz=11\rangle$

isobaric analogue states: states with $T \neq \pm Tz$

$|T=11, T_z=10\rangle = T_-|^{78}\mathrm{Ni(g.s.)}\rangle$ isospin lowering operator $T_- = \Sigma\ p_i^+ n_i$

IASs of $^{78}$Ni (N=50, Z=28, T=11) found in excited states in
$^{78}$Cu (Tz=10), $^{78}$Zn(Tz=9), $^{78}$Ga(Tz=8), $^{78}$Ge(Tz=7), ...

# Implementation to axial DFT solver HFBTHO

Sheikh et al. Phys. Rev. C **89**, 054317 (2014)

**Isobaric analogue states (IASs)**

IASs are generated by applying T- to |T Tz=T>

superposition of mean fields

**mean-field description of IAS:**

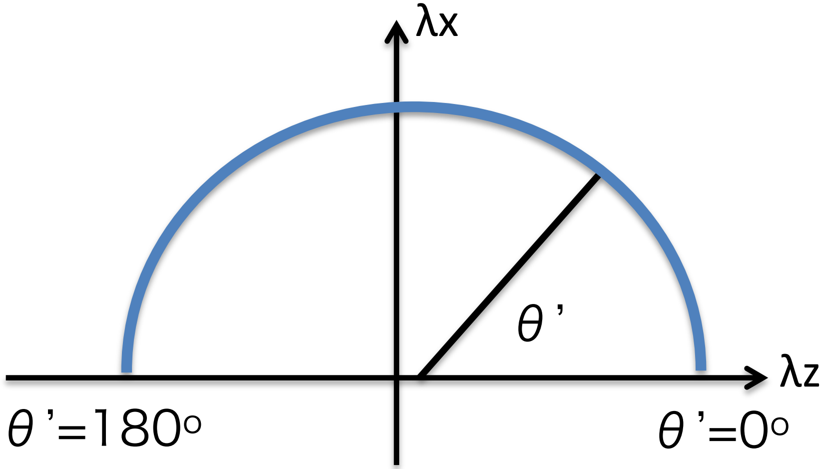
|T Tz> is the isospin T state with different orientation of the third component (orientation in isospace)

isospin rotation → neutron-proton mixing in the single-particle states

isocranking $$\hat{h}' = \hat{h} - \vec{\lambda} \circ \hat{\vec{t}},$$

isocranking term $-\vec{\lambda} \circ \hat{\vec{t}} = -\lambda_x \hat{t}_x - \lambda_z \hat{t}_z$

tx, tz: isospin operators
λx = 0 : normal HF calculation




isorotation about y axis

$$\begin{pmatrix} \rho_0(\theta') \\ \rho_1(\theta') \\ \rho_2(\theta') \\ \rho_3(\theta') \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta' & 0 & \sin\theta' \\ 0 & 0 & 1 & 0 \\ 0 & -\sin\theta' & 0 & \cos\theta' \end{pmatrix} \begin{pmatrix} \rho_0(0^\circ) \\ \rho_1(0^\circ) \\ \rho_2(0^\circ) \\ \rho_3(0^\circ) \end{pmatrix}$$

$$\rho_0(\boldsymbol{r}) = \rho_{\rm n}(\boldsymbol{r}) + \rho_{\rm p}(\boldsymbol{r})$$
$$\rho_1(\boldsymbol{r}) = \rho_{\rm np}(\boldsymbol{r}) + \rho_{\rm pn}(\boldsymbol{r})$$
$$\rho_3(\boldsymbol{r}) = \rho_{\rm n}(\boldsymbol{r}) - \rho_{\rm p}(\boldsymbol{r})$$

# A=78 isobaric analog states

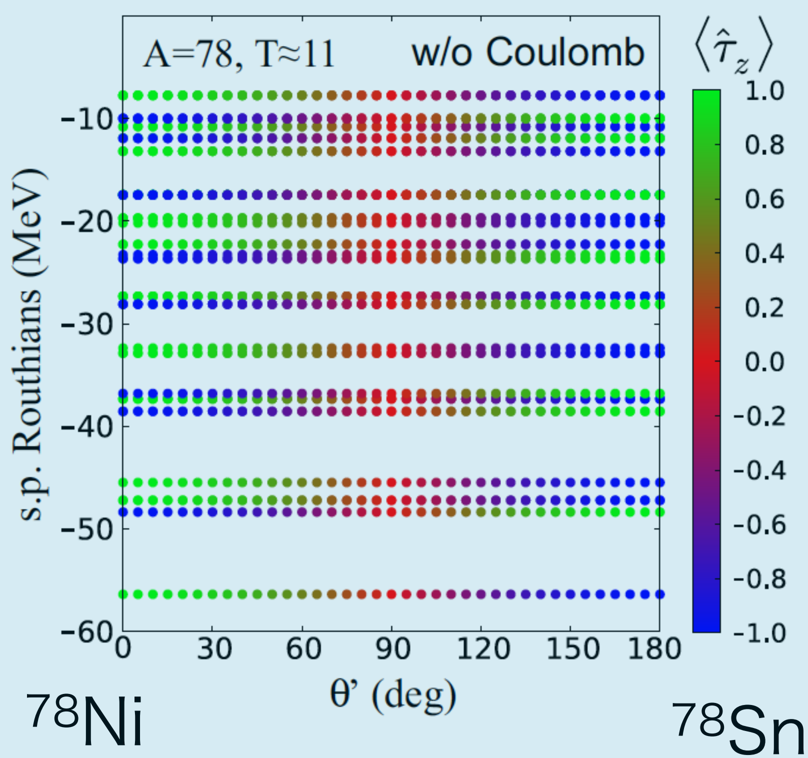
Sheikh et al. Phys. Rev. C **89**, 054317 (2014)

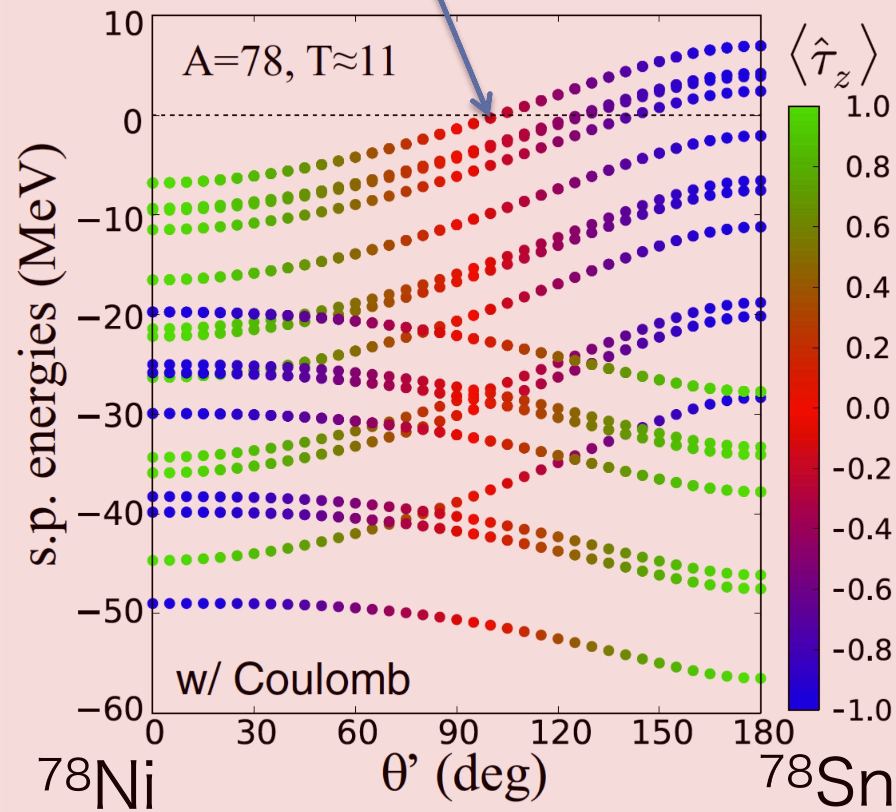
A=78 T=11 states, from $^{78}$Ni to $^{78}$Sn

without Coulomb → EDF is isospin invariant; $^{78}$Sn bound

with Coulomb → proton drip line (for T=11 IAS) at $^{78}$Zr ($T_z$=-1)

single-particle energies




mean-field description of IASs with neutron-proton mixing

neutron-proton pairing in IASs?

# pp densities in isospin representation

Perlinska et al., Phys. Rev. C **69**,014316 (2004)

pair density matrix $\hat{\kappa}(\boldsymbol{r}st,\boldsymbol{r}'s't') = \langle\Psi|a_{\boldsymbol{r}'s't'}a_{\boldsymbol{r}st}|\Psi\rangle$

$$\hat{\tilde{\rho}}(\boldsymbol{r}st,\boldsymbol{r}'s't') = -2s'\langle\Psi|a_{\boldsymbol{r}'-s't'}a_{\boldsymbol{r}st}|\Psi\rangle \quad \text{(time reversal)}$$

$$\hat{\breve{\rho}}(\boldsymbol{r}st,\boldsymbol{r}'s't') = 4s't'\langle\Psi|a_{\boldsymbol{r}'-s'-t'}a_{\boldsymbol{r}st}|\Psi\rangle \quad \text{(isospin symm.)}$$

non-local pp density $\breve{\rho}_k(\boldsymbol{r},\boldsymbol{r}') = \sum_{stt'}\hat{\breve{\rho}}(\boldsymbol{r}st,\boldsymbol{r}'st')\hat{\tau}^k_{t't}$

local pp density $\breve{\rho}_k(\boldsymbol{r}) = \breve{\rho}_k(\boldsymbol{r},\boldsymbol{r}')|_{\boldsymbol{r}=\boldsymbol{r}'}$

$$\breve{\rho}_{\mathrm{n}}(\boldsymbol{r}) = \langle\Psi|a_{\mathrm{n}}(\boldsymbol{r})a_{\mathrm{n}}(\boldsymbol{r})|\Psi\rangle \qquad \breve{\rho}_1(\boldsymbol{r}) = \breve{\rho}_{\mathrm{n}}(\boldsymbol{r}) + \breve{\rho}_{\mathrm{p}}(\boldsymbol{r})$$

$$\breve{\rho}_{\mathrm{p}}(\boldsymbol{r}) = \langle\Psi|a_{\mathrm{p}}(\boldsymbol{r})a_{\mathrm{p}}(\boldsymbol{r})|\Psi\rangle \qquad \breve{\rho}_2(\boldsymbol{r}) = i[\breve{\rho}_{\mathrm{n}}(\boldsymbol{r}) - \breve{\rho}_{\mathrm{p}}(\boldsymbol{r})]$$

isovector np pairing

$$\breve{\rho}_{np}(\boldsymbol{r}) = \langle\Psi|a_{\mathrm{n}}(\boldsymbol{r})a_{\mathrm{p}}(\boldsymbol{r}) - a_{\mathrm{p}}(\boldsymbol{r})a_{\mathrm{n}}(\boldsymbol{r})|\Psi\rangle \qquad \breve{\rho}_3(\boldsymbol{r}) = 2\breve{\rho}_{\mathrm{np}}(\boldsymbol{r})$$

$$\breve{\rho}_0(\boldsymbol{r}) = 0 \quad \text{(Pauli principle)}$$

non-local spin pp density $\breve{\boldsymbol{s}}_k(\boldsymbol{r},\boldsymbol{r}') = \sum_{ss'tt'}\hat{\breve{\rho}}(\boldsymbol{r}st,\boldsymbol{r}'s't')\boldsymbol{\sigma}_{s's}\hat{\tau}^k_{t't}$

local pp spin density
(isoscalar pairing) $\breve{\boldsymbol{s}}_0(\boldsymbol{r}) = \breve{\boldsymbol{s}}_0(\boldsymbol{r},\boldsymbol{r}')|_{\boldsymbol{r}=\boldsymbol{r}'} = \breve{\boldsymbol{s}}_{\mathrm{np}}(\boldsymbol{r})$ k=1-3 zero due to Pauli principle

# Isospin-invariant pairing EDF

Perlinska et al., Phys. Rev. C **69**,014316 (2004)

isovector pairing functional
(conventional)

$$\breve{\chi}_{\rm n}(\boldsymbol{r}) = \breve{C}^{\rho}_{\rm n}[\rho_0]|\breve{\rho}_{\rm n}(\boldsymbol{r})|^2$$

$$\breve{\chi}_{\rm p}(\boldsymbol{r}) = \breve{C}^{\rho}_{\rm p}[\rho_0]|\breve{\rho}_{\rm p}(\boldsymbol{r})|^2$$

isovector pairing functional
(isospin-rotation invariant)

$$\breve{\chi}_1(\boldsymbol{r}) = \breve{C}^{\rho}_1[\rho_0]\{|\breve{\rho}_1(\boldsymbol{r})|^2 + |\breve{\rho}_2(\boldsymbol{r})|^2 + \boxed{|\breve{\rho}_3(\boldsymbol{r})|^2}\}$$
$$= \breve{C}^{\rho}_1[\rho_0]\vec{\breve{\rho}}^*(\boldsymbol{r}) \circ \vec{\breve{\rho}}(\boldsymbol{r})$$

k= 3 neutron-proton pairing terms

isoscalar pairing functional (np)

$$\breve{\chi}_0(\boldsymbol{r}) = \breve{C}^{s}_0[\rho_0]|\breve{\boldsymbol{s}}_0(\boldsymbol{r})|^2$$

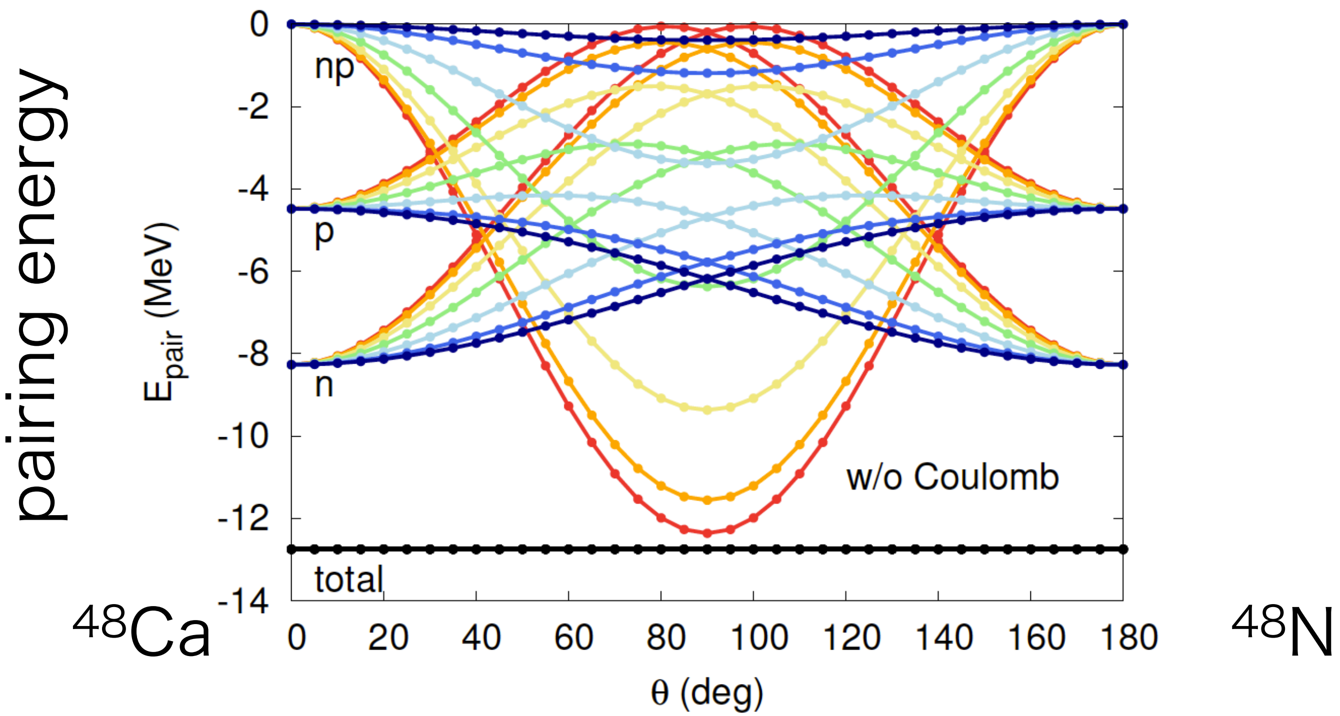
# A=48 isobaric analog states (w/o Coulomb)

A=48 T=4 states, from $^{48}$Ca to $^{48}$Ni
w/o Coulomb: with artificially strong pairing ($\Delta n$, $\Delta p \neq 0$)

calculation starting from $^{48}$Ca with proton gauge angle at $^{48}$Ca
$\phi = 0^o, 30^o, 60^o, \cdots, 150^o, 180^o$ $(\Delta_p \sim |\Delta_p| e^{i\phi})$

**gauge angle**

U(1) phase degree of freedom introduced with gauge symmetry breaking by pairing condensation



# Isospin rotation about 2-axis

ph density
$$\begin{pmatrix}\rho_0(\theta')\\ \rho_1(\theta')\\ \rho_2(\theta')\\ \rho_3(\theta')\end{pmatrix} = \begin{pmatrix}1 & 0 & 0 & 0\\ 0 & \cos\theta' & 0 & \sin\theta'\\ 0 & 0 & 1 & 0\\ 0 & -\sin\theta' & 0 & \cos\theta'\end{pmatrix}\begin{pmatrix}\rho_0(0^\circ)\\ \rho_1(0^\circ)\\ \rho_2(0^\circ)\\ \rho_3(0^\circ)\end{pmatrix}$$

$$\rho_0(\boldsymbol{r}) = \rho_\mathrm{n}(\boldsymbol{r}) + \rho_\mathrm{p}(\boldsymbol{r})$$
$$\rho_1(\boldsymbol{r}) = \rho_\mathrm{np}(\boldsymbol{r}) + \rho_\mathrm{pn}(\boldsymbol{r})$$
$$\rho_3(\boldsymbol{r}) = \rho_\mathrm{n}(\boldsymbol{r}) - \rho_\mathrm{p}(\boldsymbol{r})$$

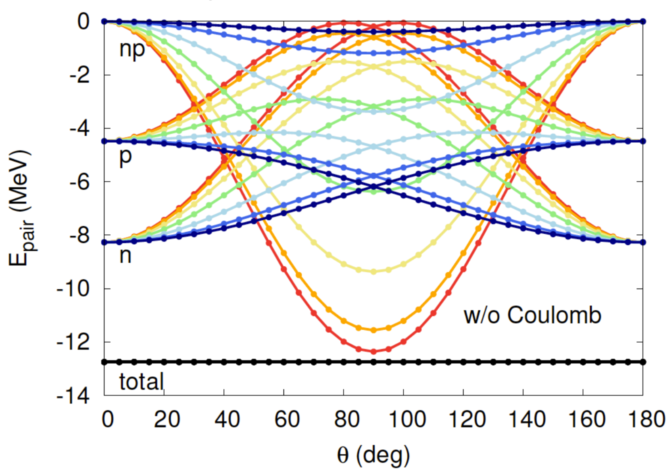
pp density
$$\begin{pmatrix}\breve\rho_0(\theta')\\ \breve\rho_1(\theta')\\ \breve\rho_2(\theta')\\ \breve\rho_3(\theta')\end{pmatrix} = \begin{pmatrix}1 & 0 & 0 & 0\\ 0 & \cos\theta' & 0 & \sin\theta'\\ 0 & 0 & 1 & 0\\ 0 & -\sin\theta' & 0 & \cos\theta'\end{pmatrix}\begin{pmatrix}\breve\rho_0(0)\\ \breve\rho_1(0)\\ \breve\rho_2(0)\\ \breve\rho_3(0)\end{pmatrix}$$

$$\breve\rho_0(\boldsymbol{r}) = 0$$
$$\breve\rho_1(\boldsymbol{r}) = \breve\rho_\mathrm{n}(\boldsymbol{r}) + \breve\rho_\mathrm{p}(\boldsymbol{r})$$
$$\breve\rho_2(\boldsymbol{r}) = i[\breve\rho_\mathrm{n}(\boldsymbol{r}) - \breve\rho_\mathrm{p}(\boldsymbol{r})]$$
$$\breve\rho_3(\boldsymbol{r}) = 2\breve\rho_\mathrm{np}(\boldsymbol{r})$$

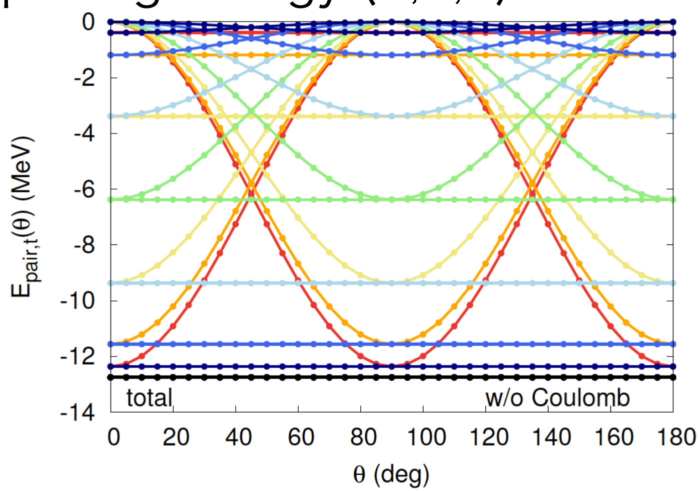
$\phi$: relative gauge angle at $T=T_z$ ($^{48}$Ca)

k=1 pair density depends on the relative gauge angle

pairing energy (n, p, np)




pairing energy (1,2,3)

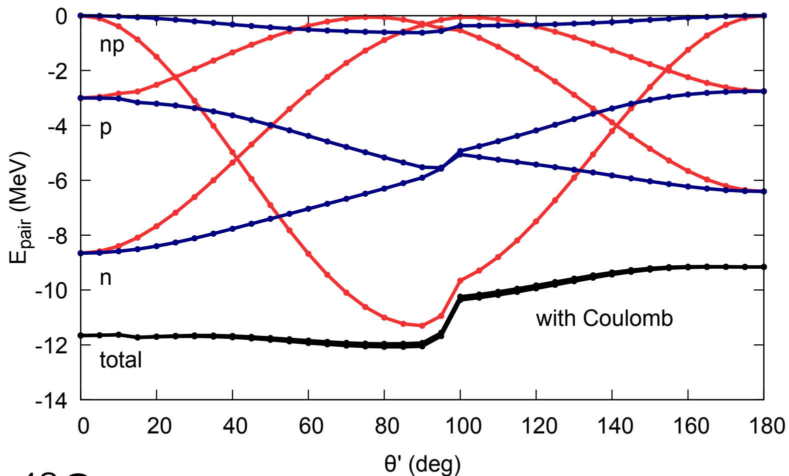



infinitesimally degenerated solutions with isospin symmetry of EDF

# A=48 isobaric analog states (w/ Coulomb)

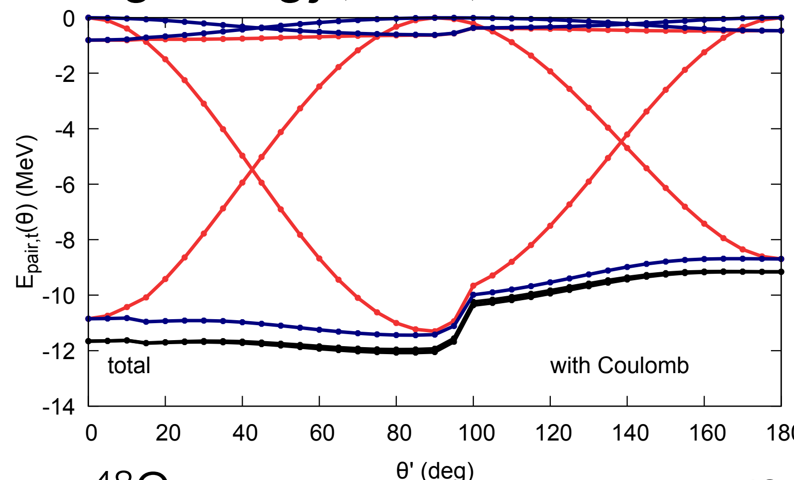
A=48 T=4 states, from $^{48}$Ca to $^{48}$Ni
w/ Coulomb: isospins symmetry of the EDF broken
solutions with $\phi$=0$^o$ and 180$^o$ survive during the isospin rotation

pairing energy(n,p,np)




$^{48}$Ca $^{48}$Ni

pairing energy(1,2,3)




$^{48}$Ca $^{48}$Ni

Almost degenerated two solutions in IASs with pairings (with Coulomb)
Coulomb prefers like-particle pairing solution
degree of freedom connecting these two would be important
kink around $\theta$'~100$^o$ is due to the pairing cutoff

energy at $\theta$'=90$^o$ $\phi$=0$^o$ -387.115215 MeV
$\phi$=180$^o$ -387.103344 MeV

# Neutron-proton condensation in ground states

results: difficult to get convergence(!)

if the potential energy surface is close to constant, finding an energy minimum is very difficult

**constraints on pairing amplitudes for PES calculations**

$$\hat{S}_k = \int d\boldsymbol{r} \sum_{stt'} 4st' \hat{a}_{\boldsymbol{r}-s-t'} \hat{a}_{\boldsymbol{r}st} \hat{\tau}^k_{t't} \qquad \langle \hat{S}_k \rangle = \int d\boldsymbol{r} \breve{\rho}_k(\boldsymbol{r})$$

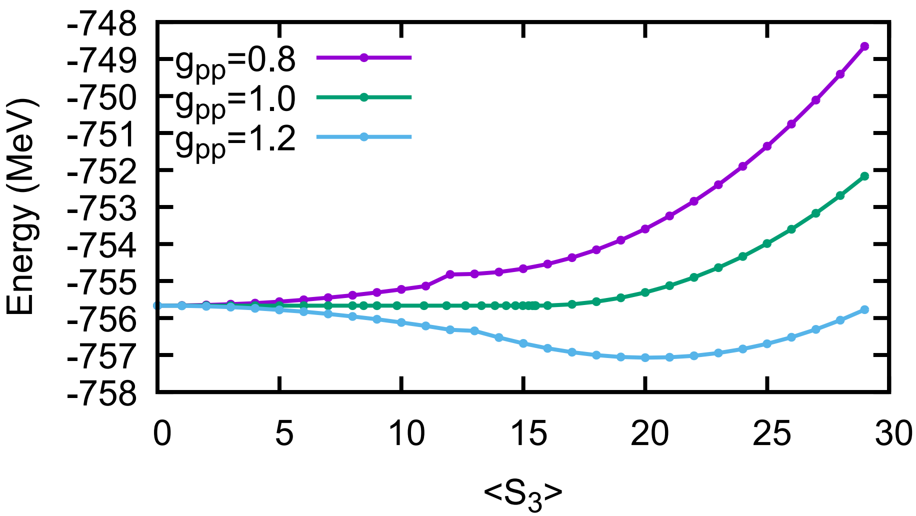
$\langle S_3 \rangle$: neutron-proton pairing amplitude

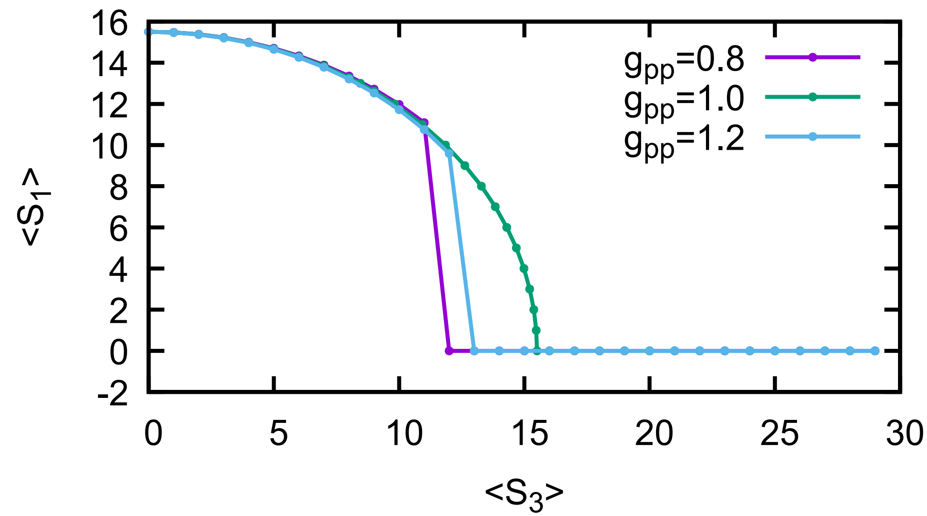
$\langle S_3 \rangle = 0$: (with $\langle S_1 \rangle$ or $\langle S_2 \rangle \neq 0$) like-particle condensation

$\langle S_3 \rangle \neq 0$: isovector np pair condensation

# isovector np pair condensation in N=Z

parameter choice: spherical and large pairing (non realistic)
A=68, $T_z$=0, without Coulomb, $Q_2$ = 0 (spherical), $\langle S_2 \rangle$=0
(SkM*, $N_{sh}$=7, $c_1^{\rho}$=-1224 MeV fm$^3$)




$g_{pp}$: $\breve{\chi}_1(\boldsymbol{r}) = \breve{C}_1[\rho_0]\{|\breve{\rho}_1(\boldsymbol{r})|^2 + |\breve{\rho}_2(\boldsymbol{r})|^2 + g_{pp}|\breve{\rho}_3(\boldsymbol{r})|^2\}$

**$g_{pp}$=1** (isospin-invariant EDF)

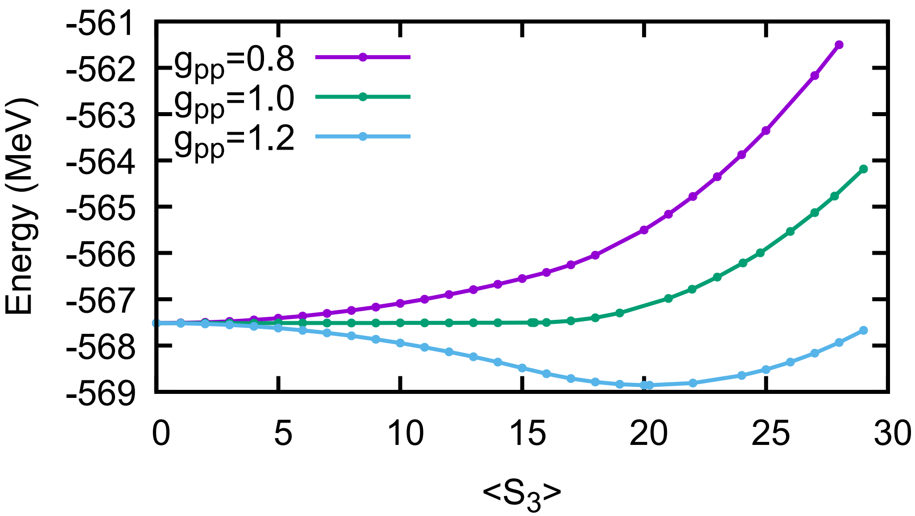
degenerated solutions along $|\langle S_1 \rangle|^2 + (|\langle S_2 \rangle|^2) + |\langle S_3 \rangle|^2 \sim 15.5^2$

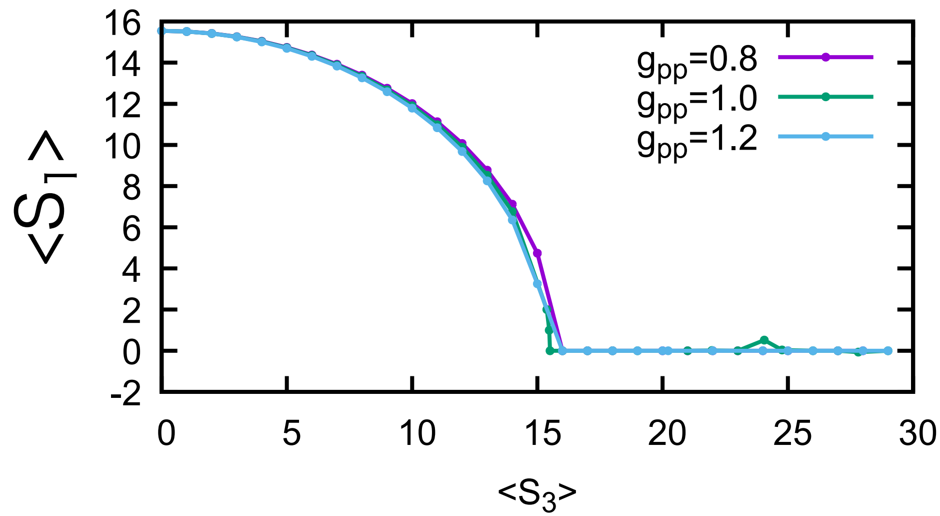
**$g_{pp}$>1:**

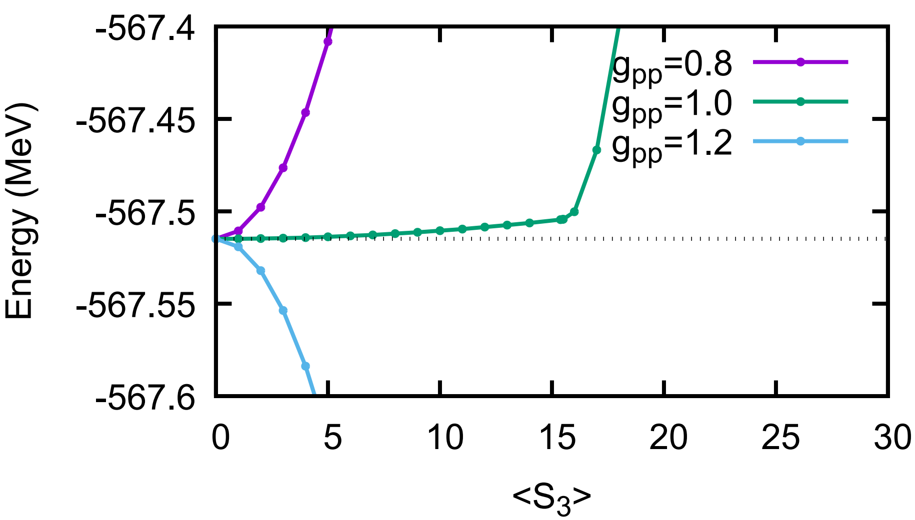
$\langle S_1 \rangle$=0 and $\langle S_3 \rangle \neq 0$ HFB solution (np condensation)

PES from $\langle S_3 \rangle$= 0 to ~15.5 corresponds to isospin rotation

# Effect of Coulomb




Coulomb does not prefer isovector np condensation

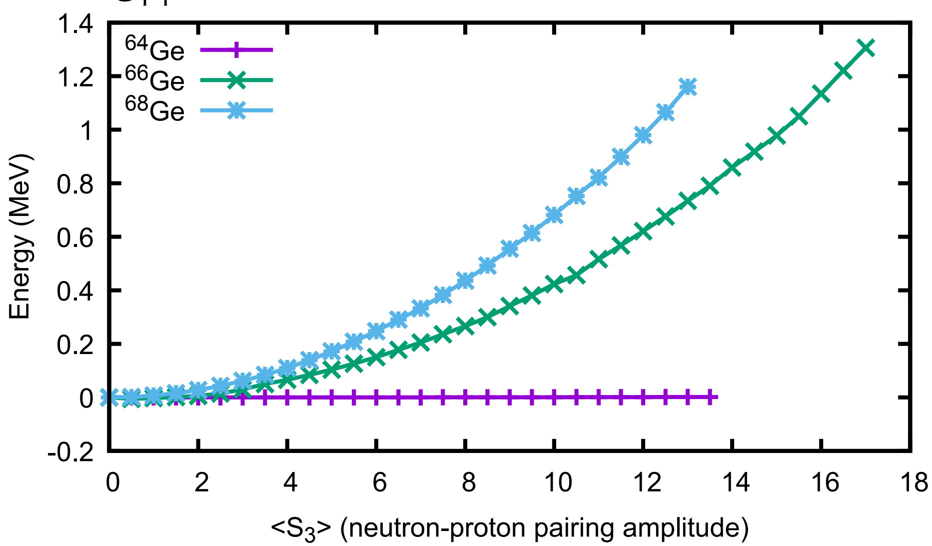
# Realistic calculations

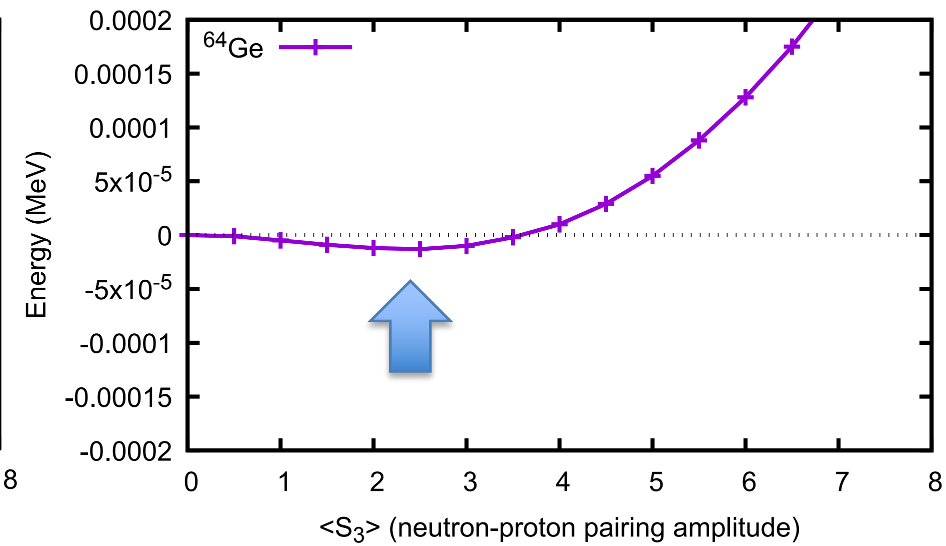
UNEDF1-HFB functional, $N_{sh}$=10
O to Sn isotopes around N=Z

Three sources of isospin symmetry breaking

1) particle-hole Coulomb functional
2) difference in nn and pp pairing strength
3) $g_{pp}$ difference between isovector nn-pp and np pairing strength

$g_{pp}$=1

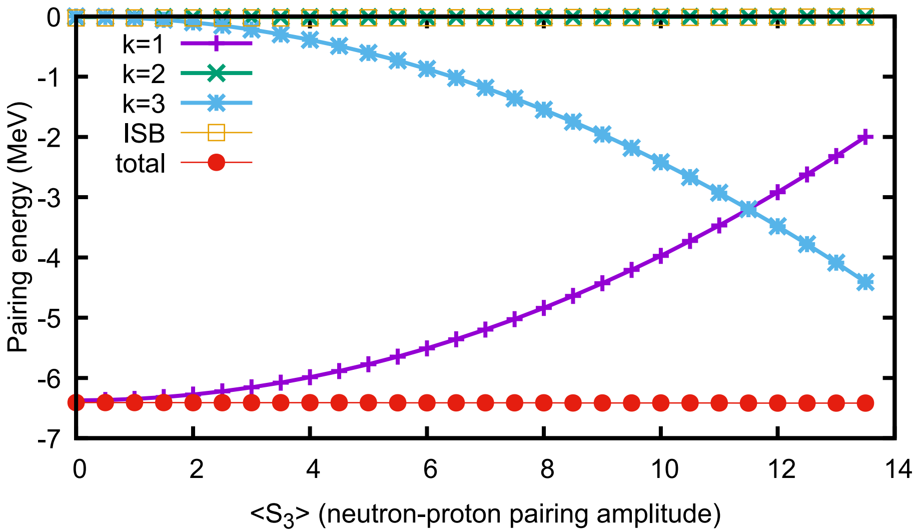


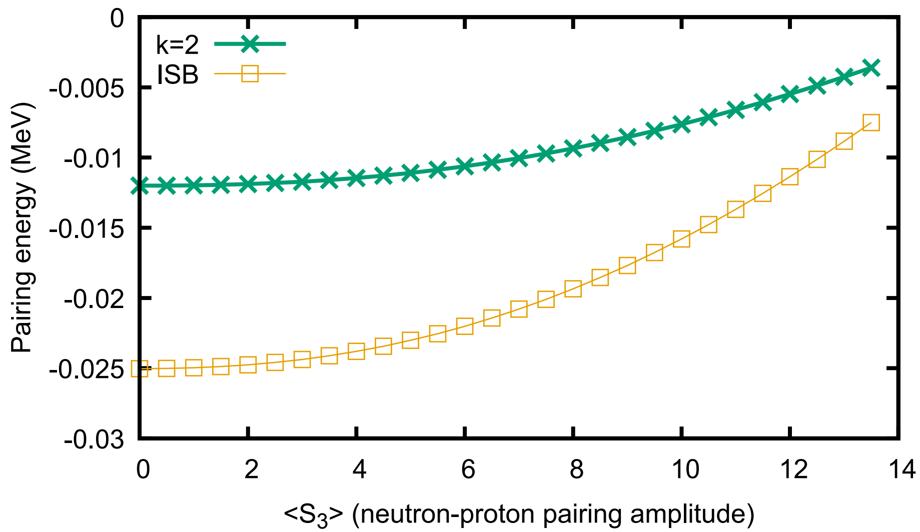




- Approximate isospin symmetry holds for isovector pairing with 1) and 2)
- Isovector np-pair condensed solution found in $^{64}$Ge ($g_{pp}$=1.0)

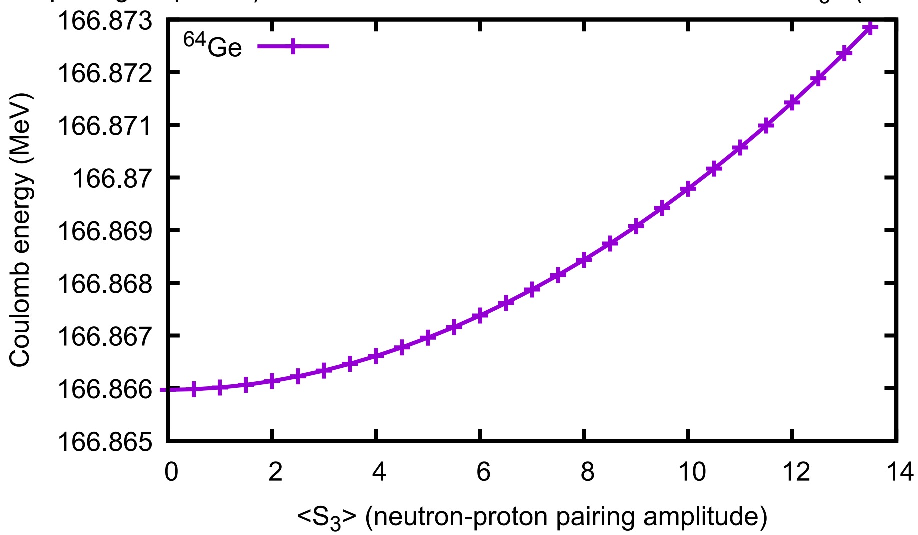
# Realistic calculations ($^{64}$Ge)

Pairing energy: isospin-symmetry breaking component very small




Coulomb energy




- Isospin symmetry breaking in isovector pairing is very small
- Coulomb provides energy shift, keeping approximate isospin symmetry in pairing
- Isospin symmetry is still valid for isospin-symmetry-breaking realistic calculation

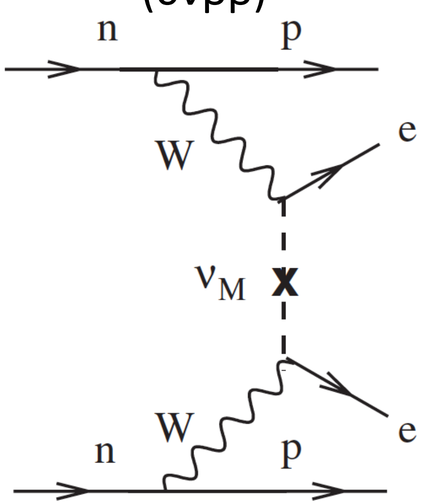
# Isoscalar pairing

isoscalar pairing

- no evidence for isoscalar np pair condensation
- strength globally fitted to beta decay rate (QRPA)

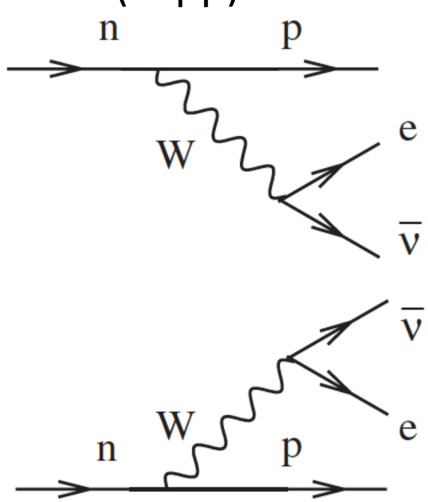
  Mustonen and Engel, Phys. Rev. C **93**, 014304 (2016)
- double-beta decay (QRPA, no global fit completed)

neutrinoless double-beta decay
($0\nu\beta\beta$)




take place if neutrino is Majorana particle
half life related to neutrino mass
**not measured yet**

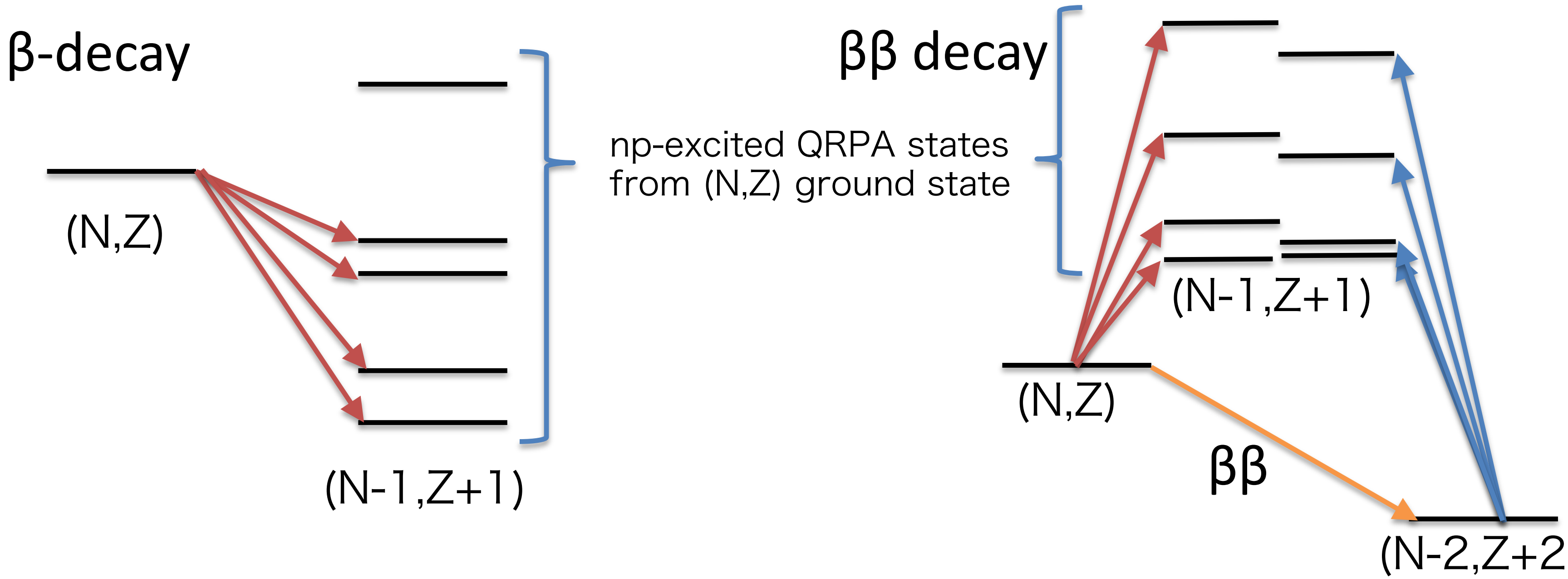
two-neutrino double-beta decay
($2\nu\beta\beta$)



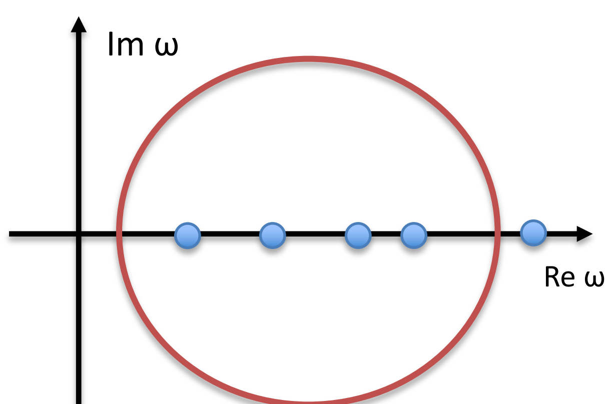

higher-order process of beta decay
experimental data available

both rates (nuclear matrix element) depend strongly on isoscalar pairing
→include available $2\nu\beta\beta$ data for global fit of isoscalar pairing strength

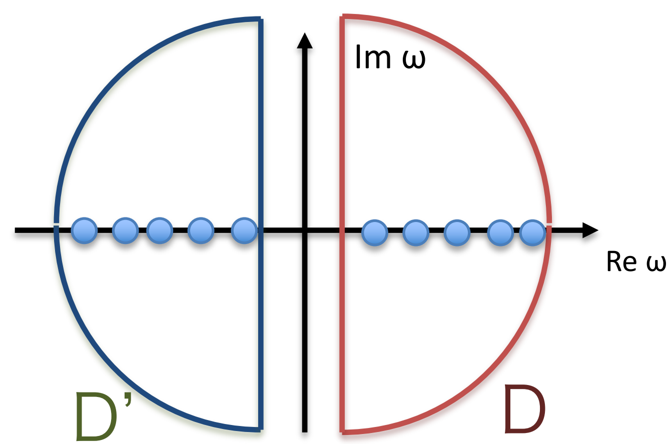
# Double-beta decay in nuclear DFT




standard DFT approach: diagonalization (computationally demanding)
finite-amplitude method (linear response theory, iteration)





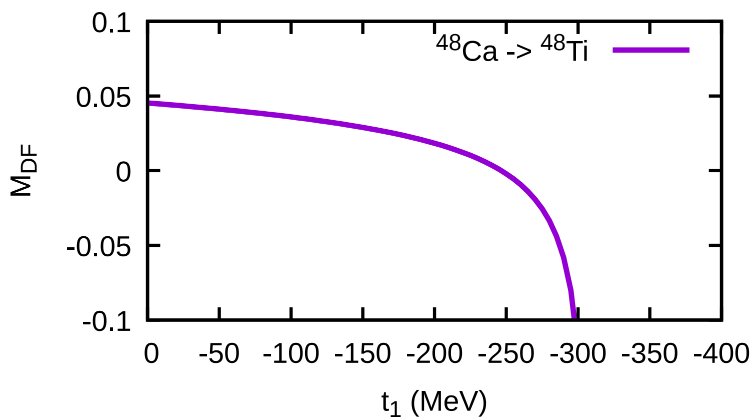


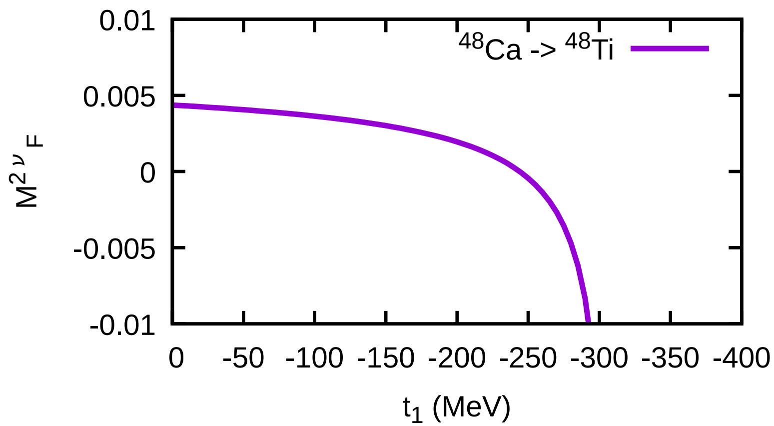

NH, Kortelainen, Nazarewicz, Phys. Rev. C **87**, 064309 (2013)
Mustonen et al., Phys. Rev. C **90**, 024308(2014)

NH, AIP Conf. Proc. **2165**, 020010 (2019)

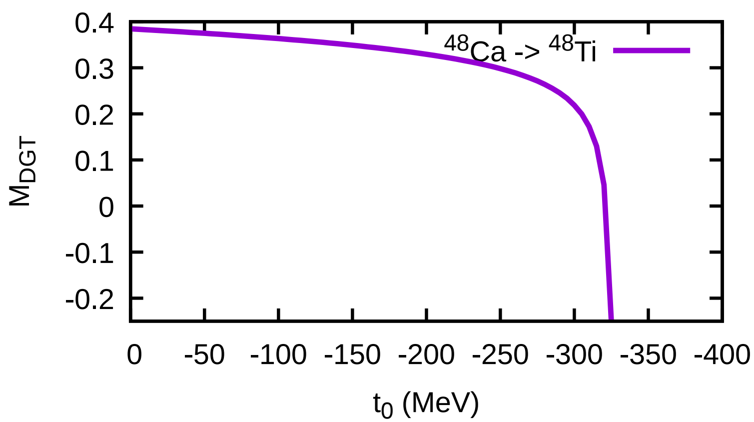
# Test calculation for $^{48}Ca \rightarrow ^{48}Ti$

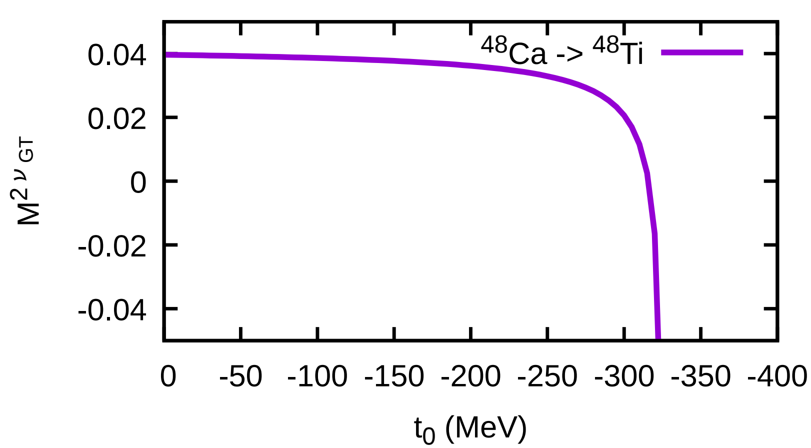
isovector np pairing dependence on double-Fermi, 2νββ Fermi matrix element




isoscalar np pairing dependence on double-GT, 2νββ Gamow-Teller matrix element




NH, AIP Conf. Proc. **2165**, 020010 (2019)

2νββ experimental value：

$M_{2\nu} = 0.046 \pm 0.004$ MeV$^{-1}$ (Barabash, Nucl. Phys. A **935**, 52 (2015), with $g_A$=1.27)

# Summary

- Neutron-proton DFT based on HFBTHO has been implemented
  - degenerated solutions are found in isobaric analogue states with isovector pairing
  - $^{64}Ge$ ground state is found to be isovector np pairing condensation in UNEDF1-HFB functional
- Isospin symmetry is still approximately good with Coulomb and like-particle pairing (with $V_n \neq V_p$)
- Beyond mean field treatment to include isospin rotation degrees of freedom is necessary to include correlation (isospin projection?)
- New technique to calculation $2\nu\beta\beta$ nuclear matrix element to be used to determine the strength of isoscalar pairing
- Collaborators: Javid Sheikh(Kashmir, India), Witek Nazarewicz (MSU), Jacek Dobaczewski (York, UK), Jon Engel (UNC-CH)