Short range correlation physics in low resolution pictures

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Sushant More

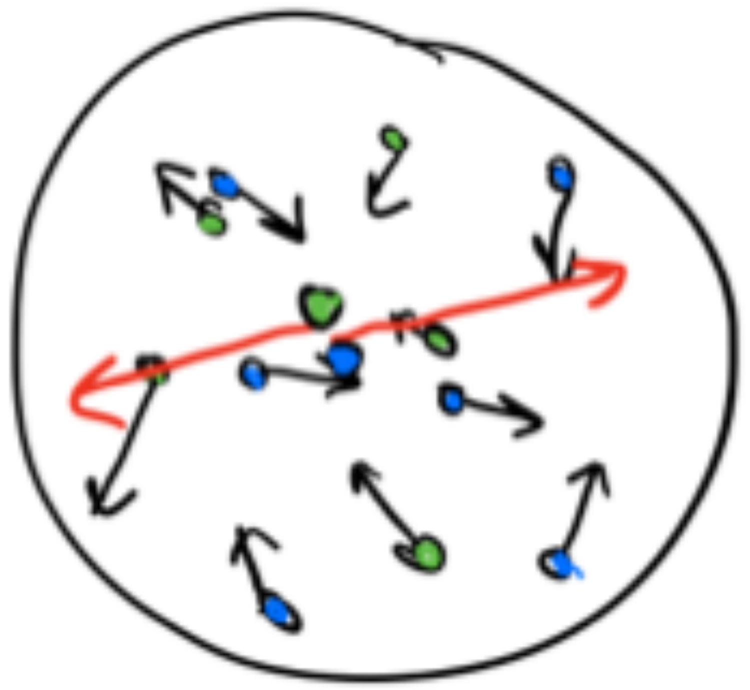
Low and High Resolution Scale Pictures

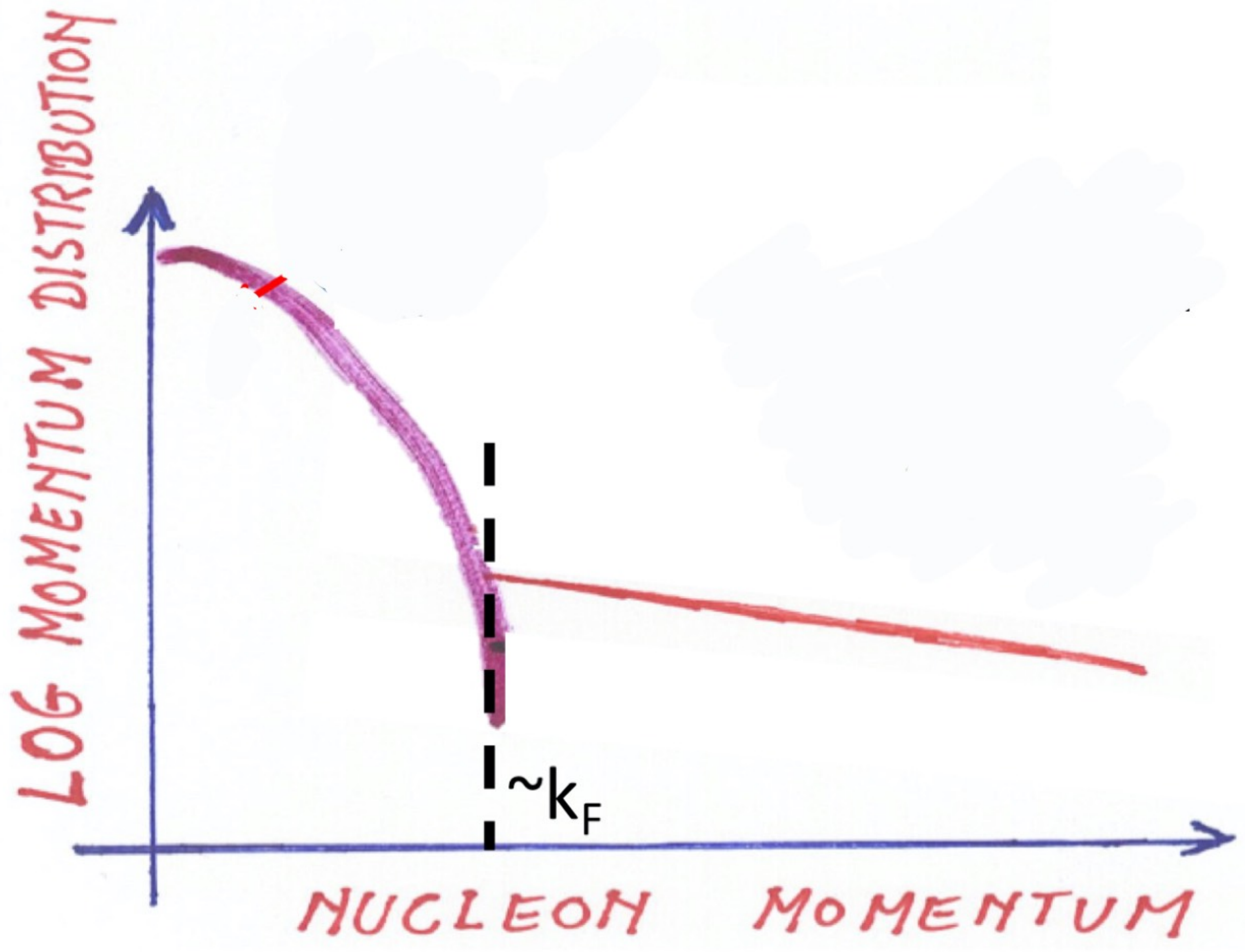
(RG) Resolution Scale $H = H(\Lambda)$ ➡ max. momenta in low-energy wf's ~ Λ

Low and High Resolution Scale Pictures

(RG) Resolution Scale $H = H(\Lambda)$ ➜ max. momenta in low-energy wf's ~ Λ

High resolution picture:





correlated pairs w/large ~ back-to-back momenta

high-k tails (k >> k_F) present

Low and High Resolution Scale Pictures

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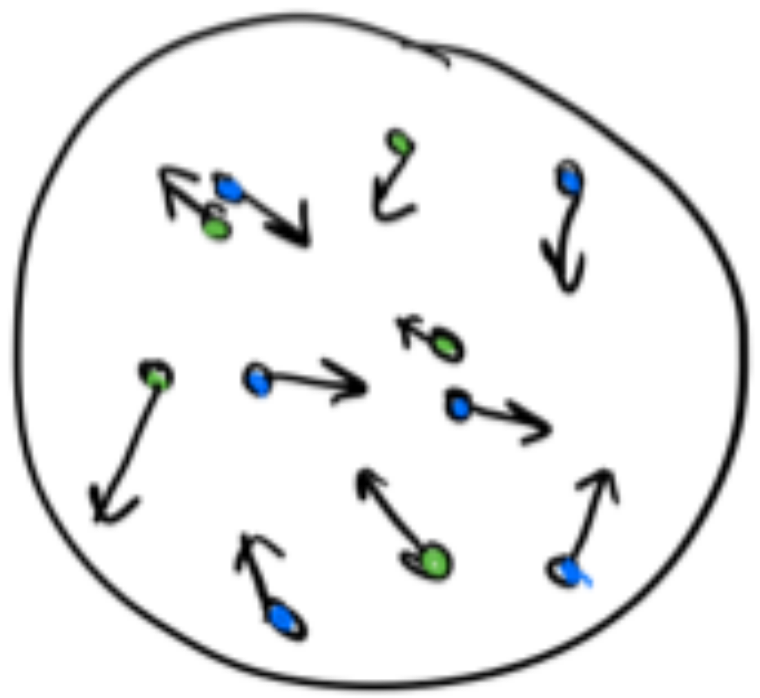

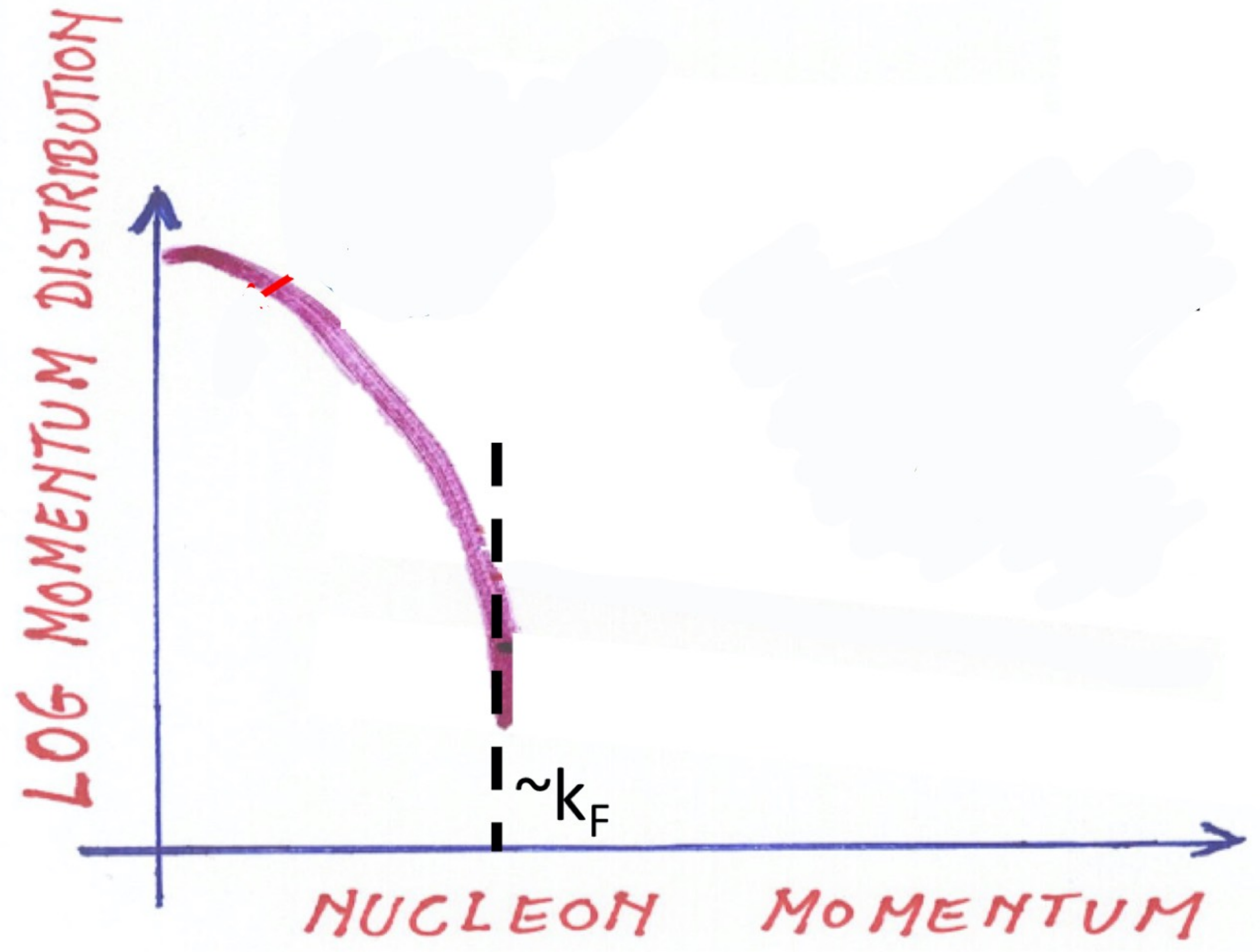

resembles "mean field" picture

no high-k tails ($k \gg k_F$)

Low and High Resolution Scale Pictures

Examples of **Low resolution** pictures

Low and High Resolution Scale Pictures

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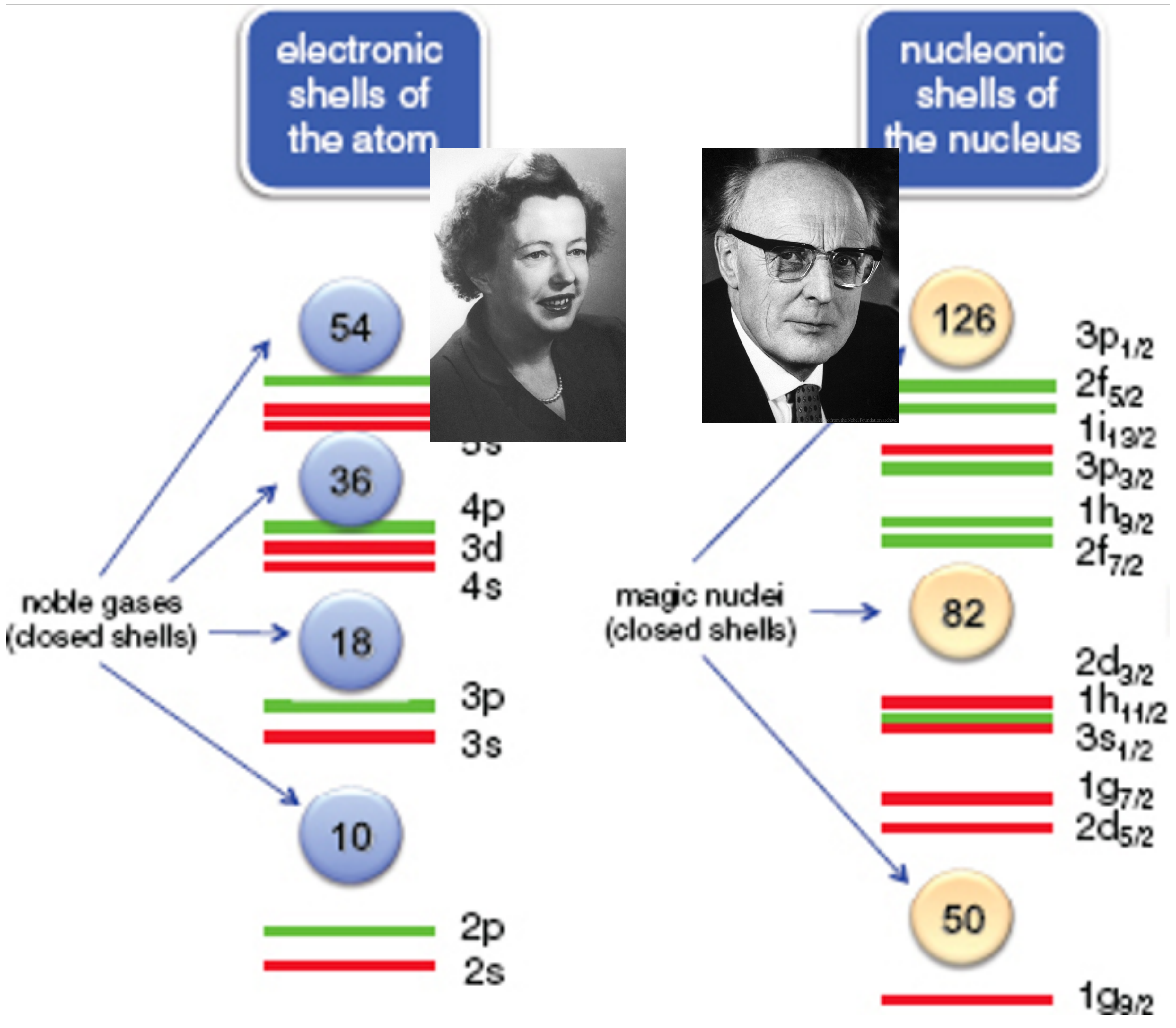

Nuclear Shell model

Low and High Resolution Scale Pictures

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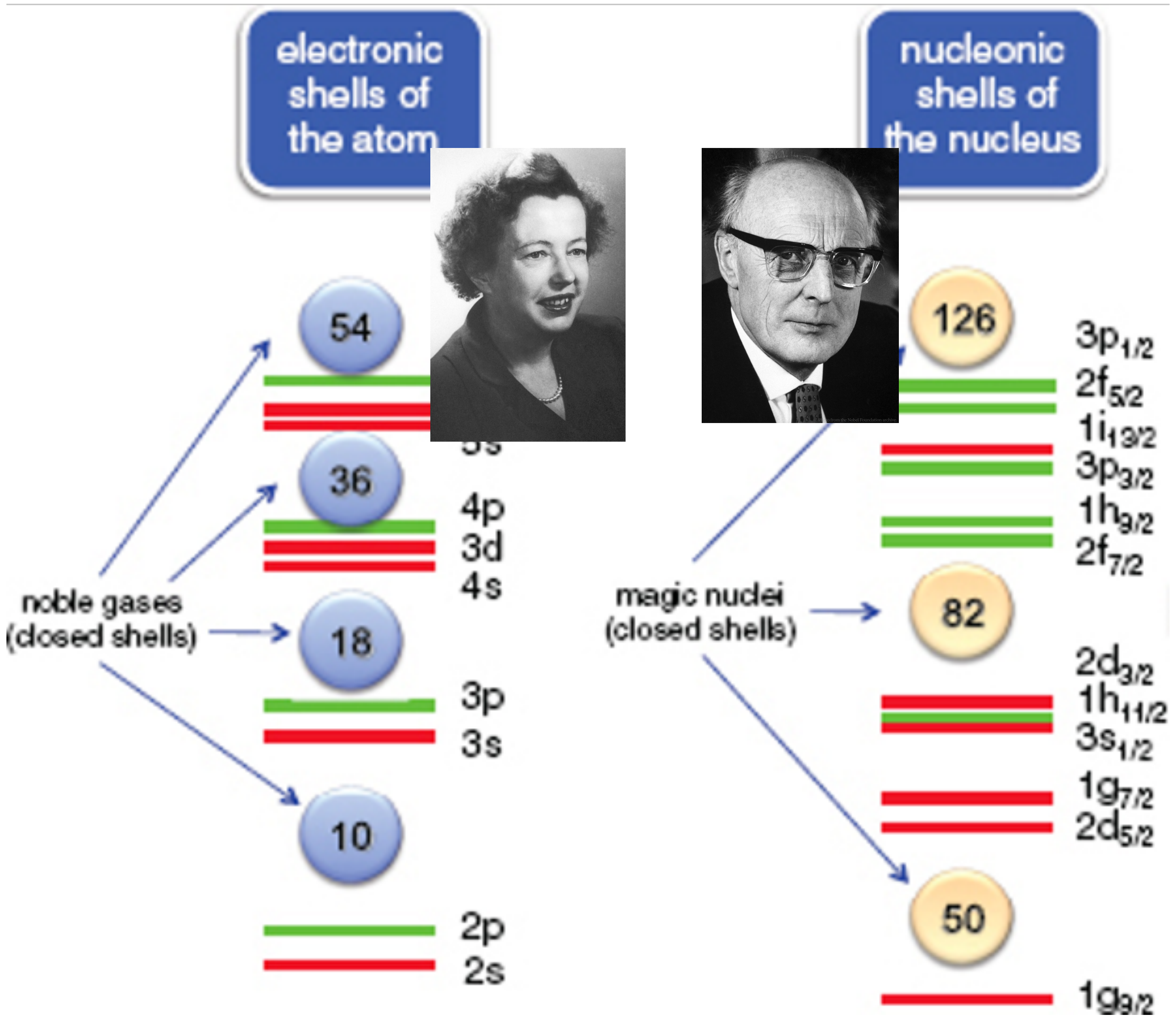

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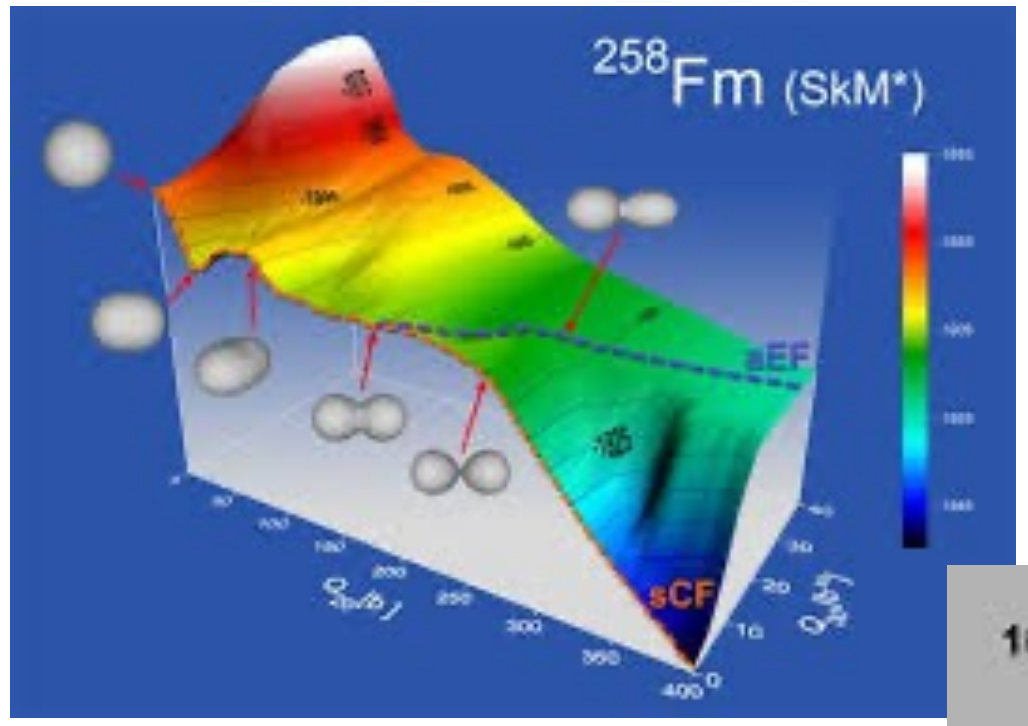
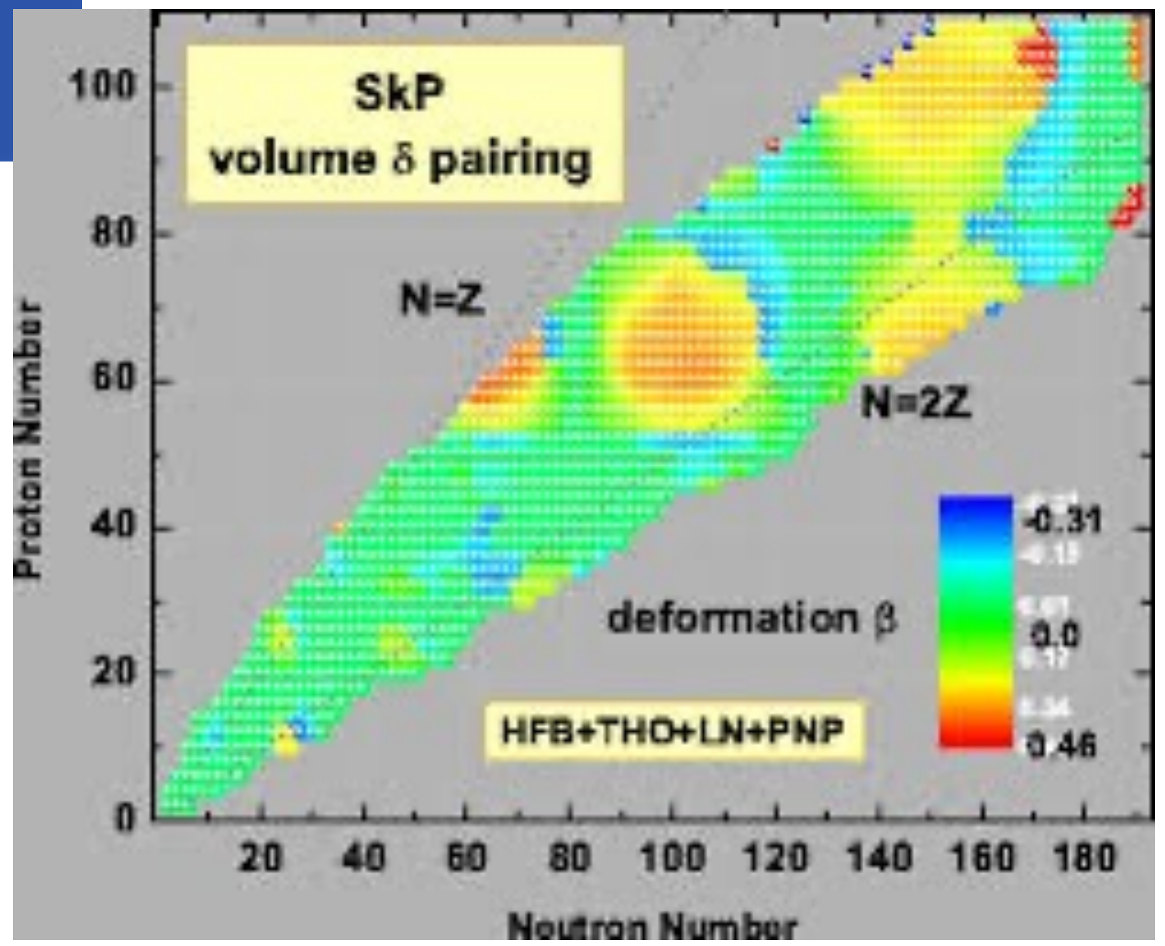


$$
\begin{aligned}
V_{Skyrme} = {} & t_0(1 + x_0\hat{P}_\sigma)\delta(\vec{r}) \\
& + \frac{1}{2}t_1(1 + x_1\hat{P}_\sigma)[\delta(\vec{r})\vec{k}^2 + \vec{k}'^2\delta(\vec{r})] \\
& + t_2(1 + x_2\hat{P}_\sigma)\vec{k}'.\delta(\vec{r})\vec{k} \\
& + \frac{1}{6}t_3(1 + x_3\hat{P}_\sigma)\rho^\alpha(\vec{R})\delta(\vec{r}) \\
& + it_4\vec{k}'.\delta(\vec{r})(\vec{\sigma}_i + \vec{\sigma}_j)\vec{k}\,,
\end{aligned}
$$



Nuclear density functional

Low and High Resolution Scale Pictures

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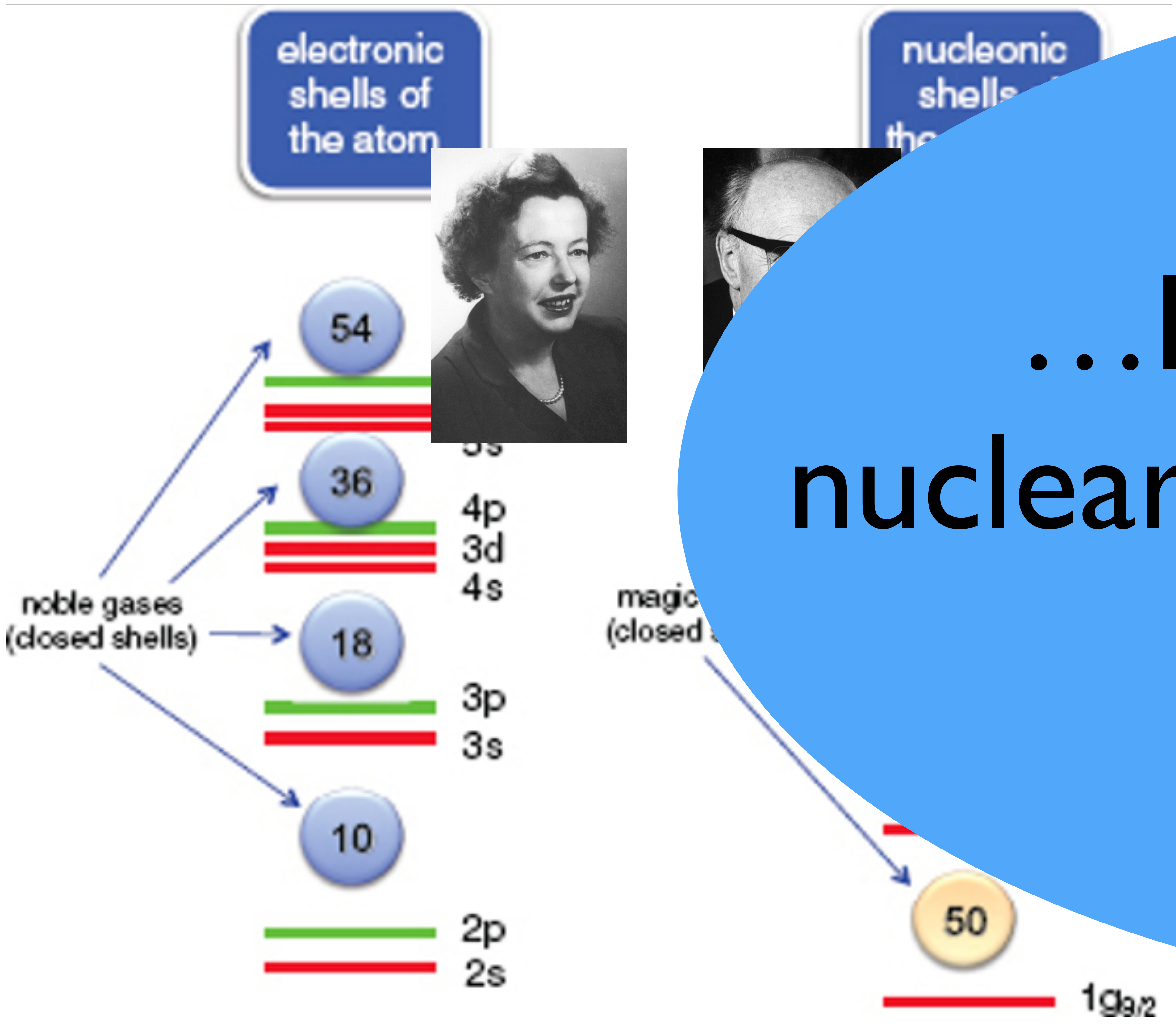

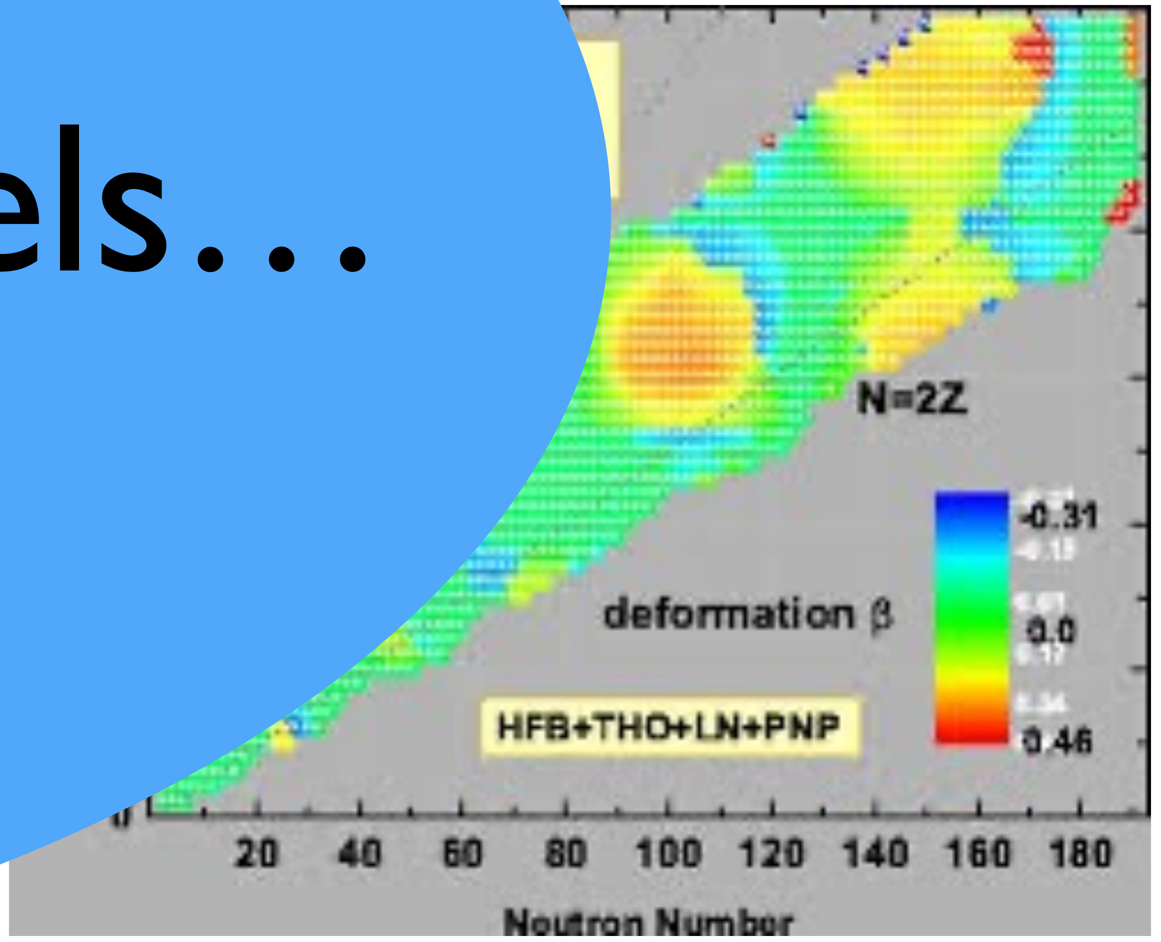

…Pretty much all our nuclear **<u>structure</u>** models…

Nuclear Shell model

Nuclear density functional

Short History: High Resolution Scale Picture

All nuclear structure models <==> **Low resolution** pictures

How did the high resolution picture arise?

exhibit A: NN scattering (1950s-60s)

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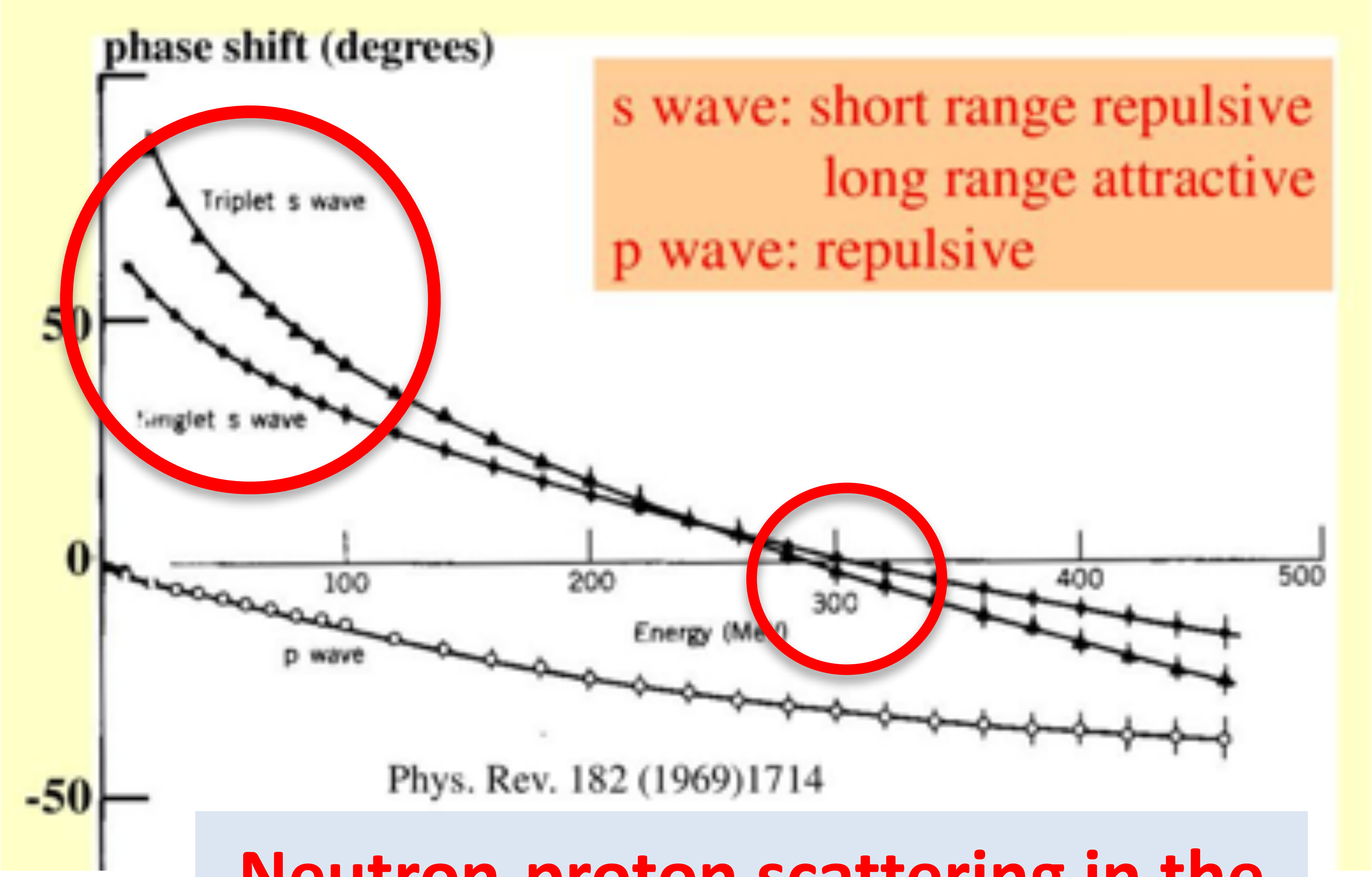

Neutron-proton scattering in the S-waves changes sign

<u>IF</u> you insist on a local $V(r)$:

strong short-range repulsive core needed to get s-wave sign change

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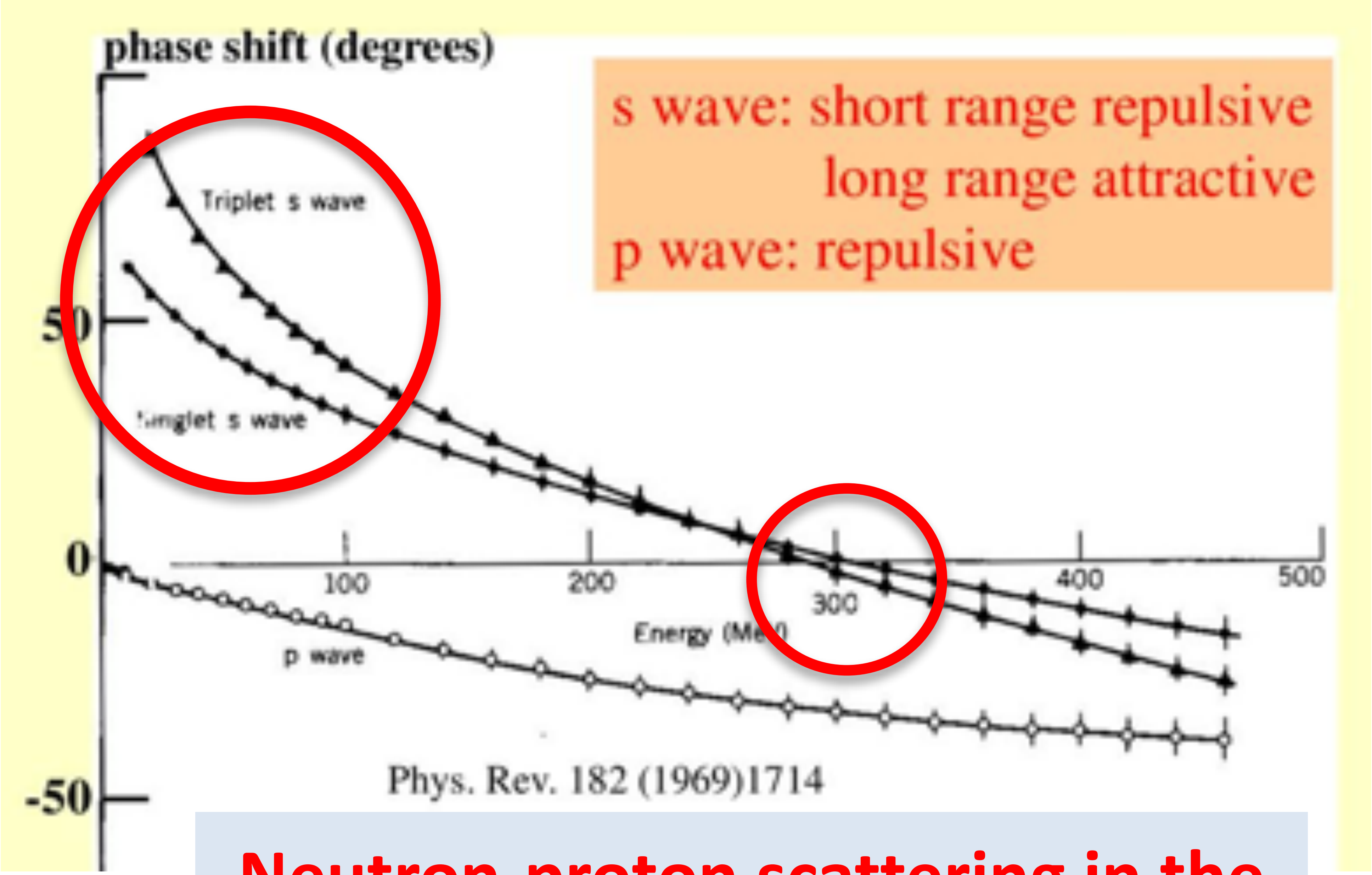

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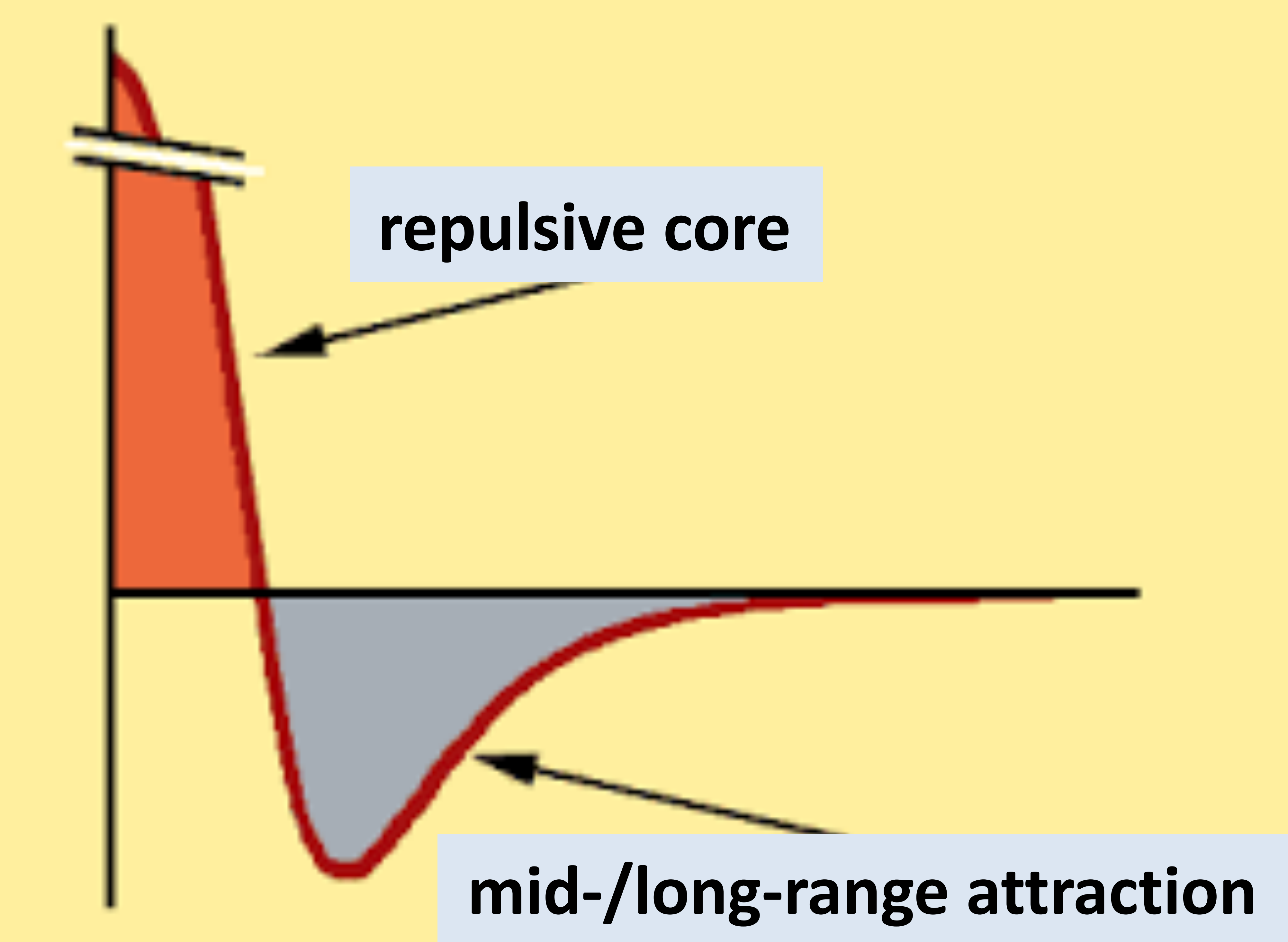

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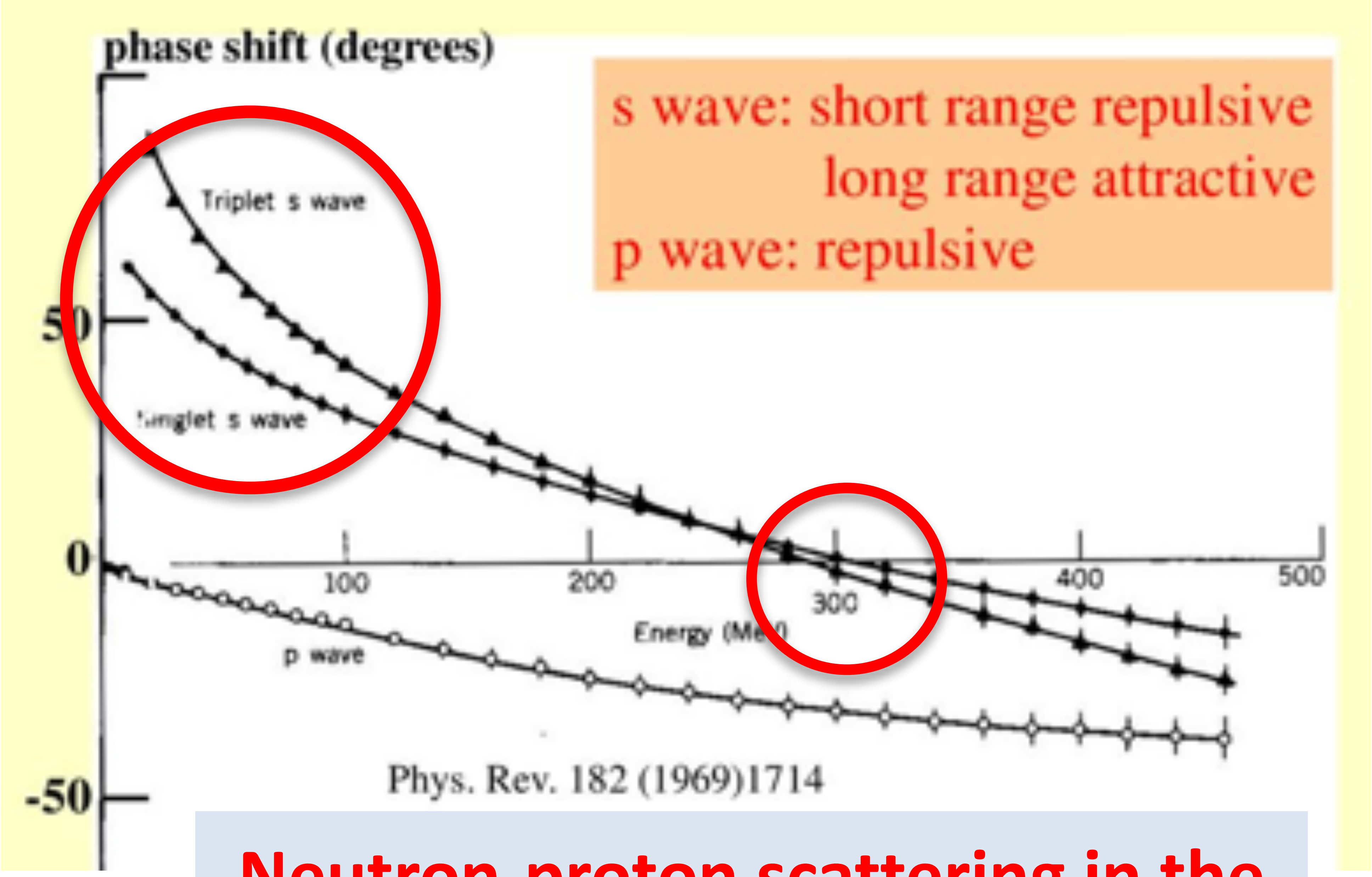

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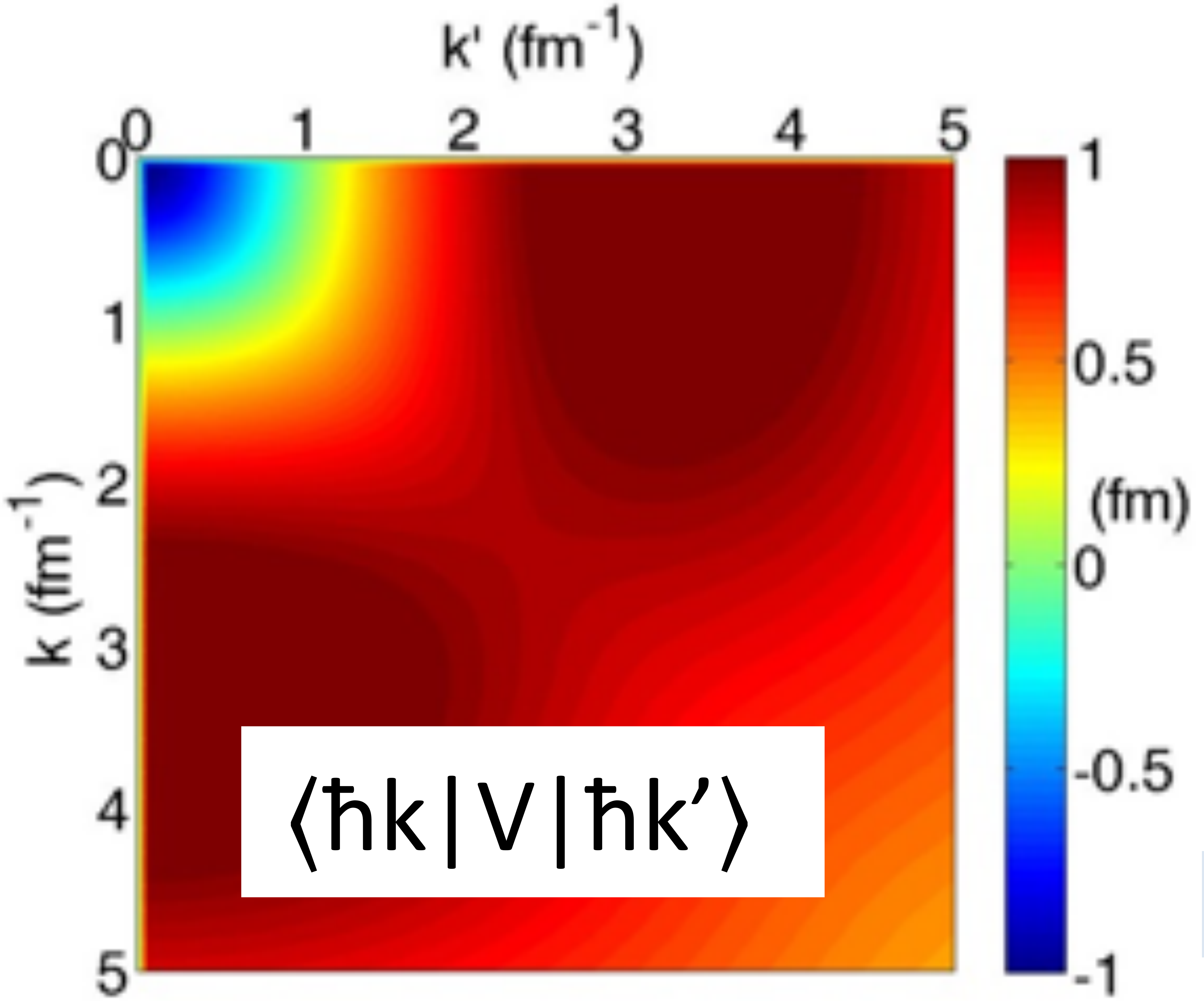
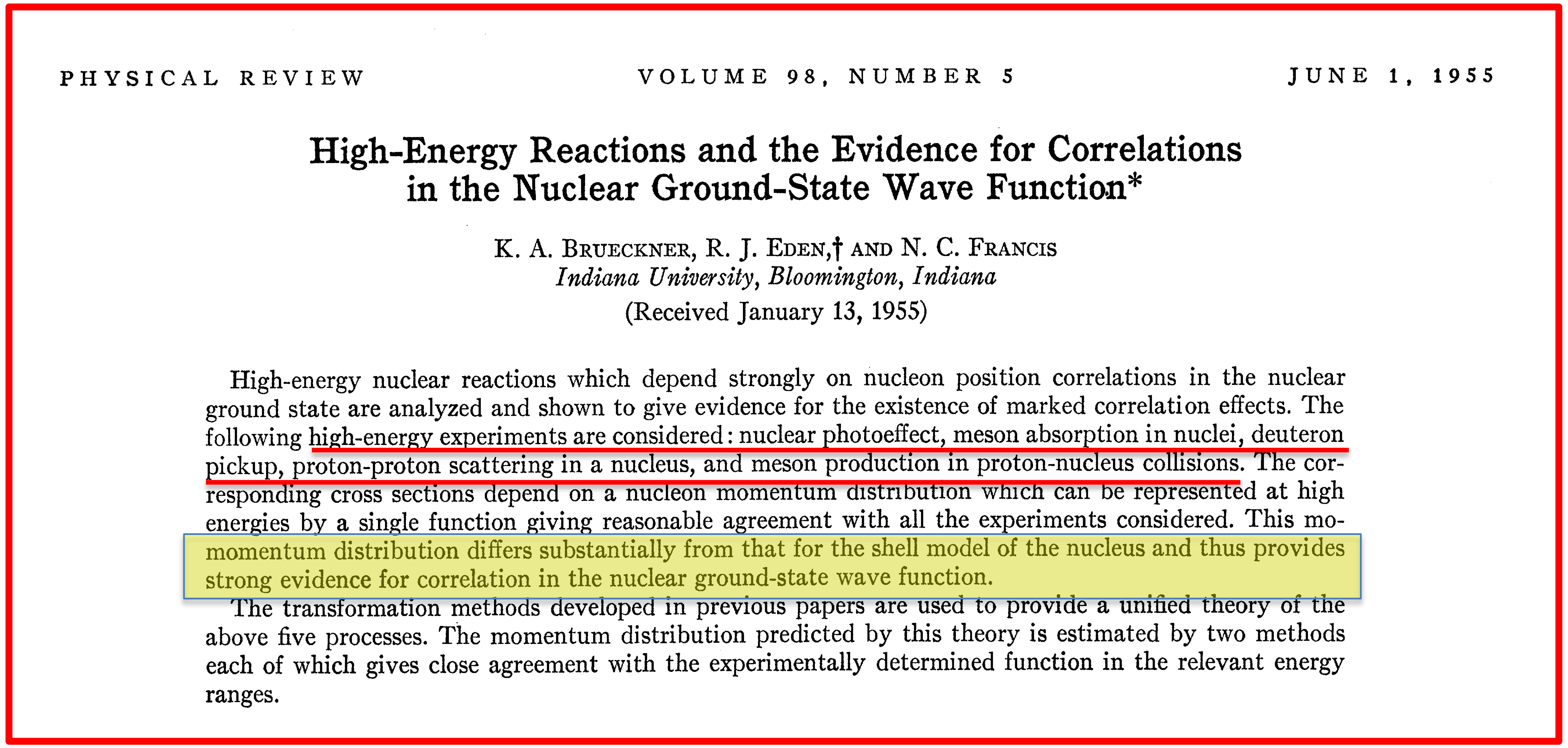


Short History: High Resolution Scale Picture

exhibit B: Brueckner 1955 paper



PHYSICAL REVIEW VOLUME 98, NUMBER 5 JUNE 1, 1955

High-Energy Reactions and the Evidence for Correlations in the Nuclear Ground-State Wave Function*

K. A. BRUECKNER, R. J. EDEN,† AND N. C. FRANCIS
Indiana University, Bloomington, Indiana

(Received January 13, 1955)

High-energy nuclear reactions which depend strongly on nucleon position correlations in the nuclear ground state are analyzed and shown to give evidence for the existence of marked correlation effects. The following high-energy experiments are considered: nuclear photoeffect, meson absorption in nuclei, deuteron pickup, proton-proton scattering in a nucleus, and meson production in proton-nucleus collisions. The corresponding cross sections depend on a nucleon momentum distribution which can be represented at high energies by a single function giving reasonable agreement with all the experiments considered. This momentum distribution differs substantially from that for the shell model of the nucleus and thus provides strong evidence for correlation in the nuclear ground-state wave function.

The transformation methods developed in previous papers are used to provide a unified theory of the above five processes. The momentum distribution predicted by this theory is estimated by two methods each of which gives close agreement with the experimentally determined function in the relevant energy ranges.

Short History: High Resolution Scale Picture

exhibit B: Brueckner 1955 paper

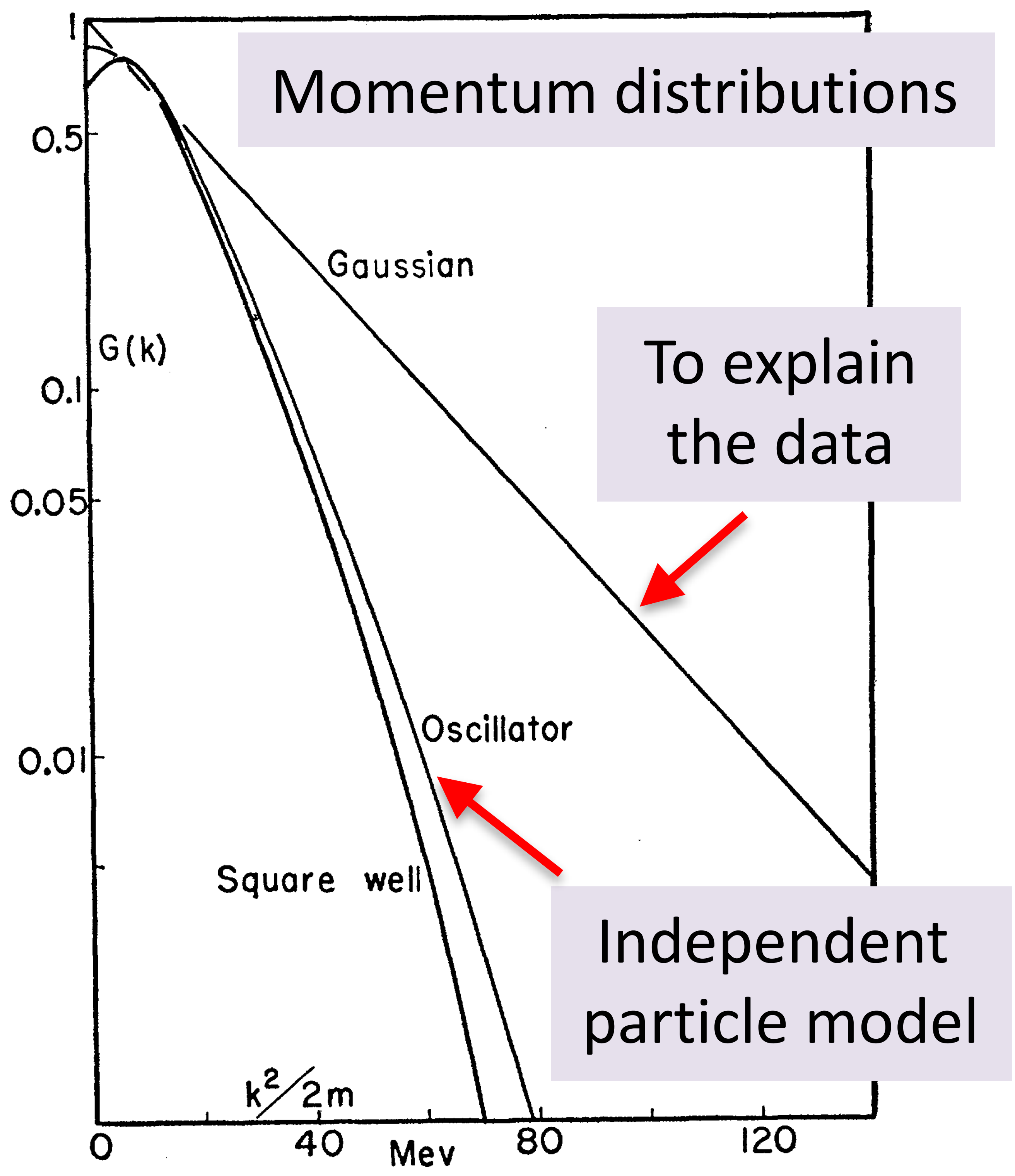


FIG. 1. Momentum distribution $G(k)$ of 8 neutrons and 8 protons in the independent-particle states of a square well with infinite walls and of a harmonic oscillator well. For comparison the Gaussian distribution of Eq. (3) is also given.

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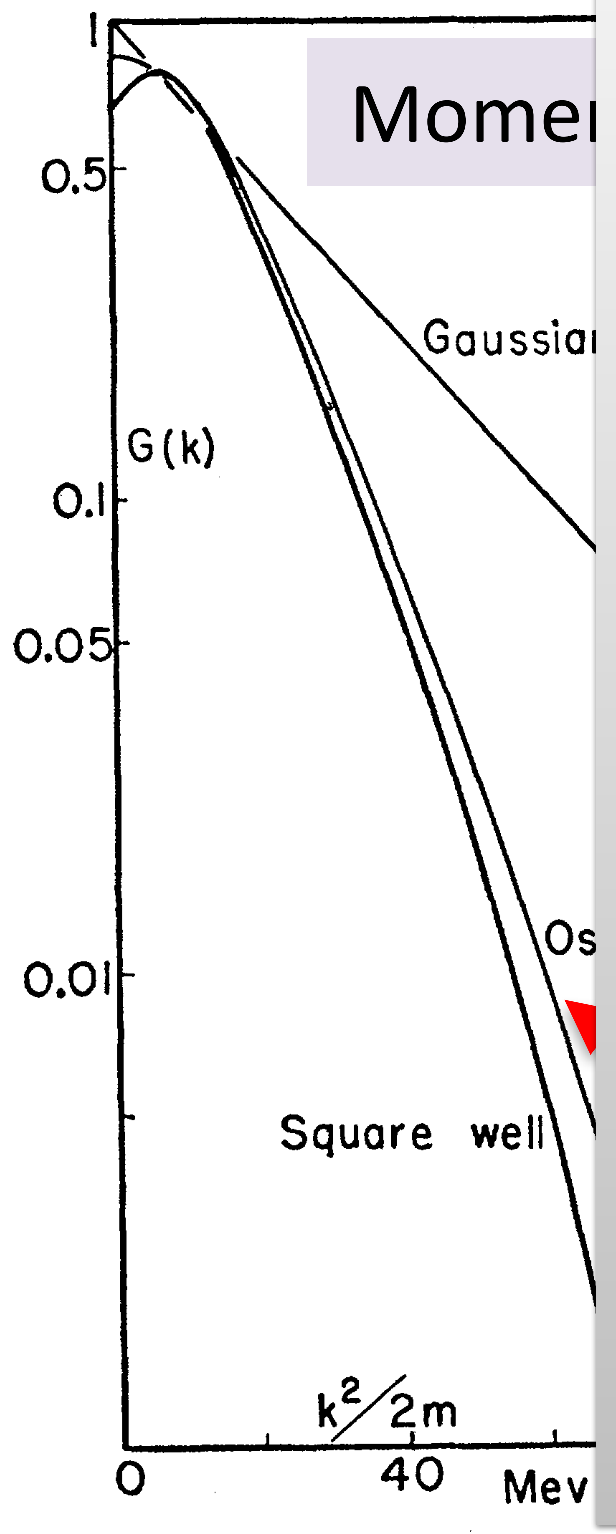
"Consequently it follows that the usual assumptions of the shell-model theory of the nucleus, that the particles move independently in a uniform potential, cannot be other than very approximately correct."

Short History: High Resolution Scale Picture

NSCL FRIB

exhibit B: Brueckner 1955 paper



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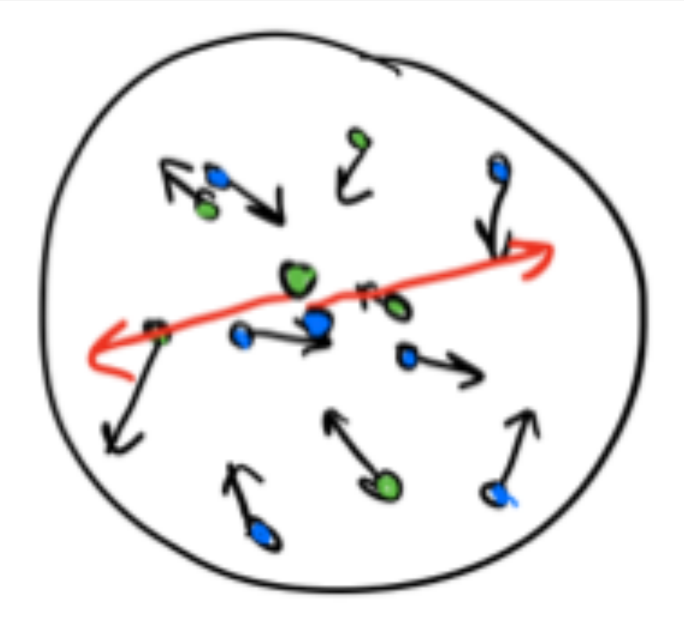
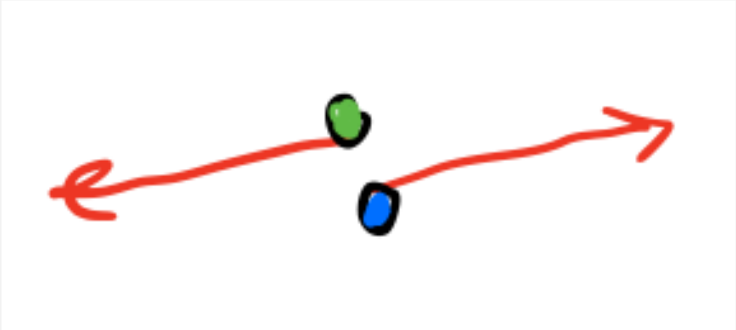
Key configurations in Brueckner's analysis:





This is a *short-range correlation* or SRC

$$\Psi(k_1, k_2, k_3, \ldots, k_A) \sim \phi(k_1 - k_2)\,\chi_A(K_{12}, k_3, \ldots, k_A)$$

JUNE 1, 1955

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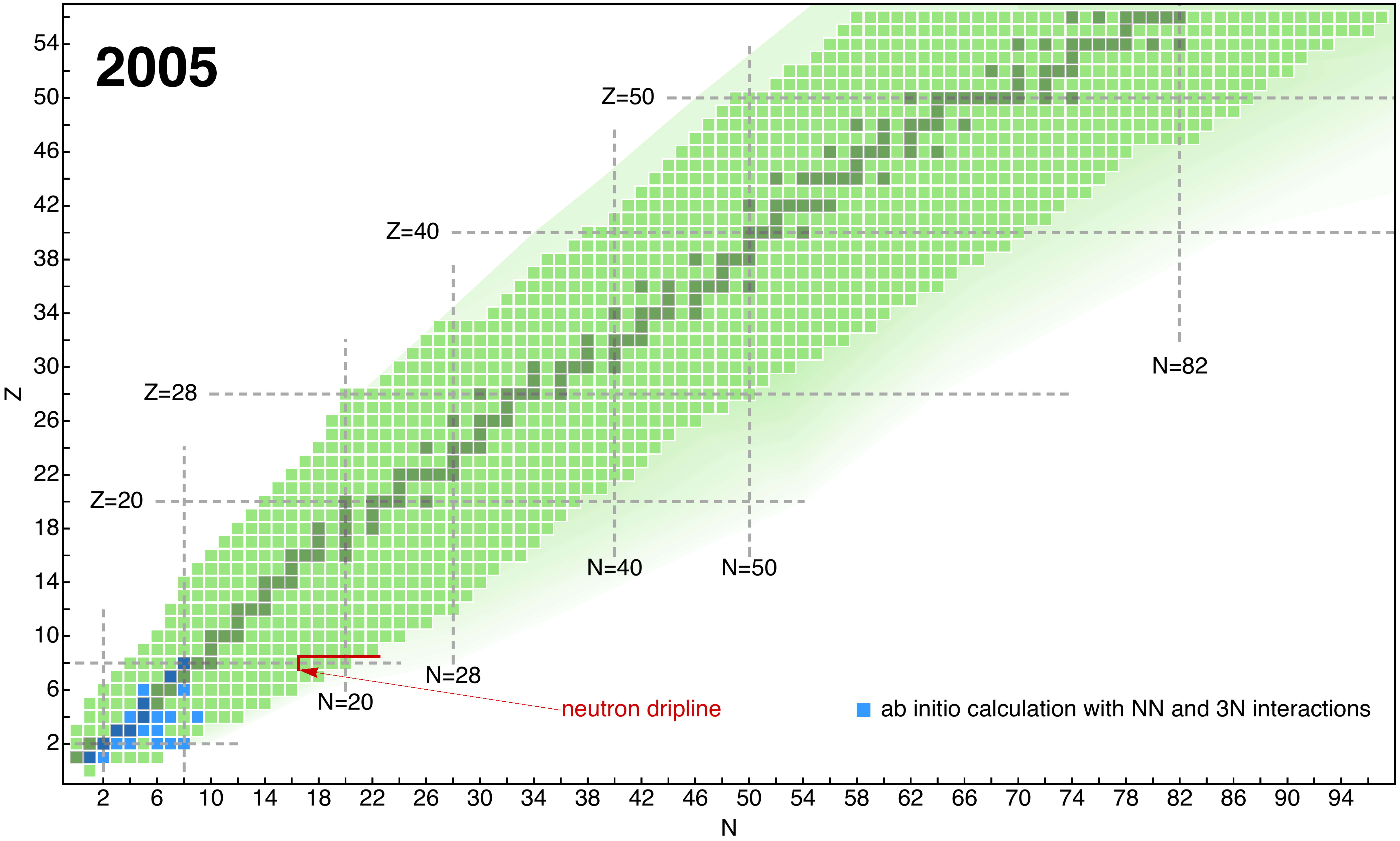
Ab Initio with **high-resolution** NN + NNN



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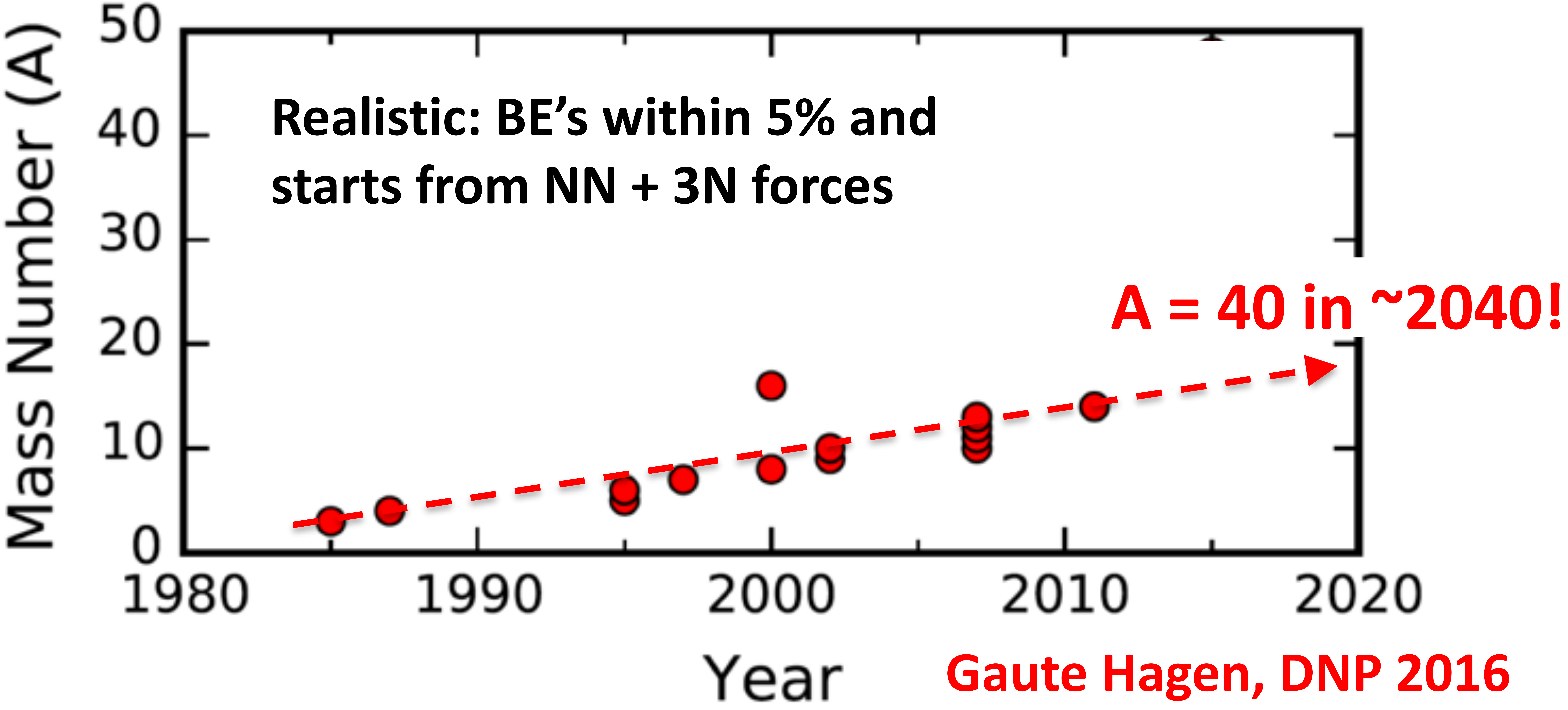
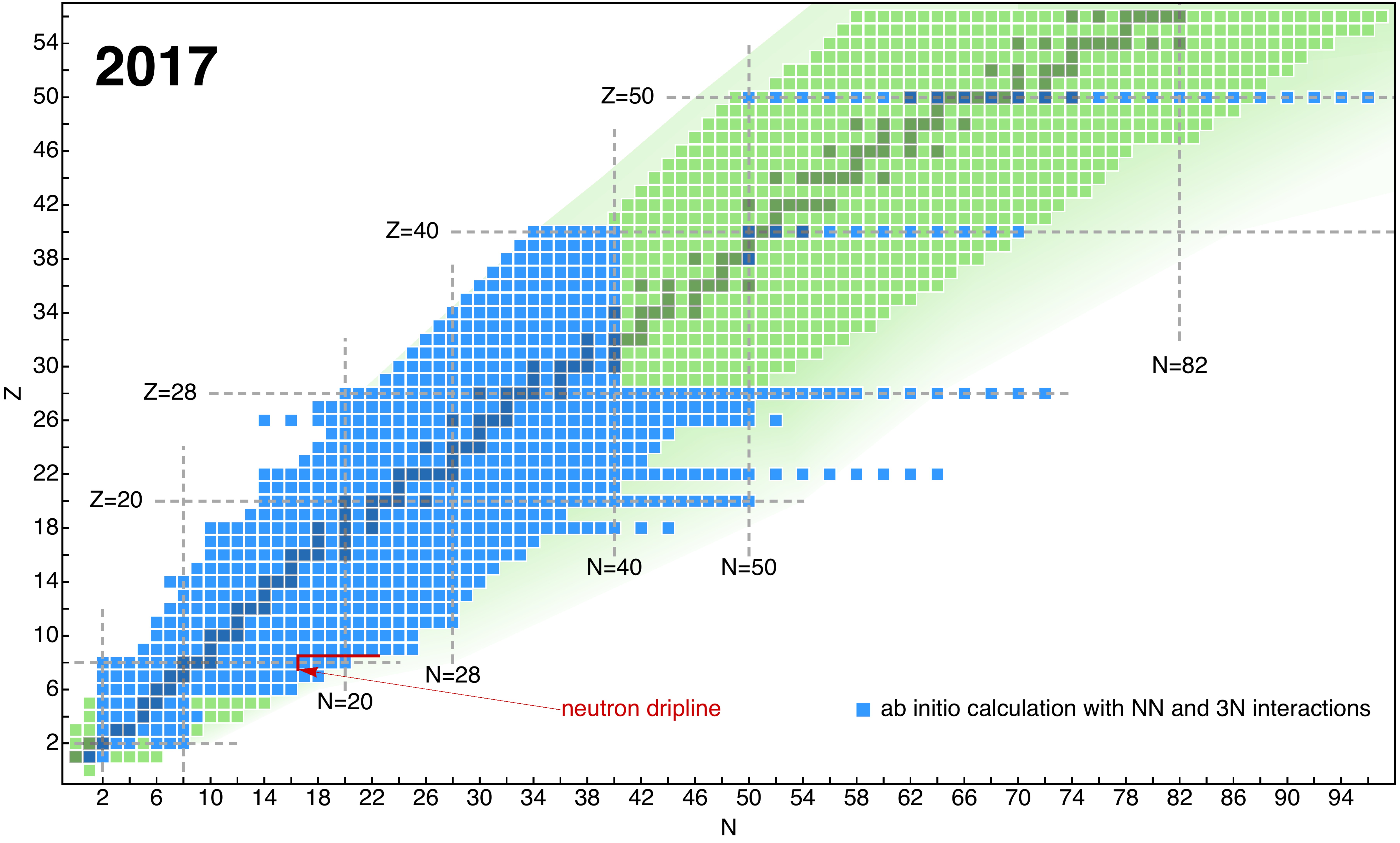


Gaute Hagen, DNP 2016

Ab Initio with **low-resolution** NN + NNN



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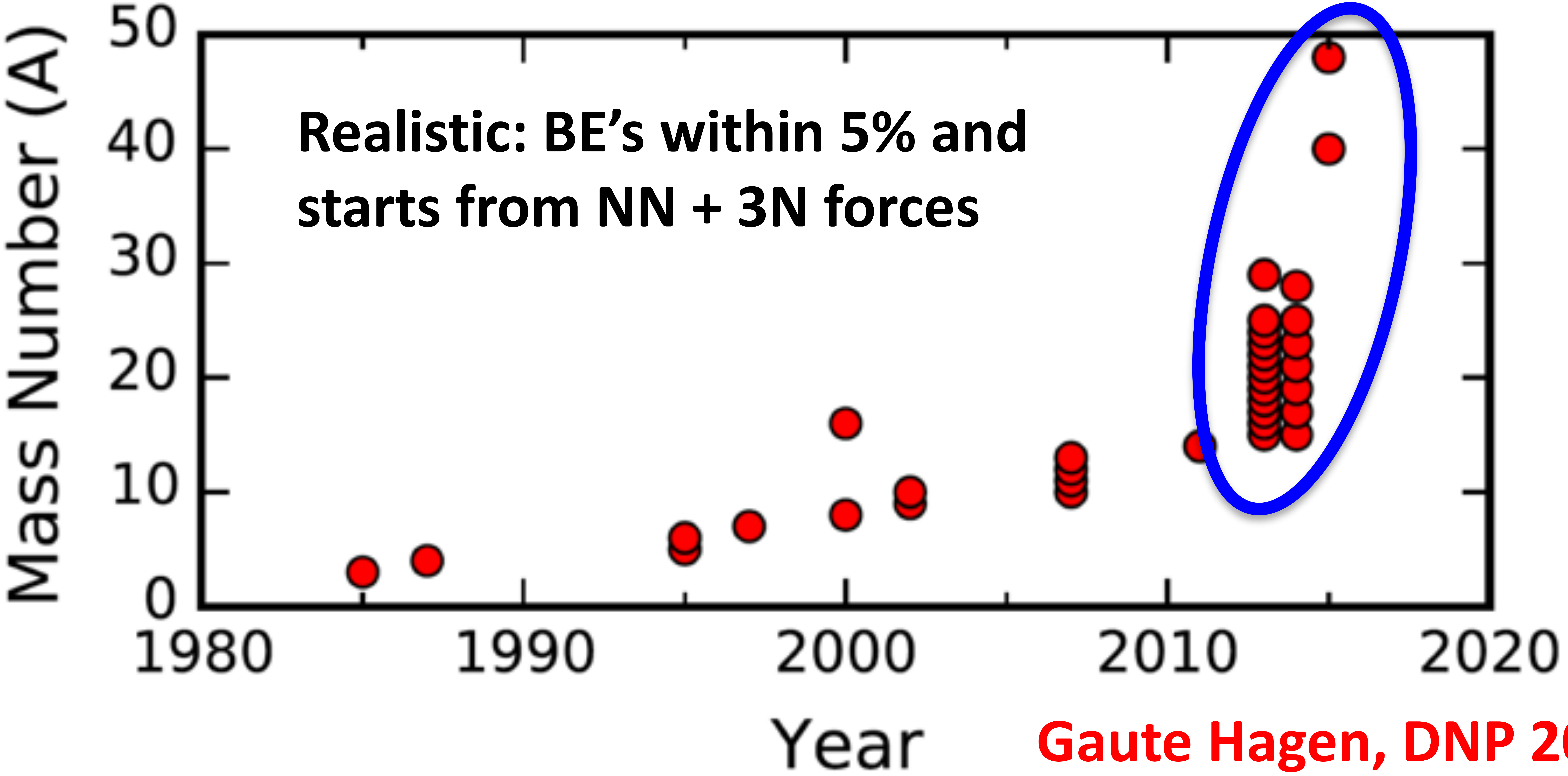

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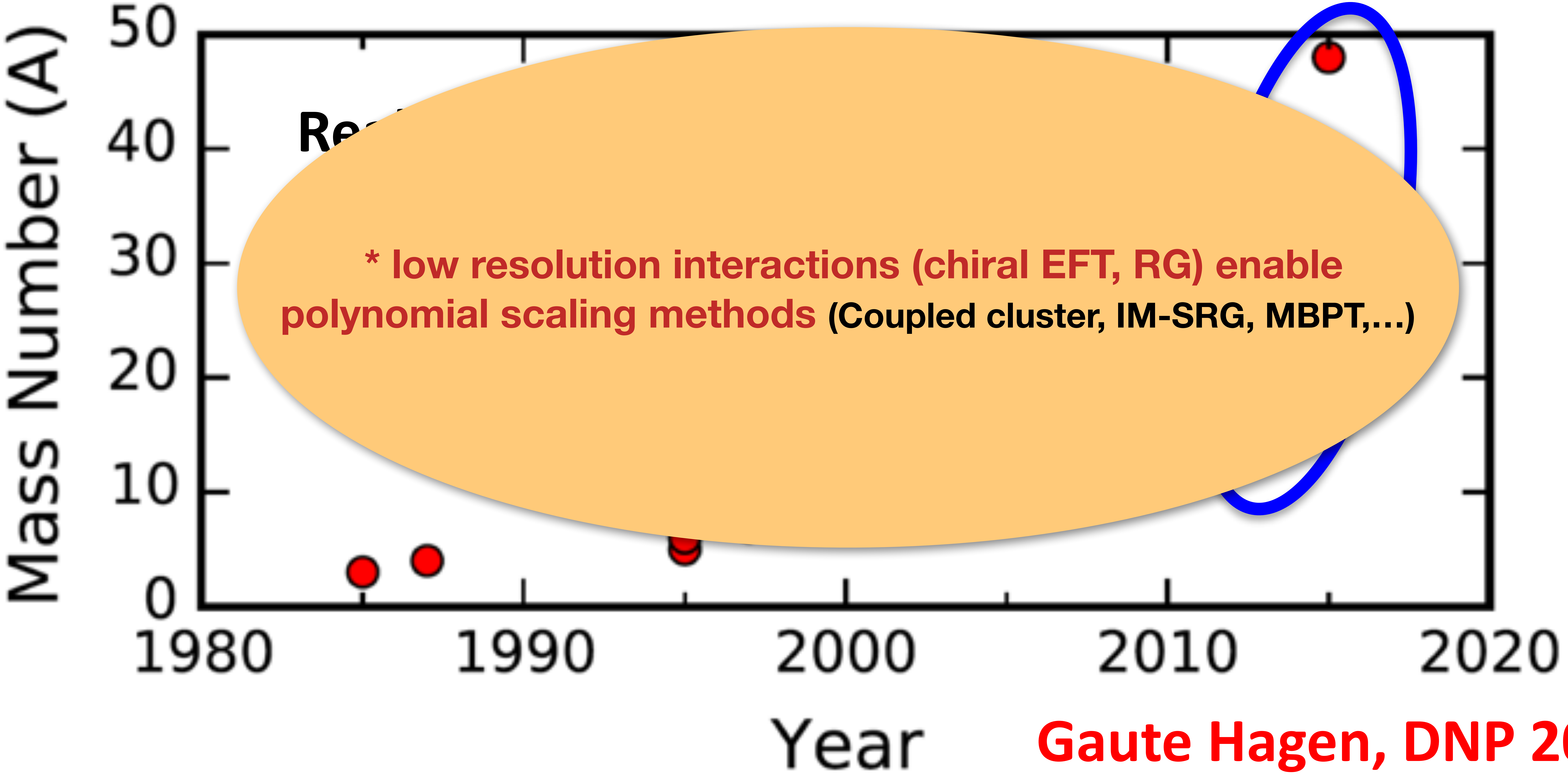

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Modern SRC Phenomenology (2000's - present)

Experiments at BNL and JLab to detect knocked-out nucleons from an SRC *pair*

Breakup the pair =>
Detect ***both*** nucleons =>
Reconstruct 'initial' state

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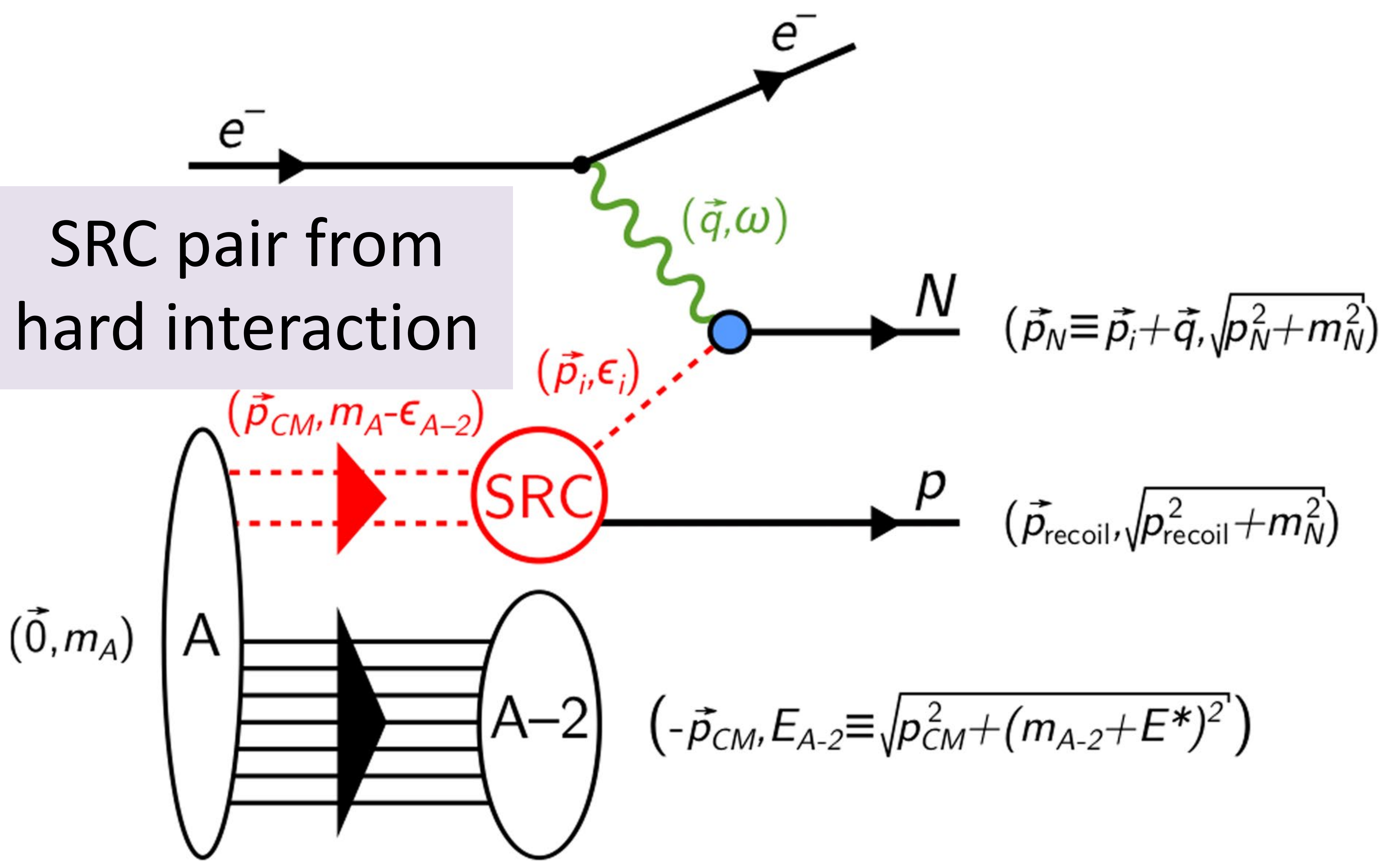
Kinematics chosen to minimize ambiguities from MECs, FSI, etc.

SRC pair from hard interaction



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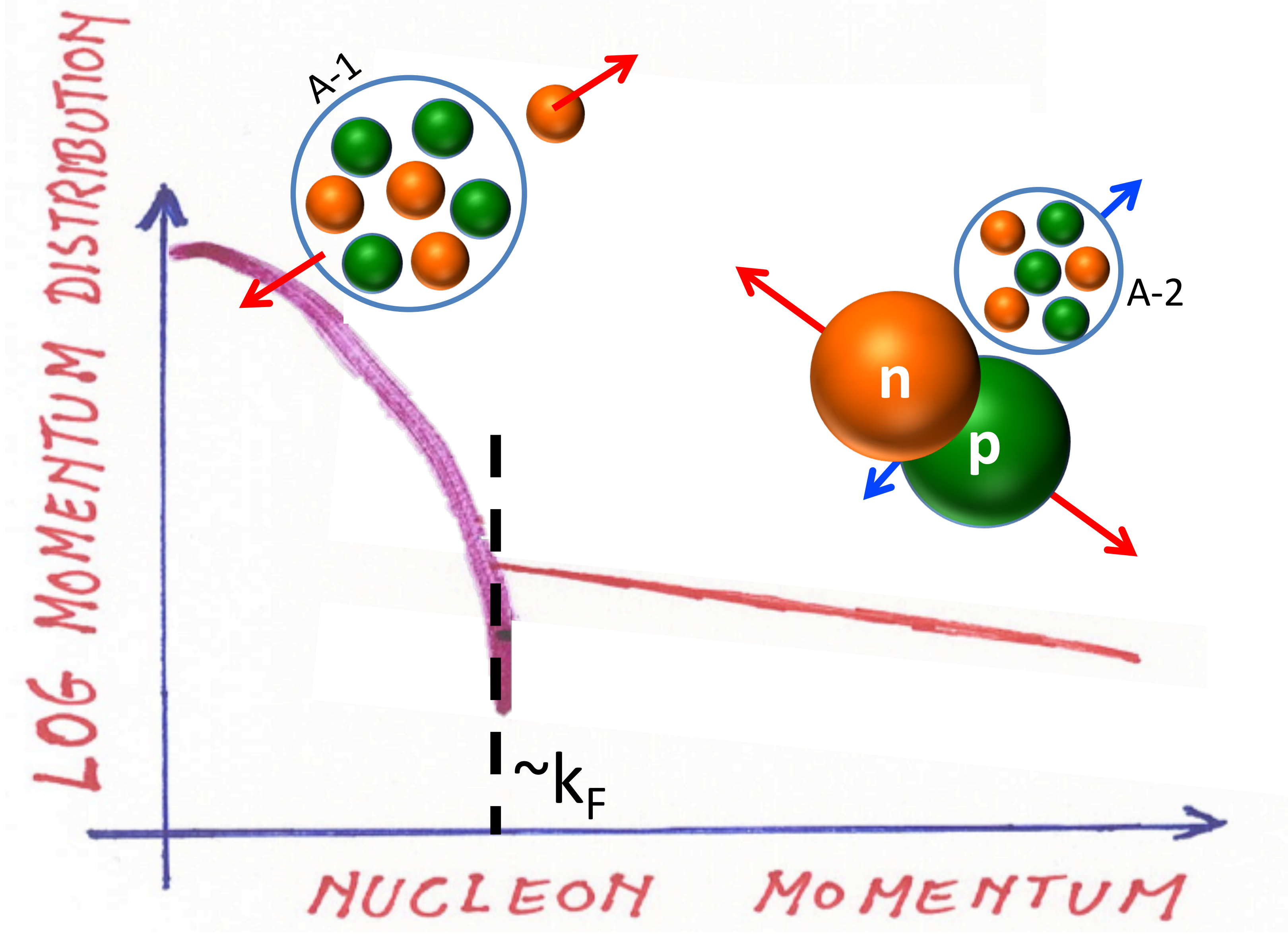
Interpretation (high resolution picture)

2 regions of momenta in nuclei

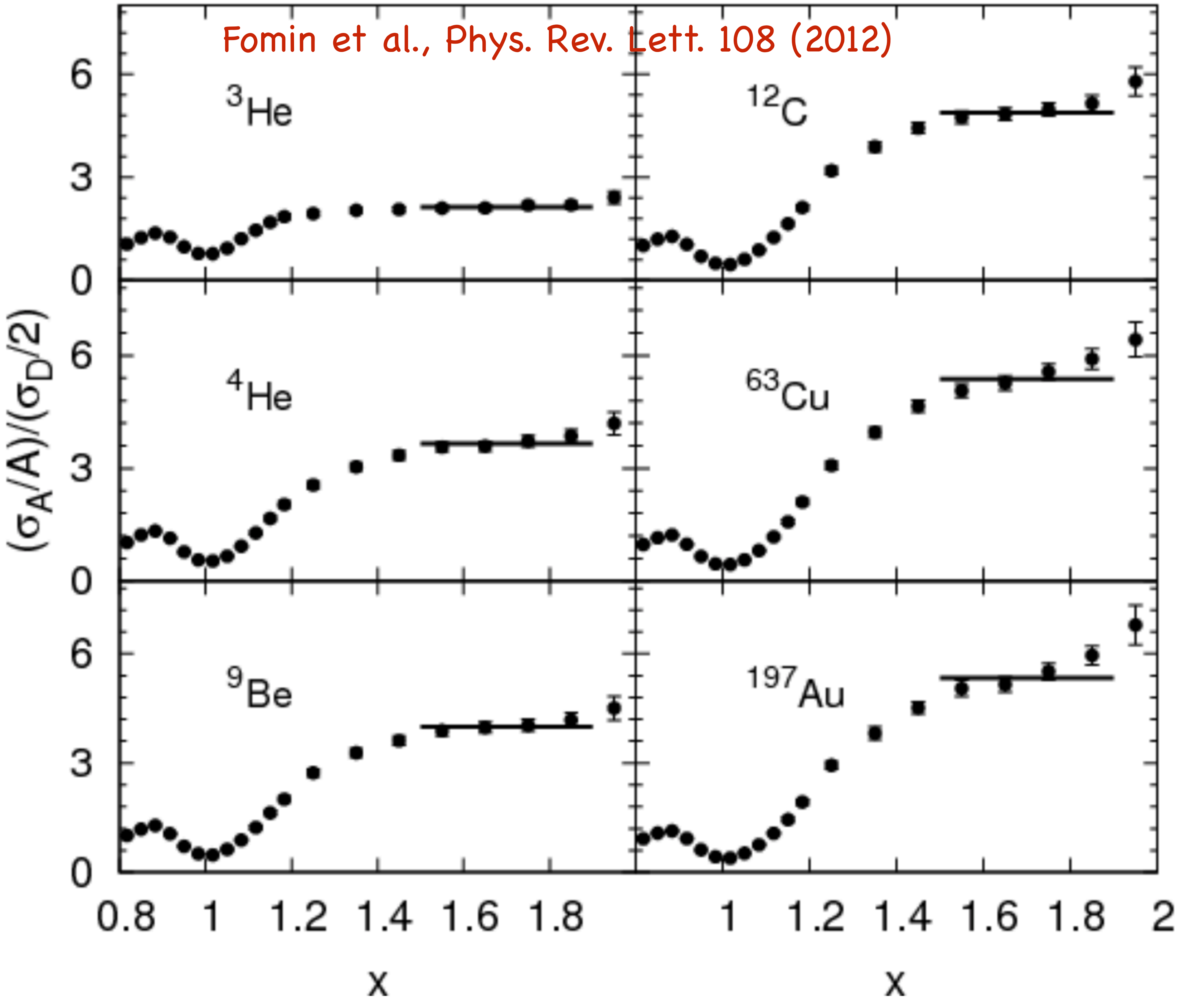
~ 20% of nucleons in SRC pairs
~ 70% of KE from SRC pairs



Modern SRC Phenomenology (2000's - present)

1) Universal high-momentum tails

inclusive ratios



$$a_2(A) = \frac{2}{A}\frac{\sigma_A(x_B, Q^2)}{\sigma_d(x_B, Q^2)}$$

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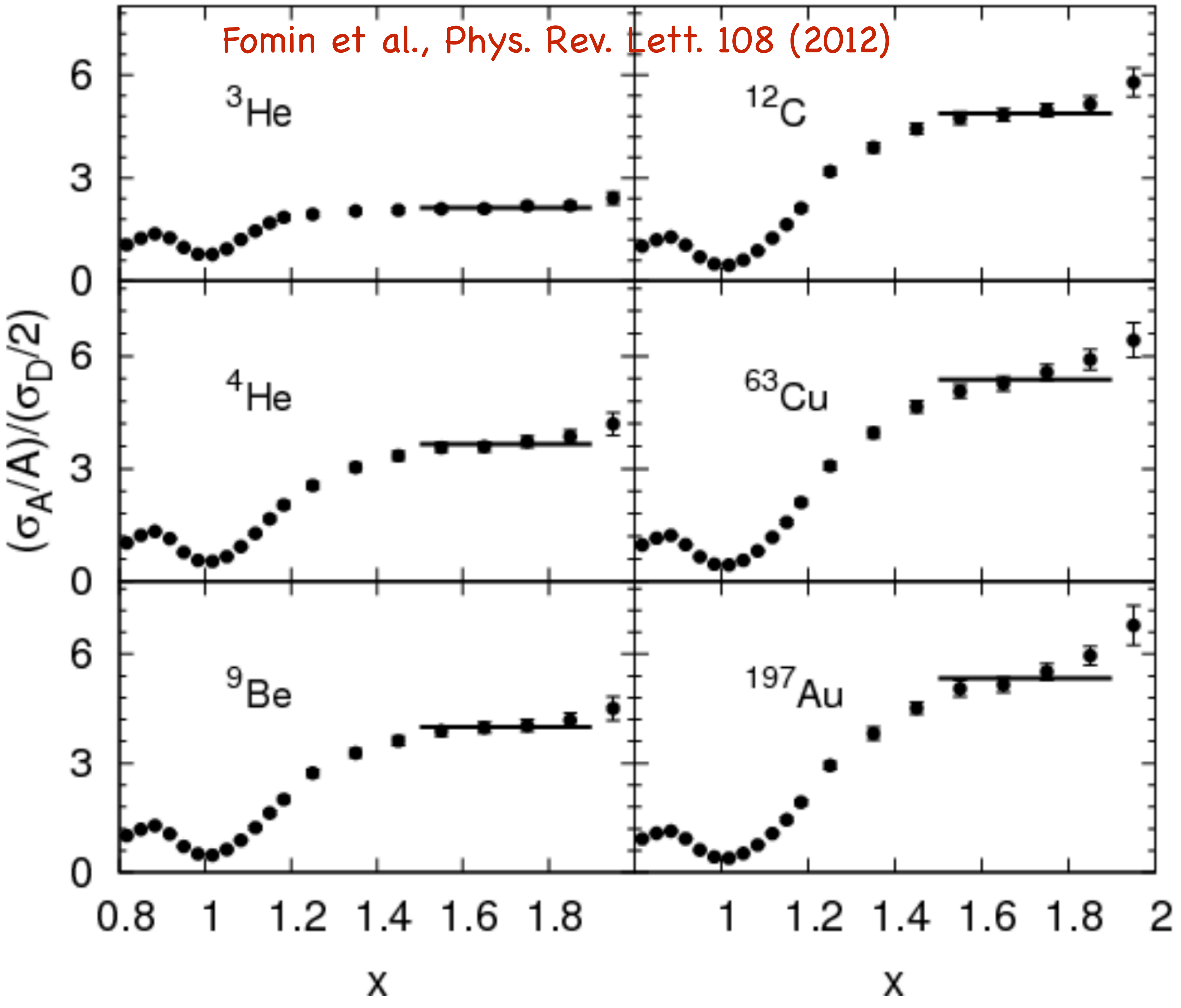

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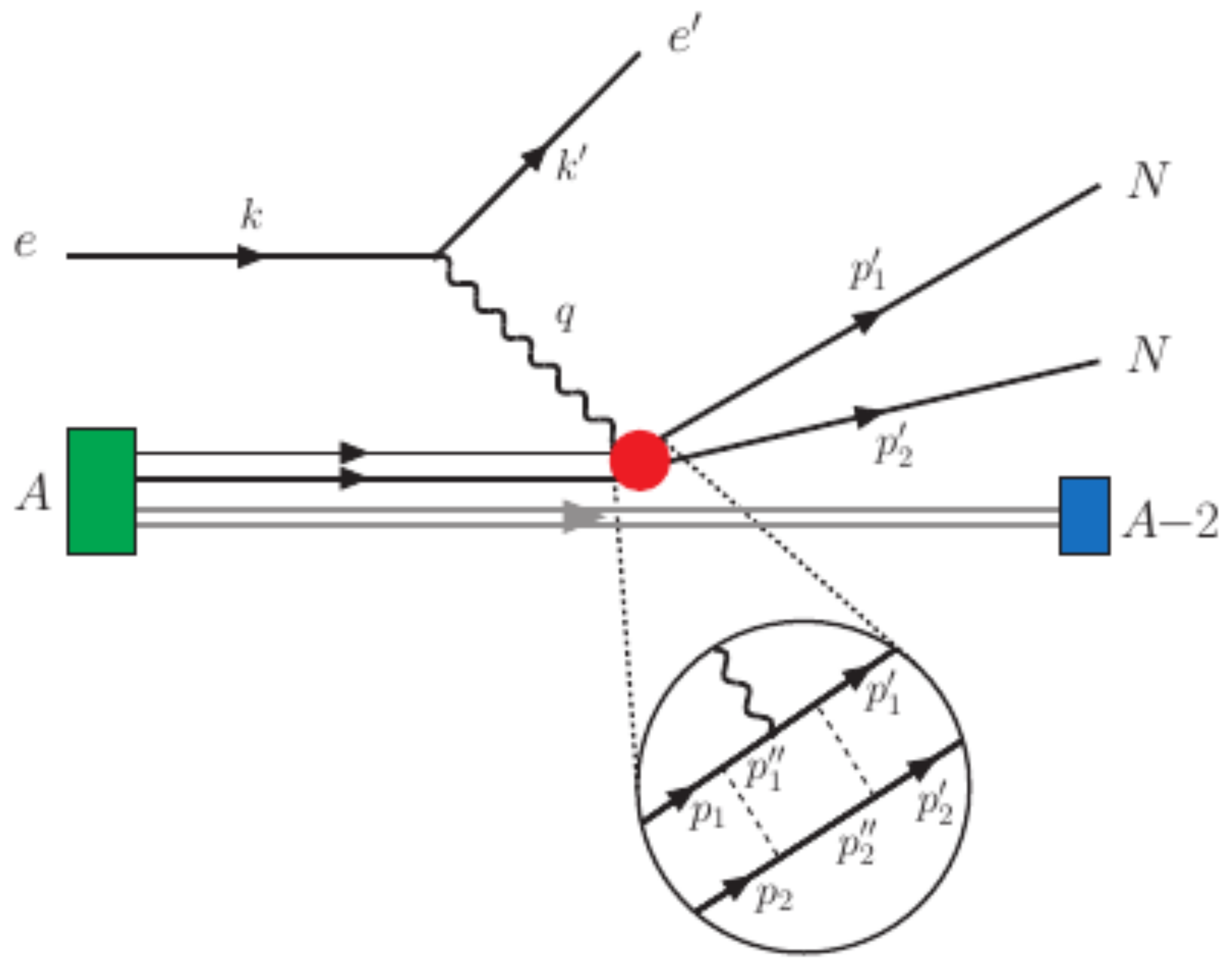

SRC interpretation:
NN interaction scatters pair $p_1, p_2 < k_F$
to intermediate-state momenta $>> k_F$
which are then knocked out by photon

Modern SRC Phenomenology (2000's - present)

NSCL FRIB

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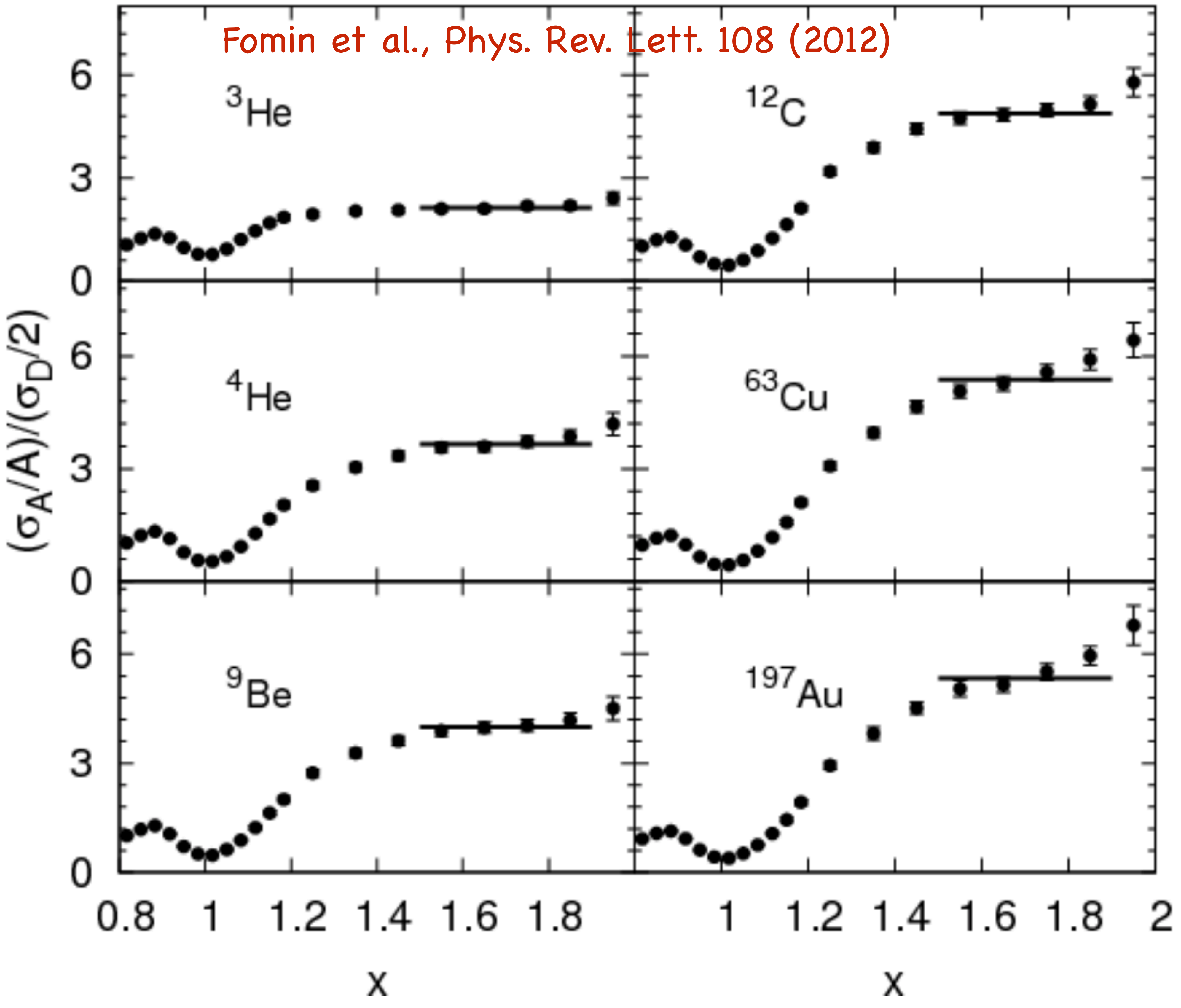

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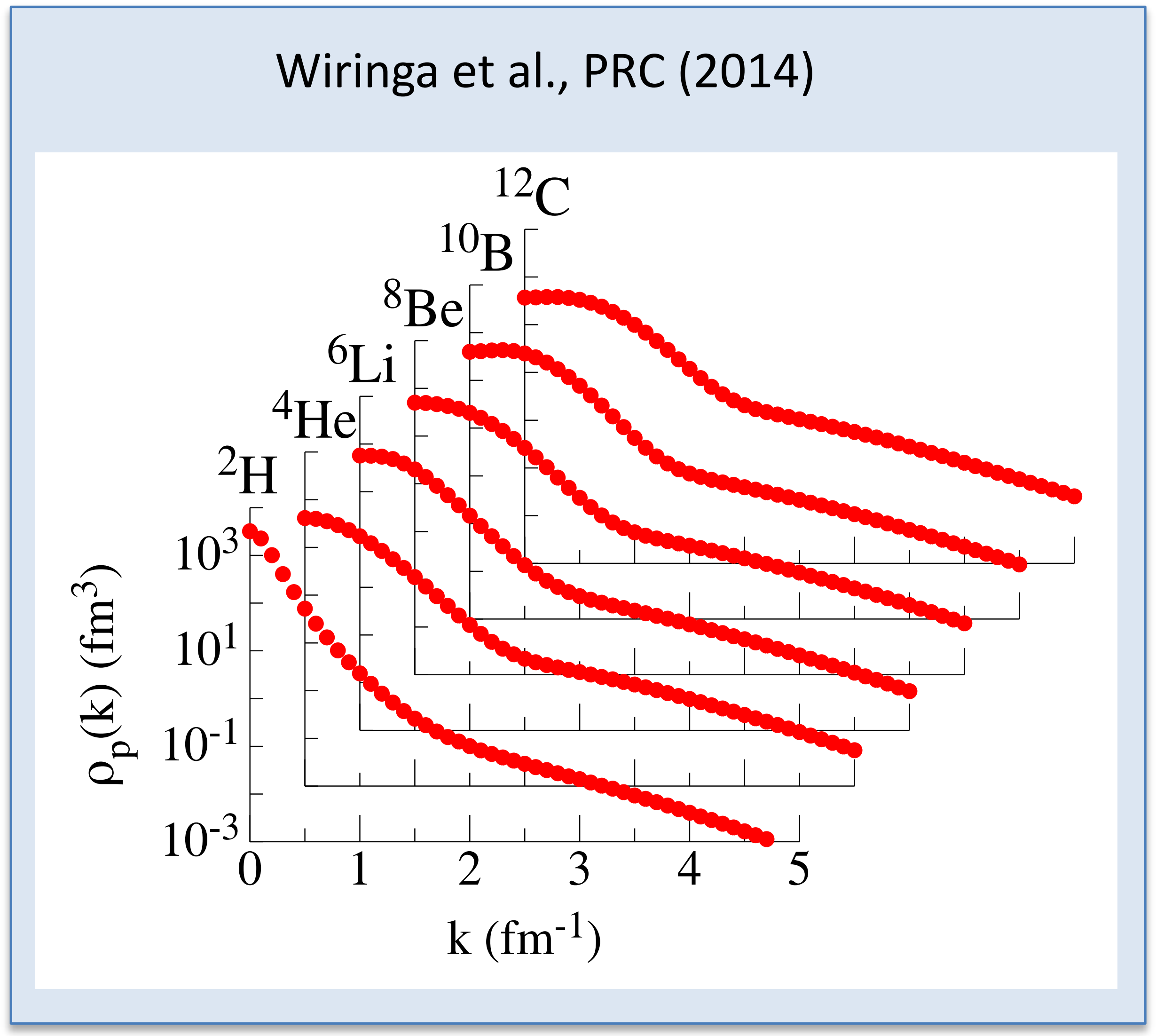
plateaus in x => universal (all nuclei) high-q momentum distributions



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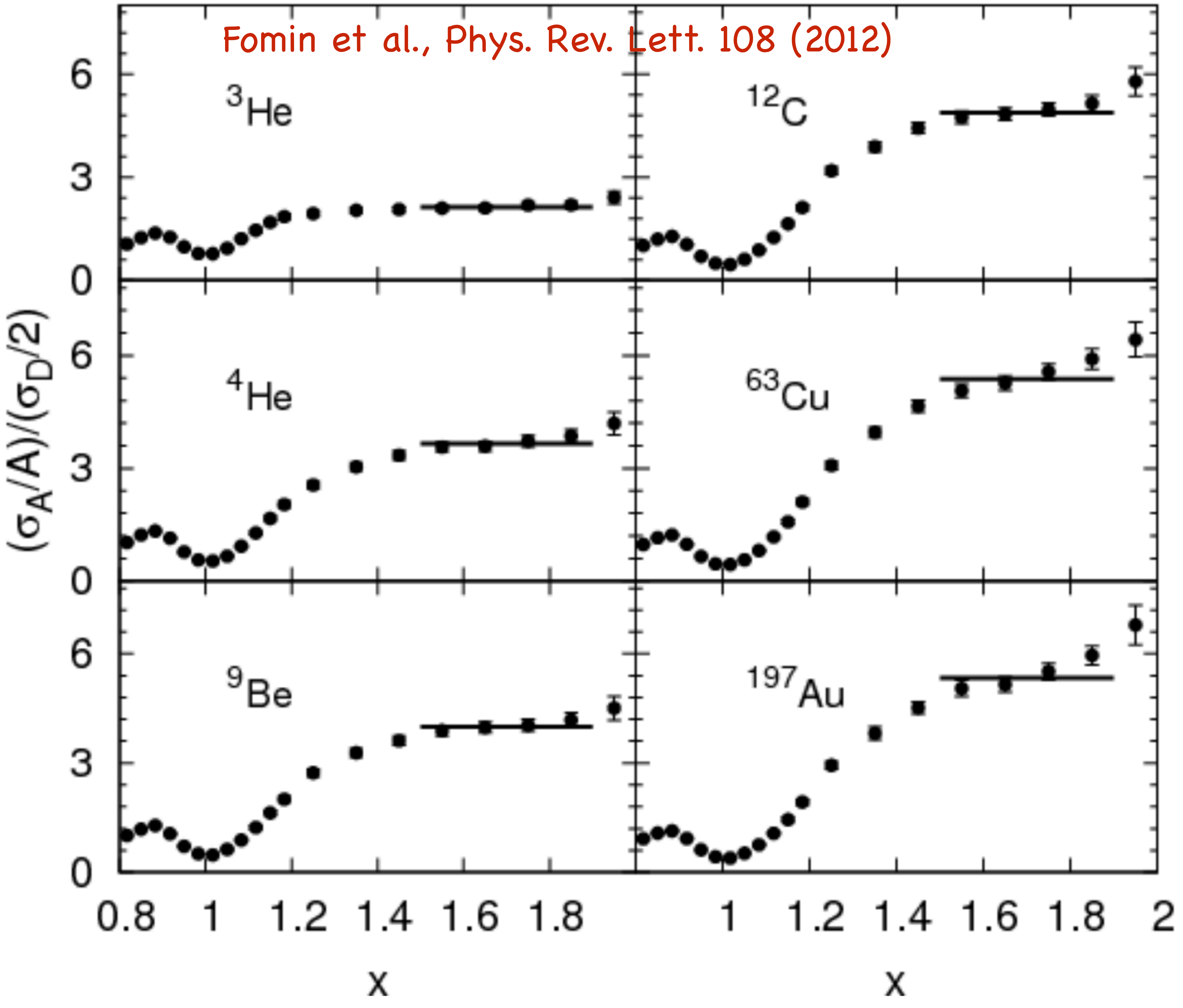

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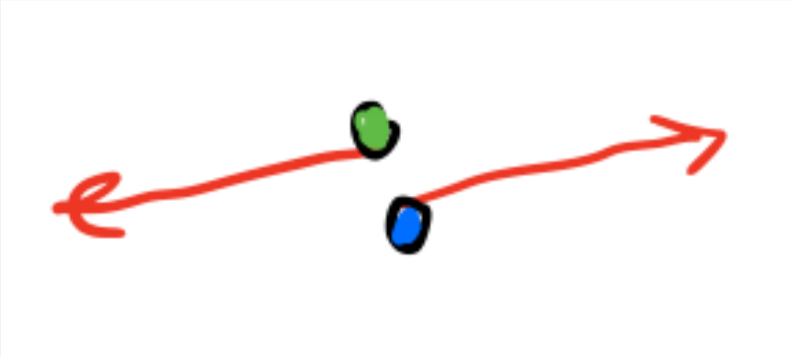
plateaus in x => universal (all nuclei) high-q momentum distributions

relative plateau height => relative prob. of finding 2N SRC

Modern SRC Phenomenology (2000's - present)

2) Kinematics of knocked-out nucleons



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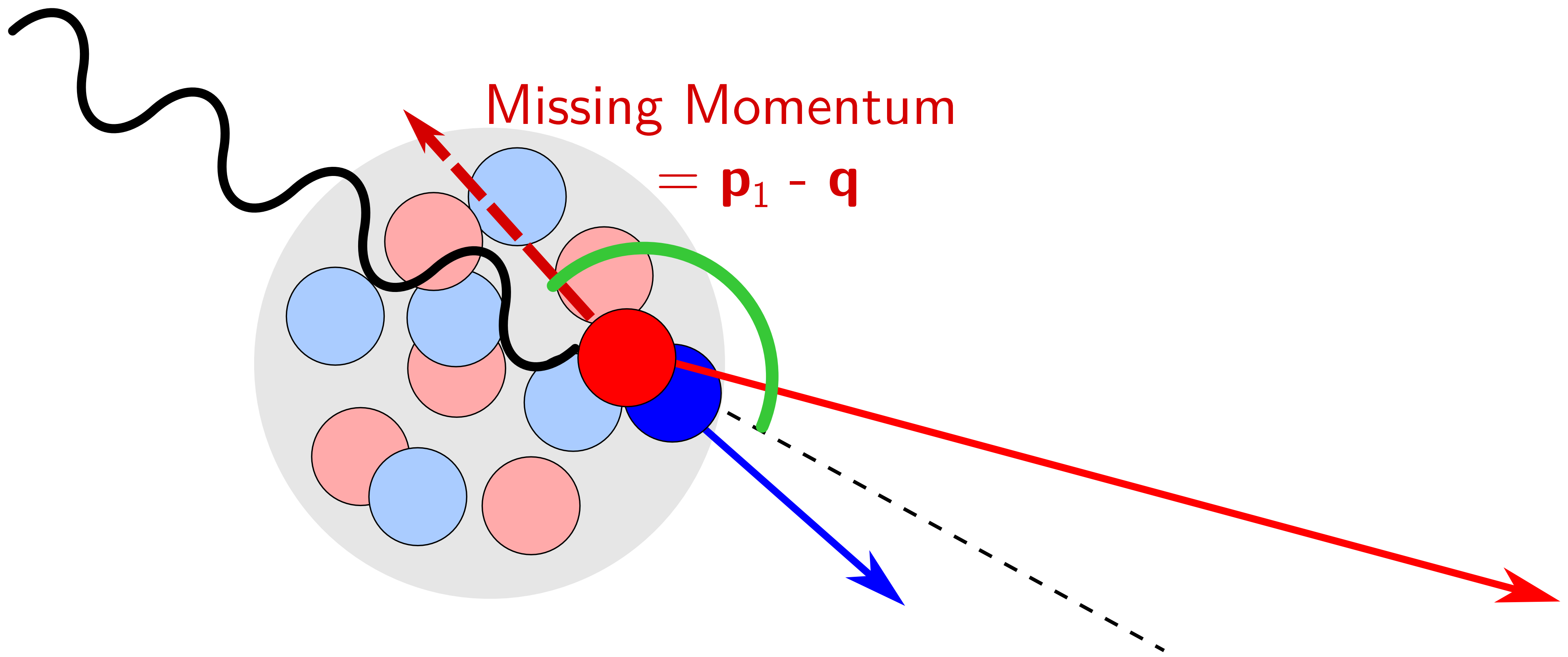

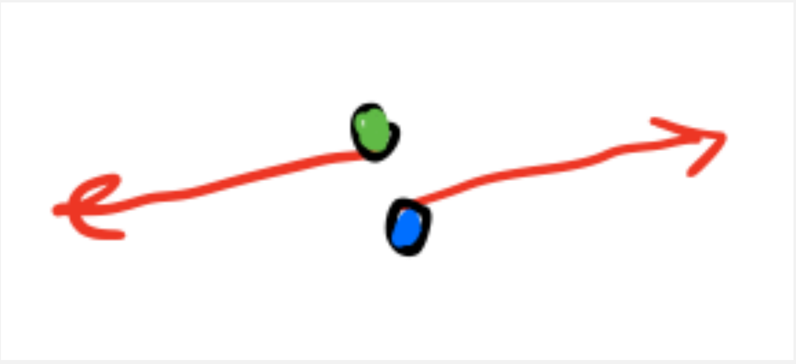

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(relative s-wave pairs)

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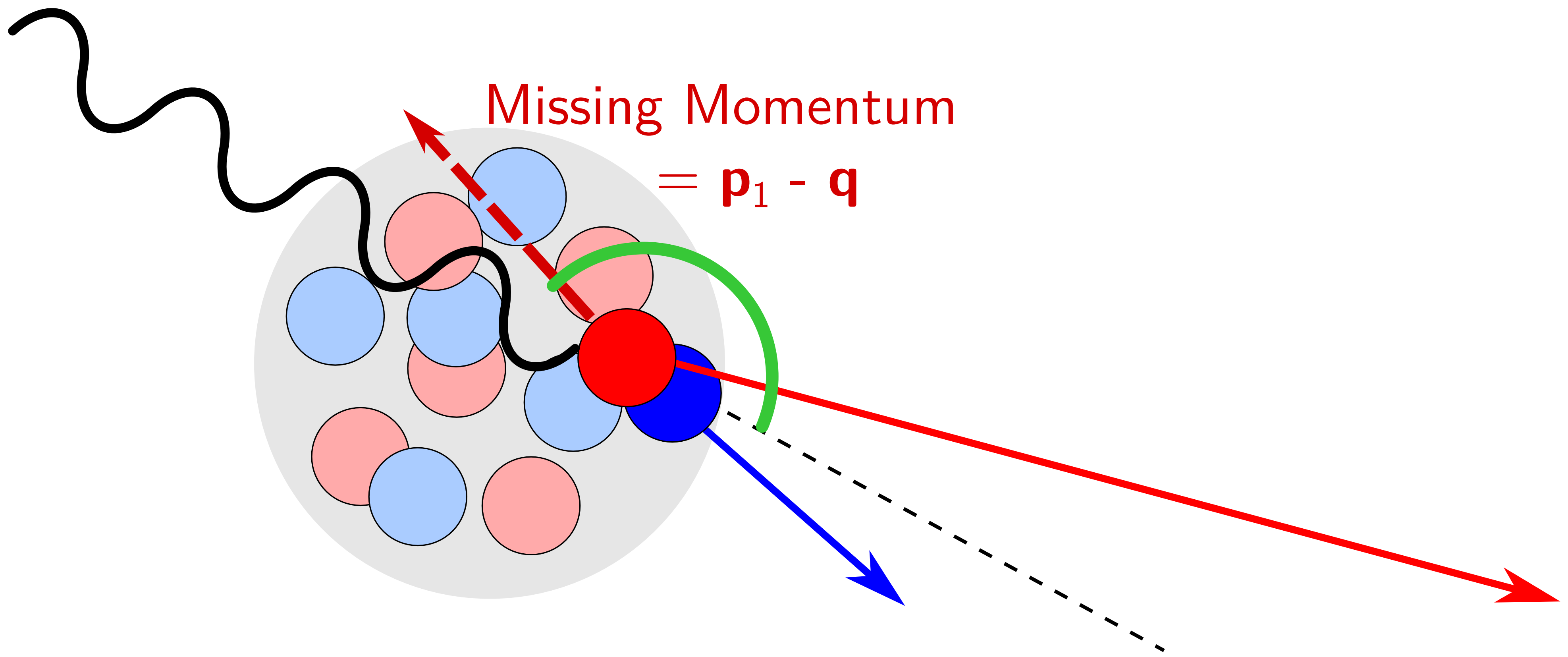

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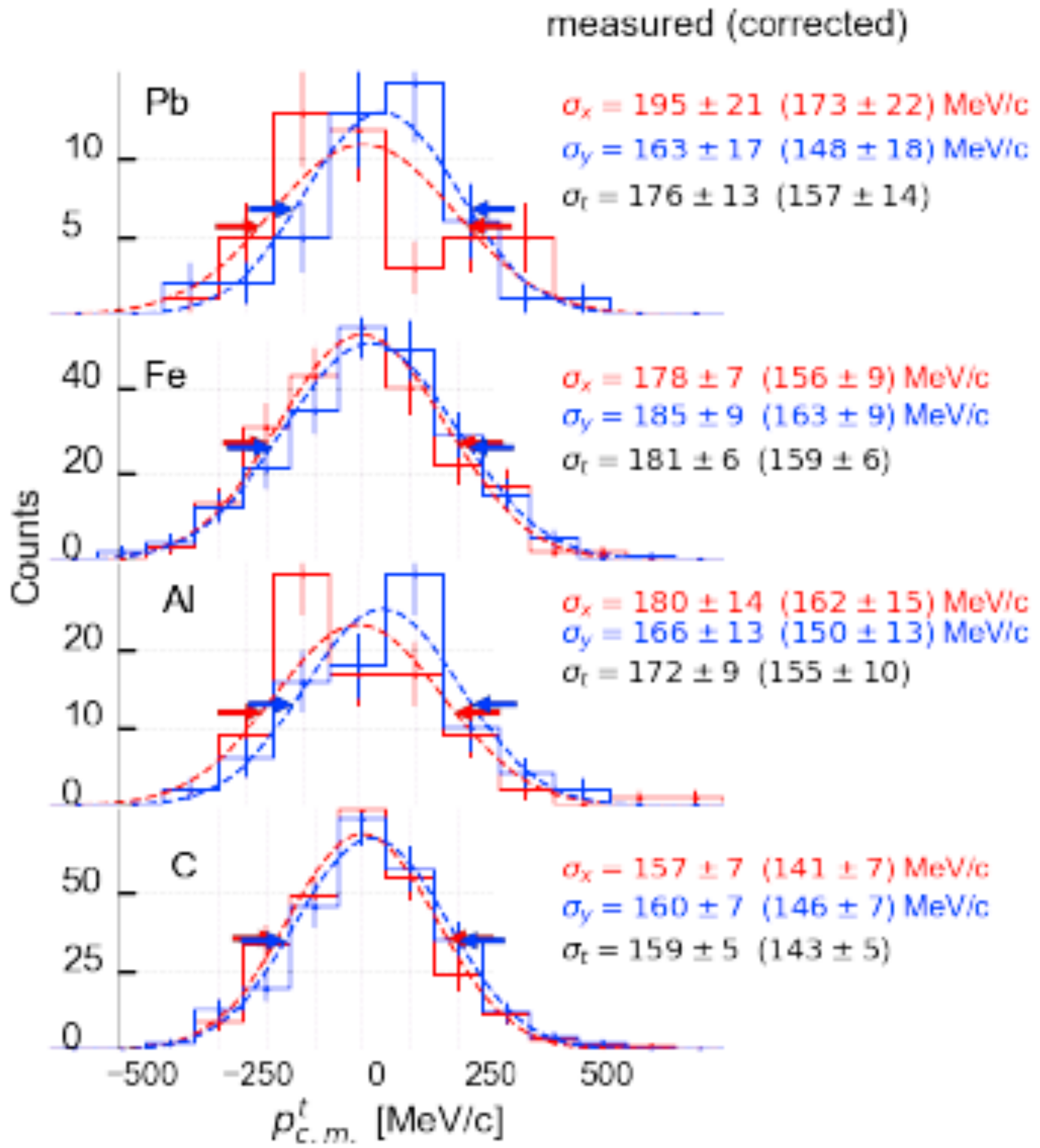

pair CM momentum distribution gaussian of width ~ k_F

Modern SRC Phenomenology (2000's - present)

3) np dominance at intermediate (300-500 MeV) relative momenta

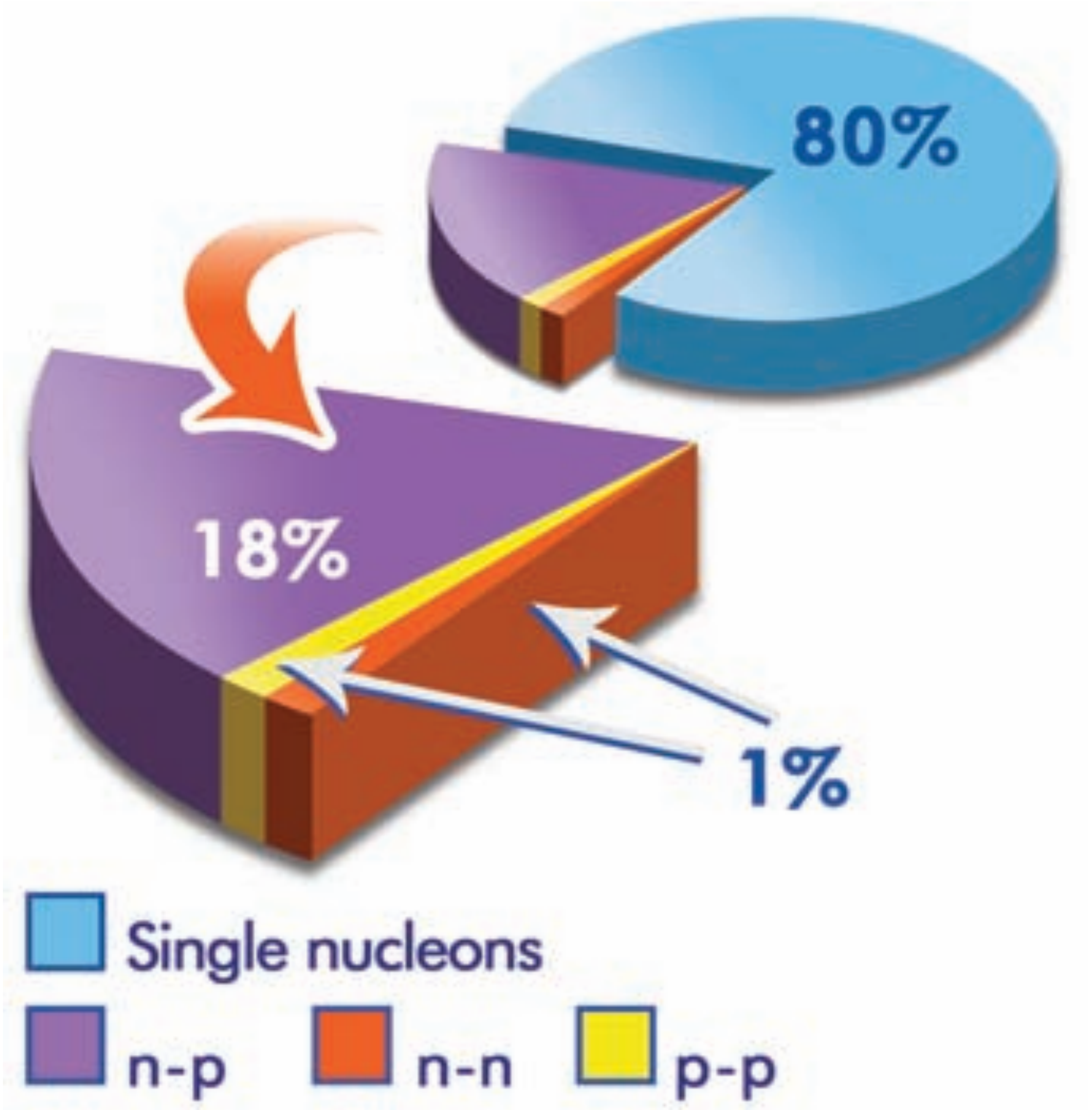

Fig. 3. The average fraction of nucleons in the various initial-state configurations of ^{12}C.

R. Subedi et al., Science (2008)

20% of nucleons in SRC pairs
but mostly neutron-proton

Modern SRC Phenomenology (2000's - present)

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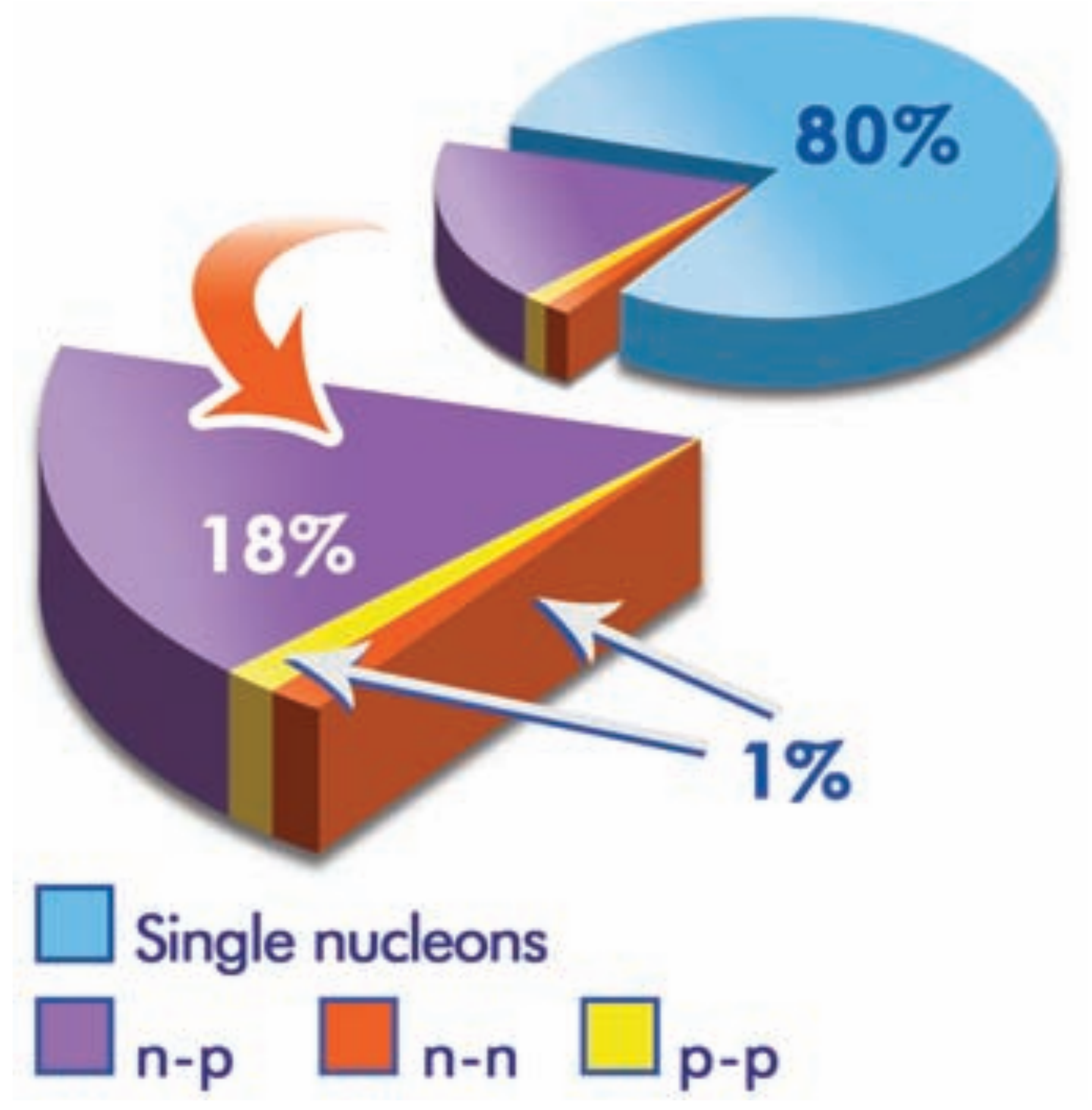

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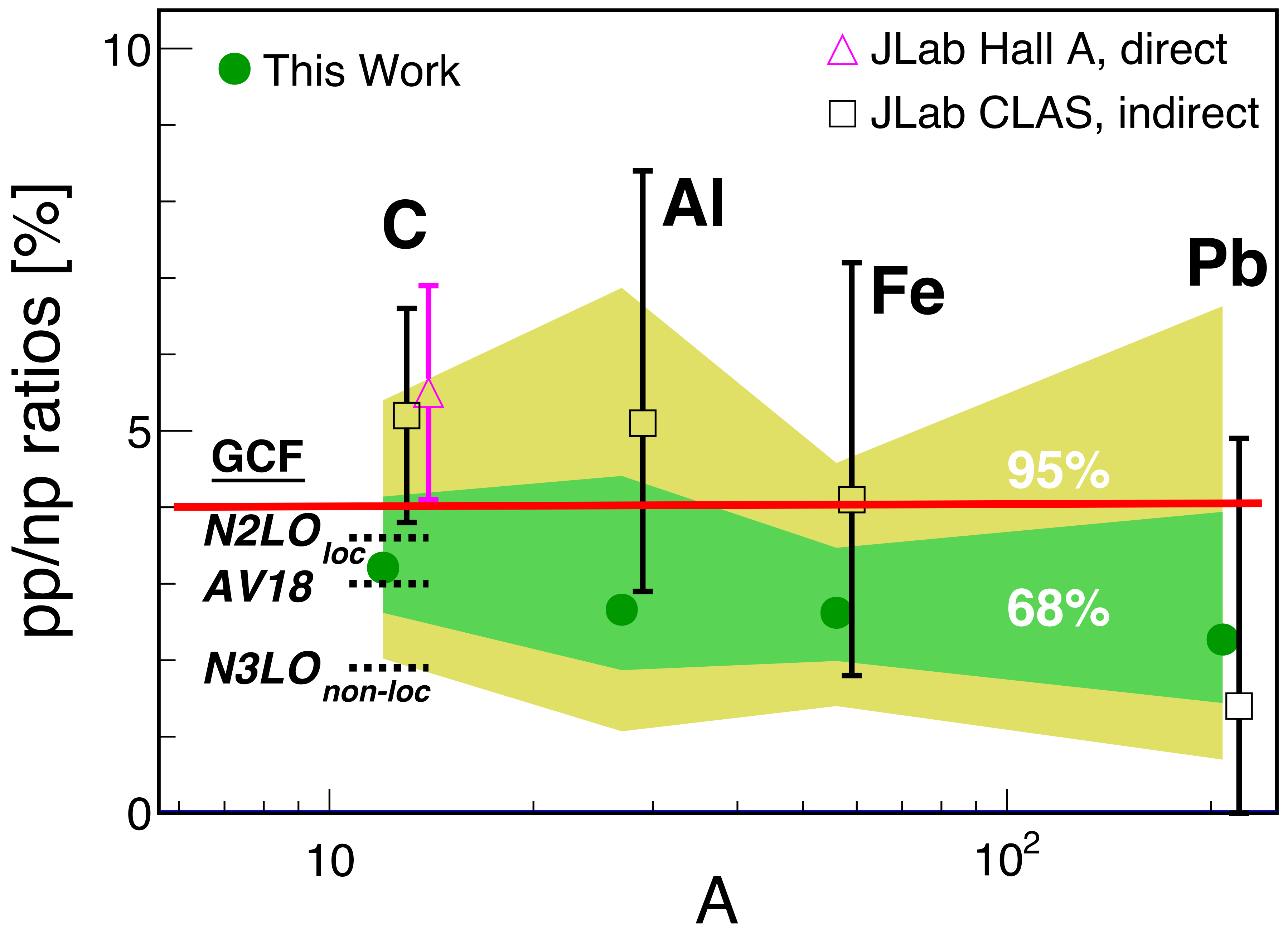


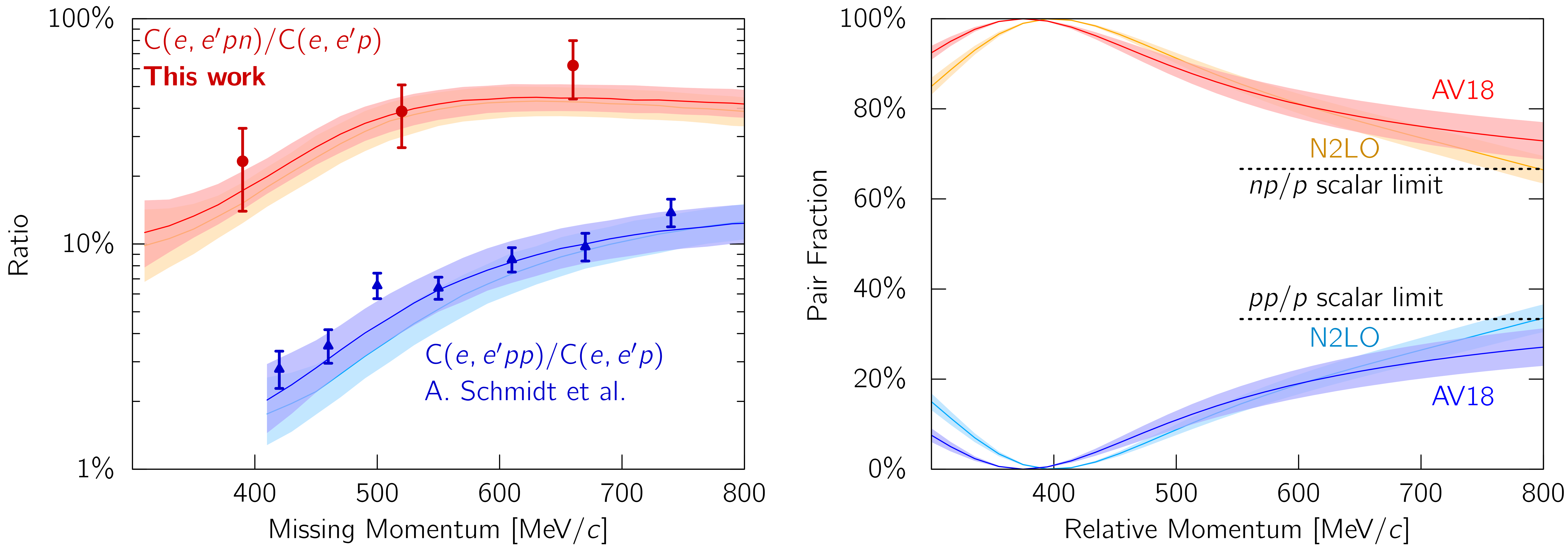
~ 4%

Proton-proton

Duer, PRL (2019); Duer, Nature (2018); Hen, Science (2014); Korover, PRL (2014); Subedi, Science (2008); Shneor, PRL (2007); Piasetzky, PRL (2006); Tang, PRL (2003); Review: Hen RMP (2017);

Modern SRC Phenomenology (2000's - present)

4) transition to scalar counting at higher relative momentum



np dominance goes away at high momenta => probe repulsive core

fraction of SRC pairs (nn,np,pp) agrees with naive pair counting

Modern SRC Phenomenology (2000's - present)

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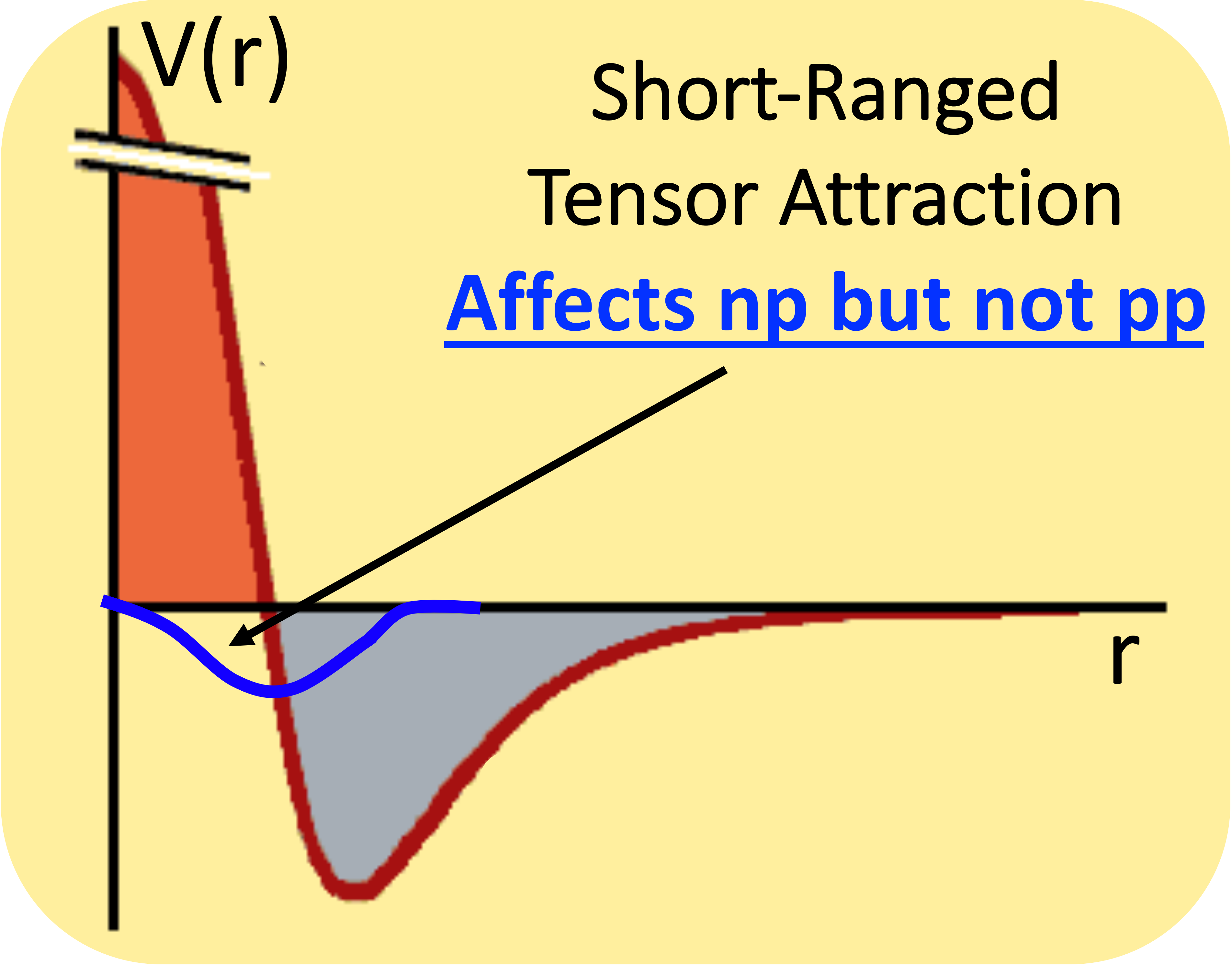

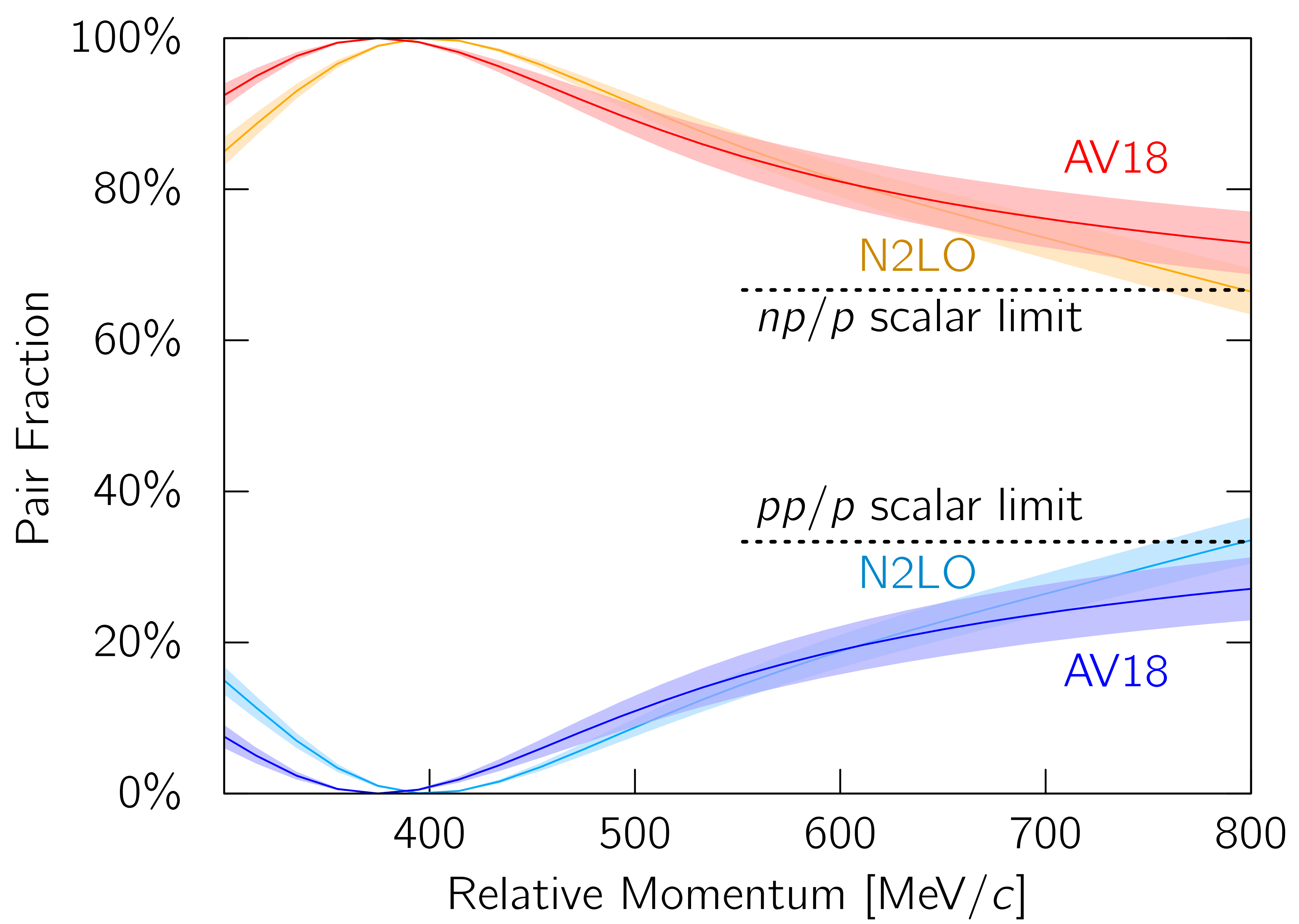

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Modern SRC Phenomenology (2000's - present)

5) Protons "speed up" in neutron rich nuclei

Experiments with increasingly neutron-rich nuclei:

→ excess neutrons correlate with core protons

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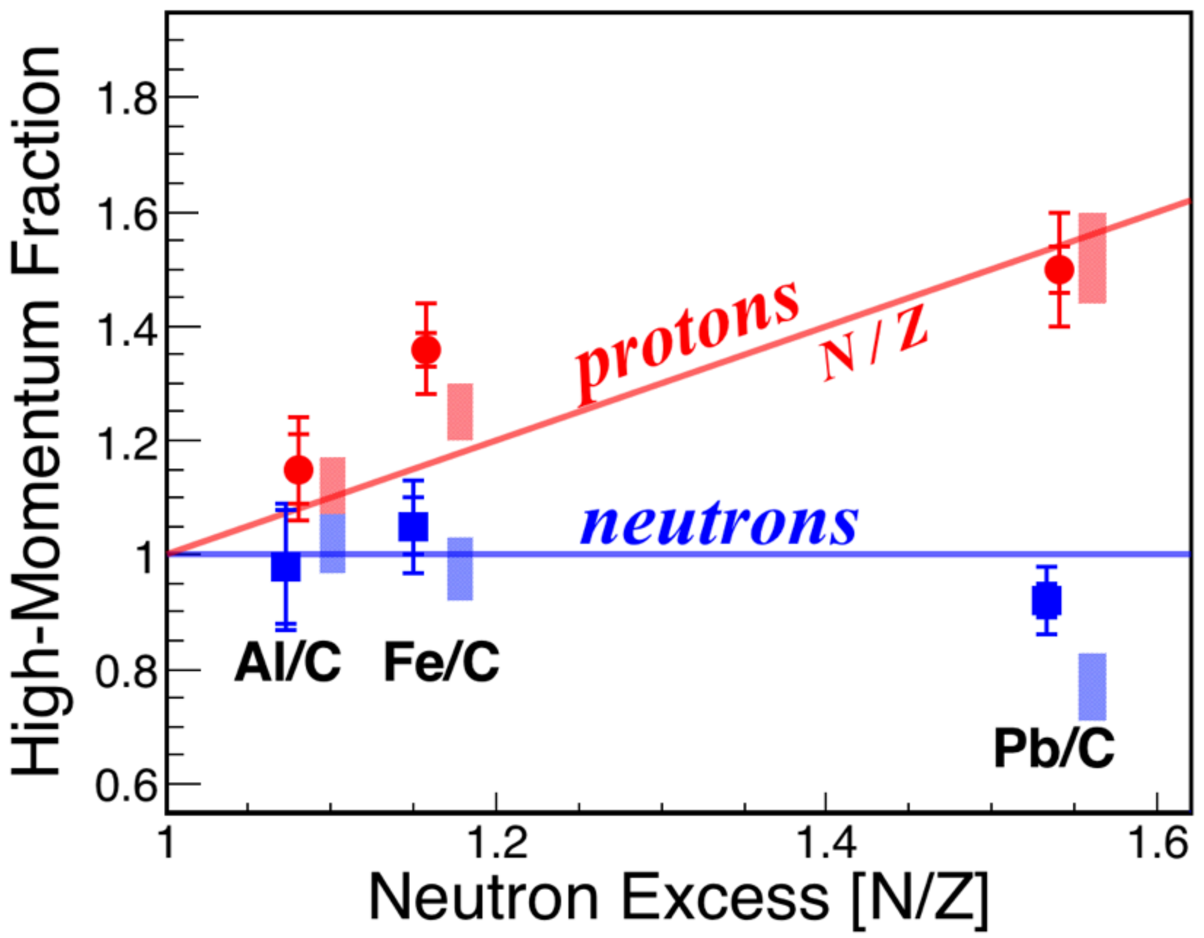
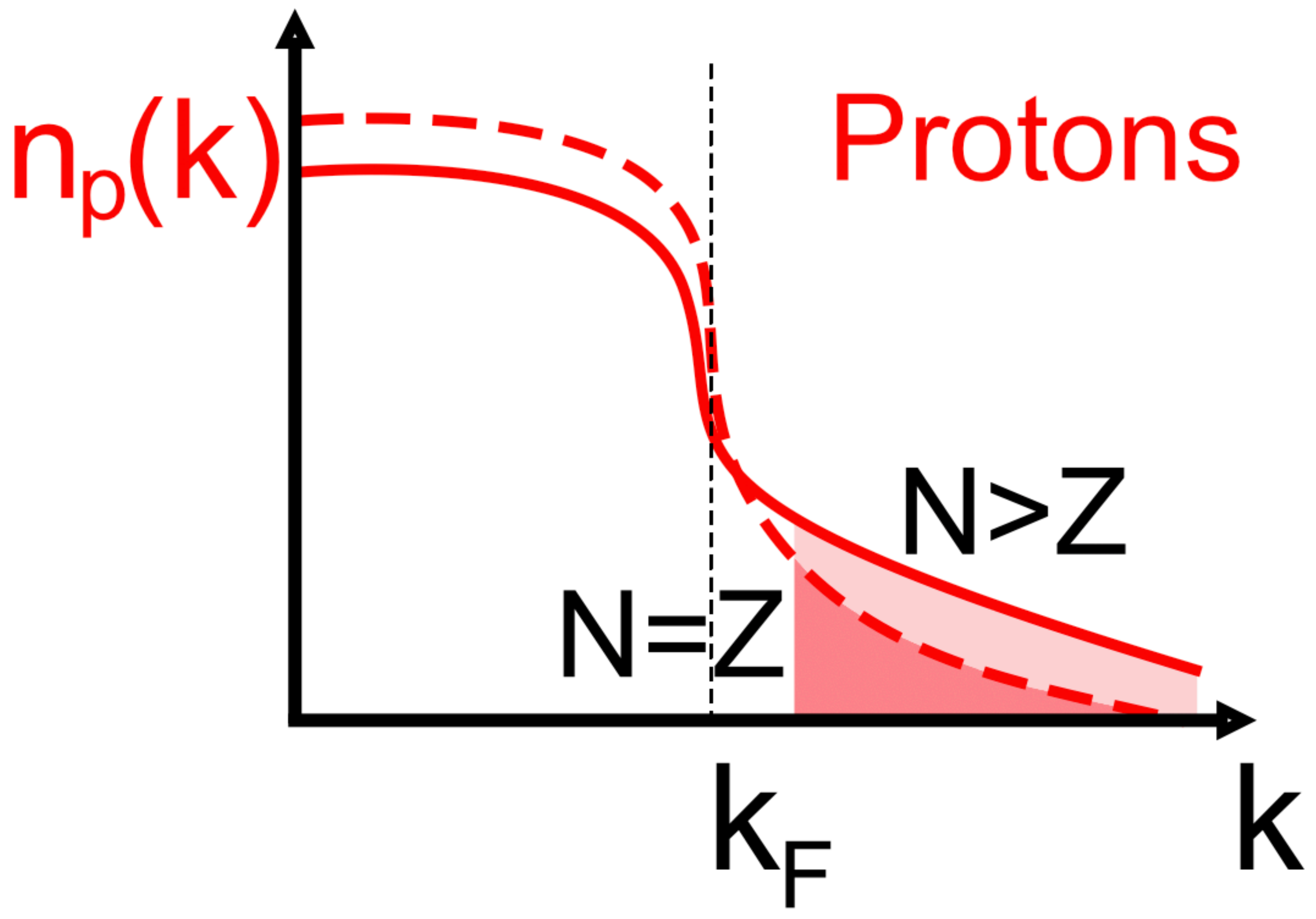
Correlation Probability:
Neutrons saturate Protons grow



Duer Nature (2018)

Protons 'Speed-Up' In Neutron-Rich Nuclei

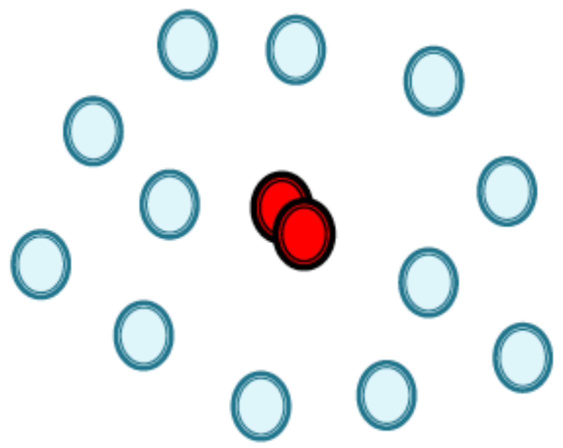


Duer Nature (2018)

Modern SRC Phenomenology (2000's - present)

6) Generalized Contact Formalism (GCF)

Cruz-Torres et al. arXiv:1907.03658
and earlier papers of Weiss/Barnea/et al.



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GCF has ***factorized*** small-r / large-k approximation to many-body wave function:

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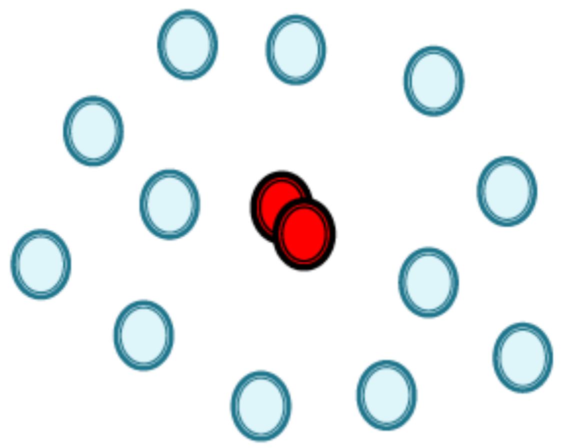

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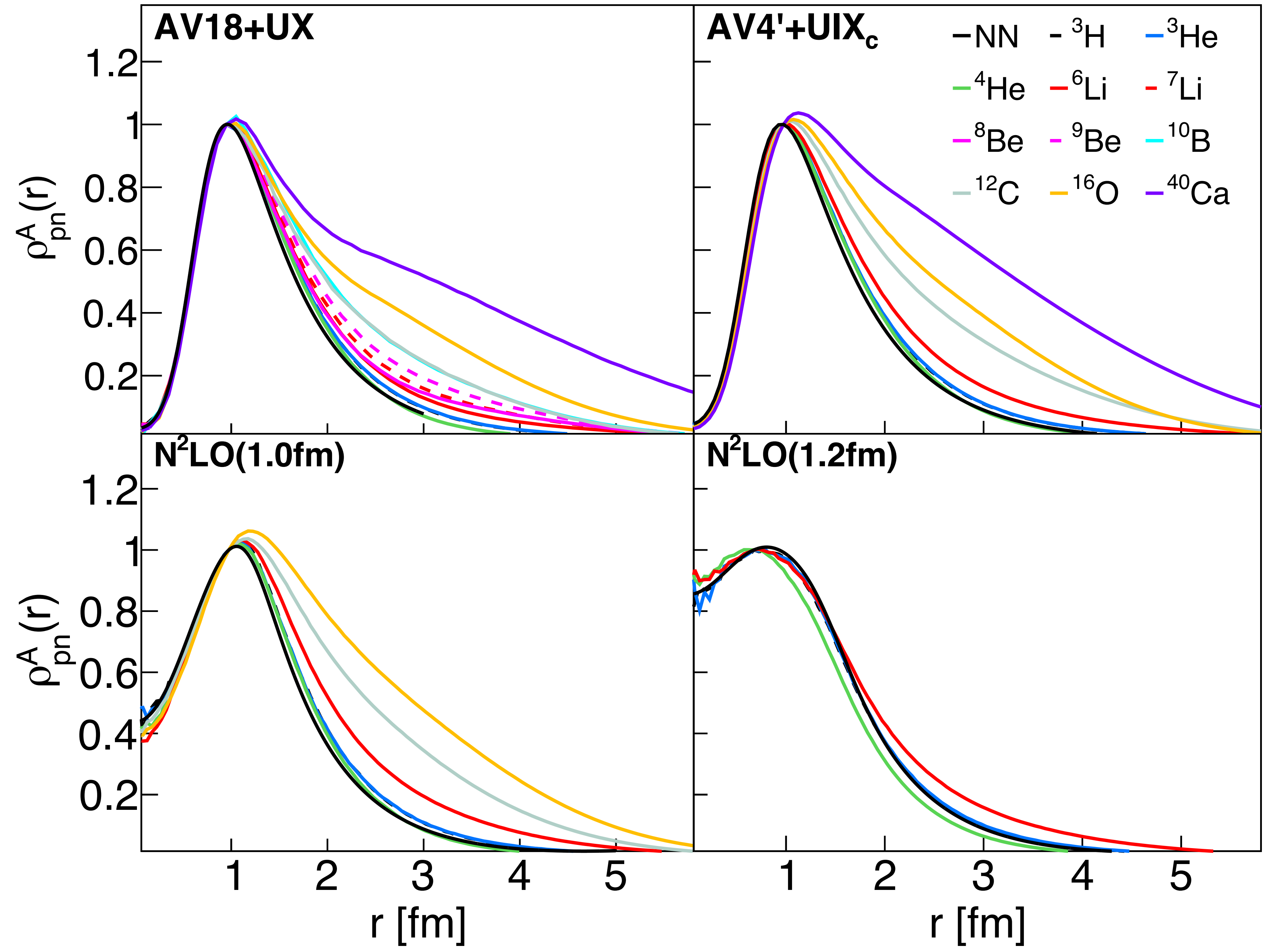
$$\rho_A^{NN,\alpha}(\mathbf{r}) = \langle \Psi | \sum_{i<j} \delta(\mathbf{r} - \mathbf{r}_{ij}) P_\alpha | \Psi \rangle$$



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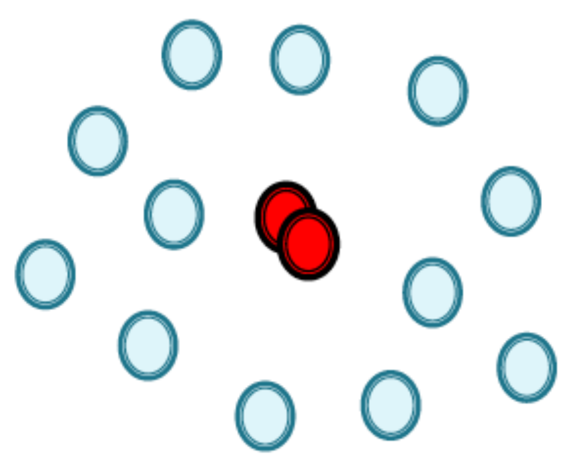

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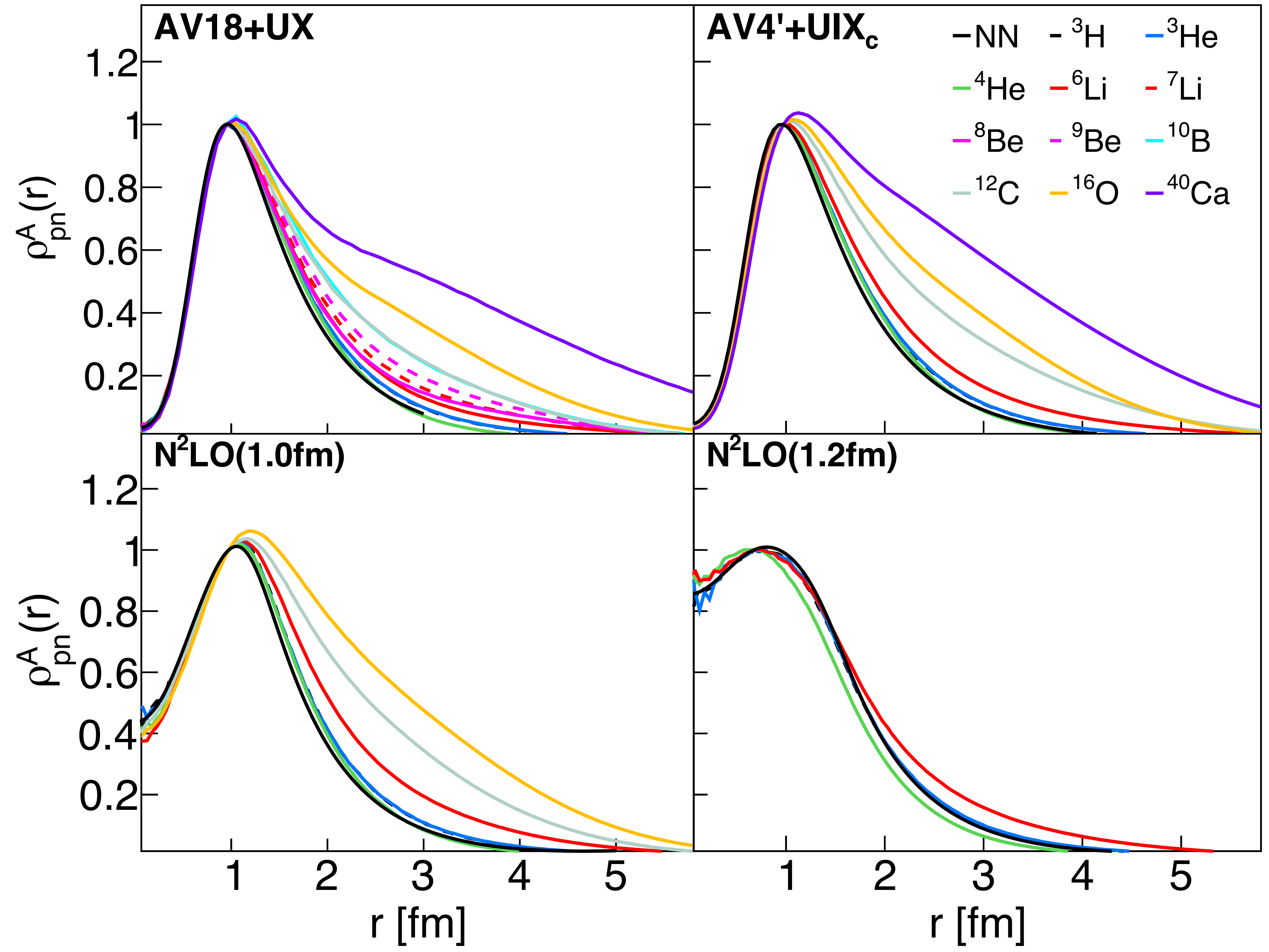

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A-dep scale factors ("nuclear contacts") $C_A \sim < \chi | \chi >$

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NSCL FRIB

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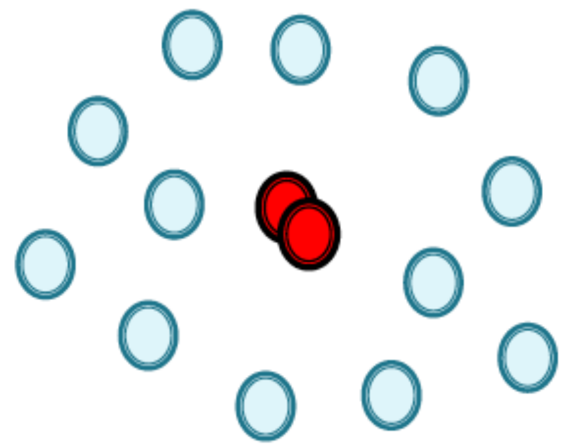

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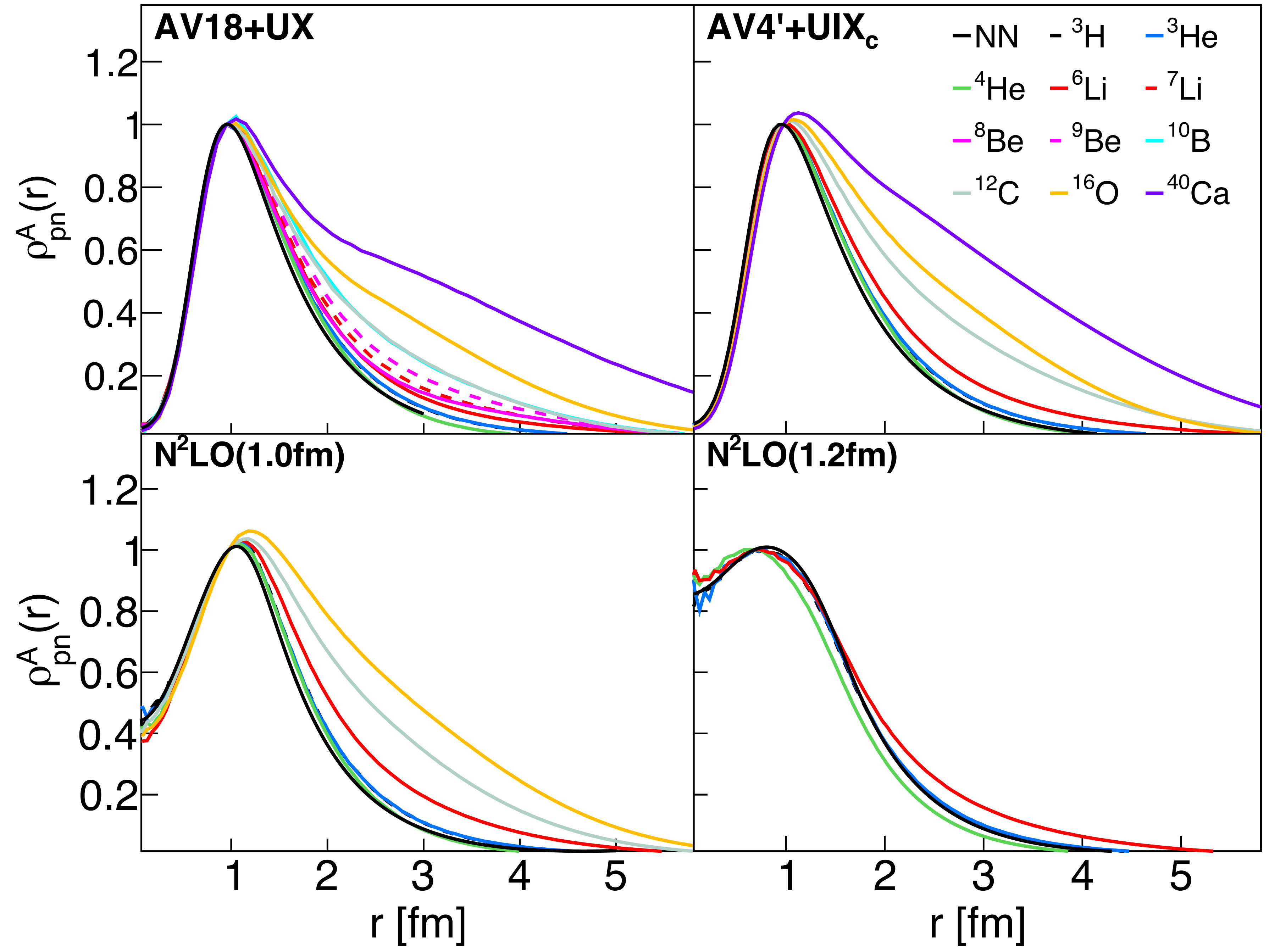

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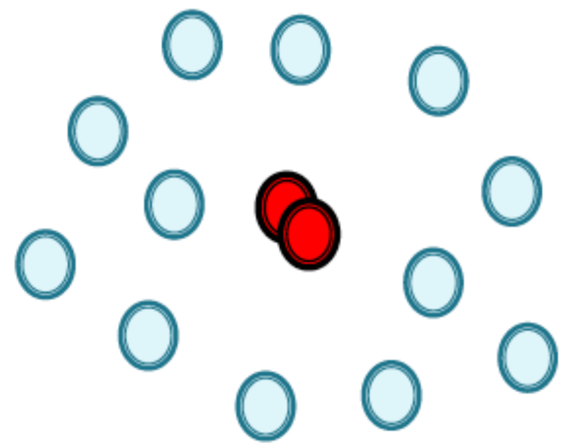

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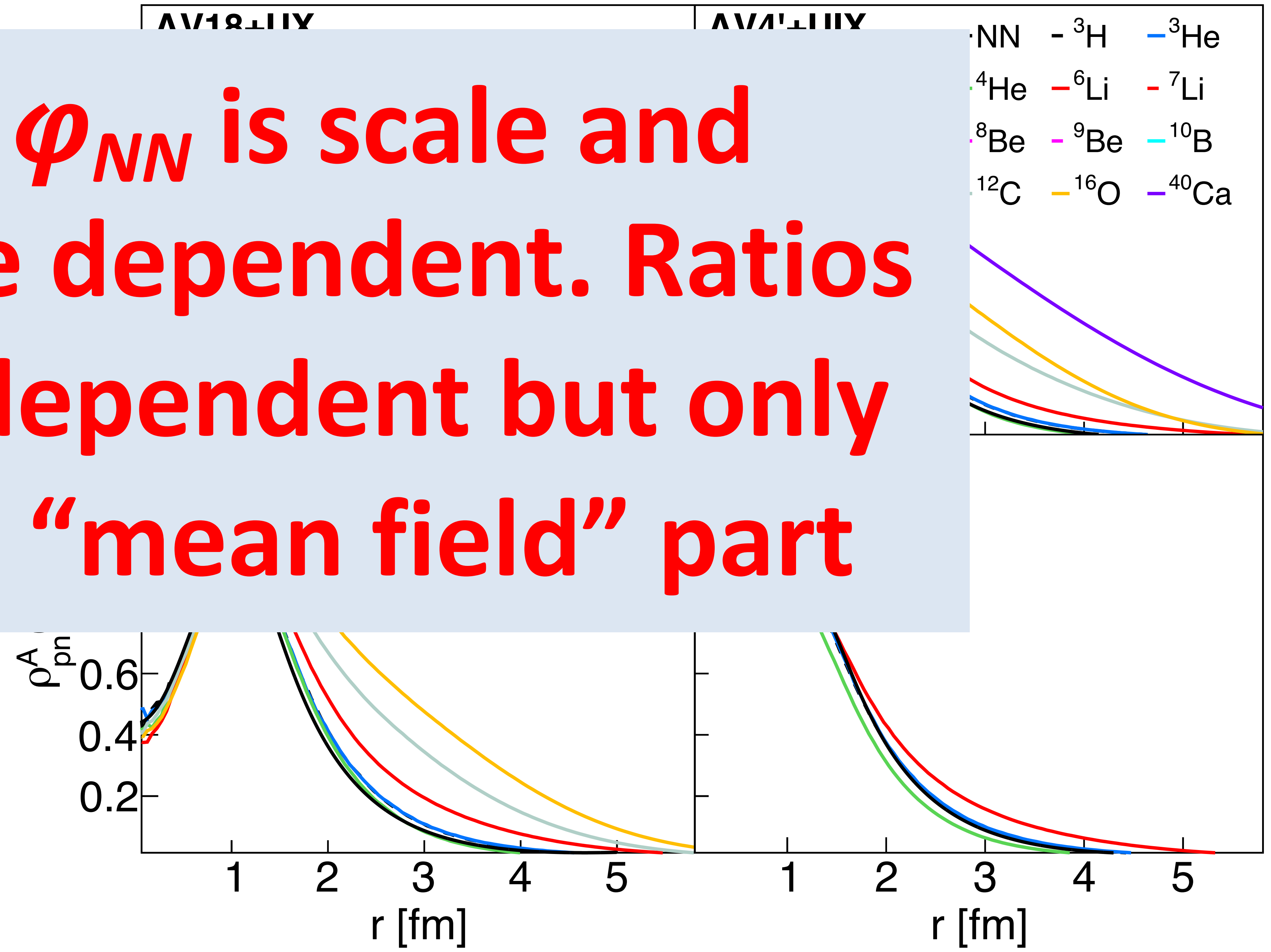

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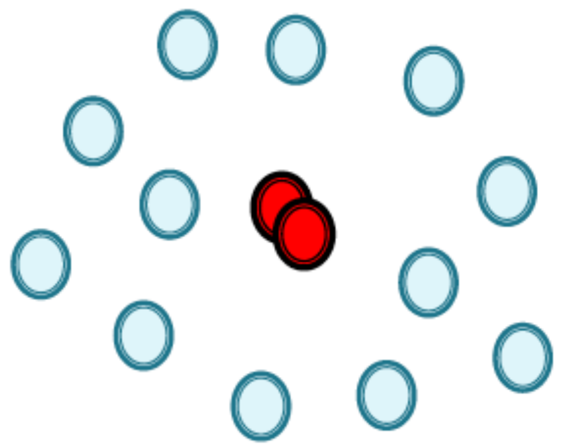

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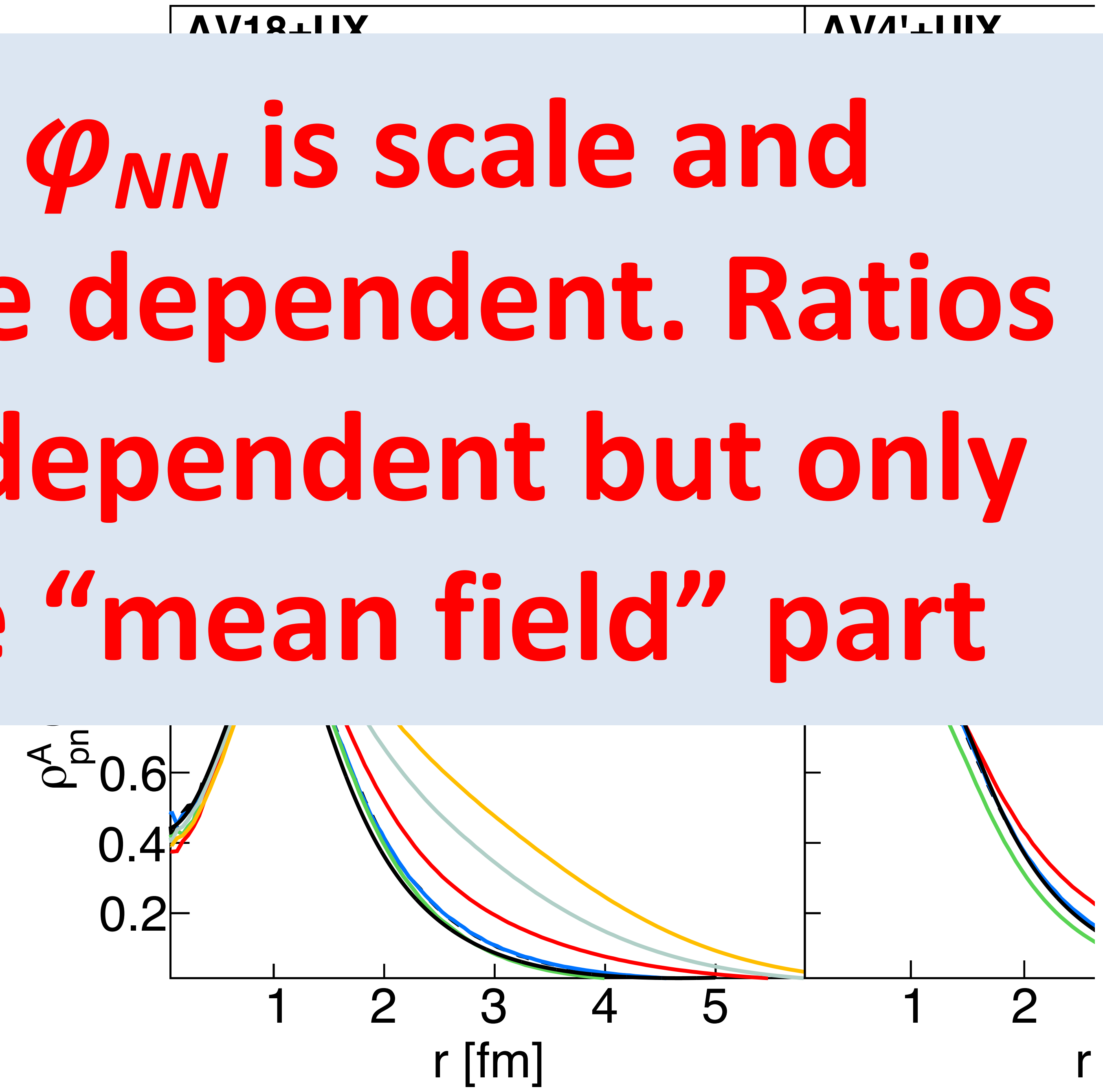
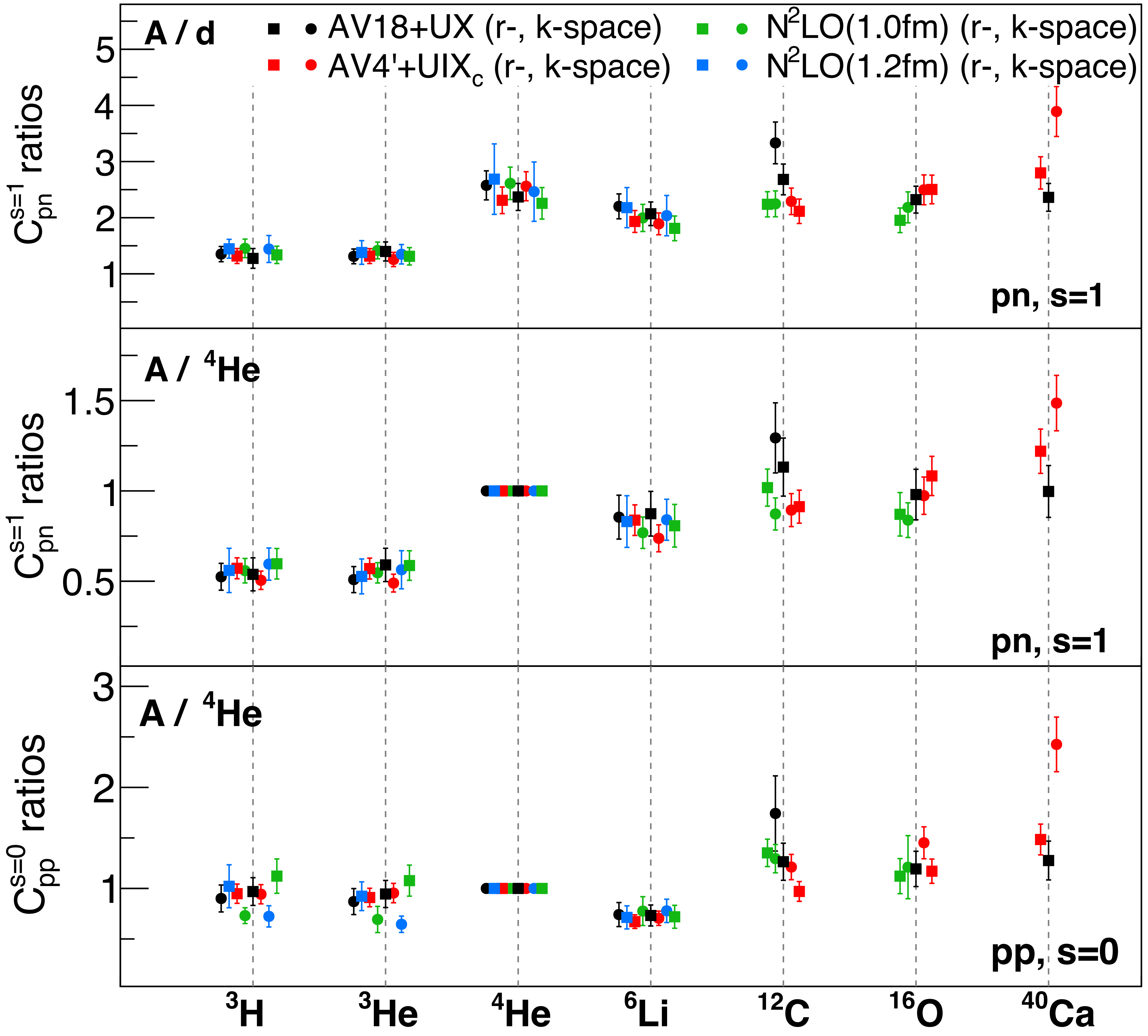
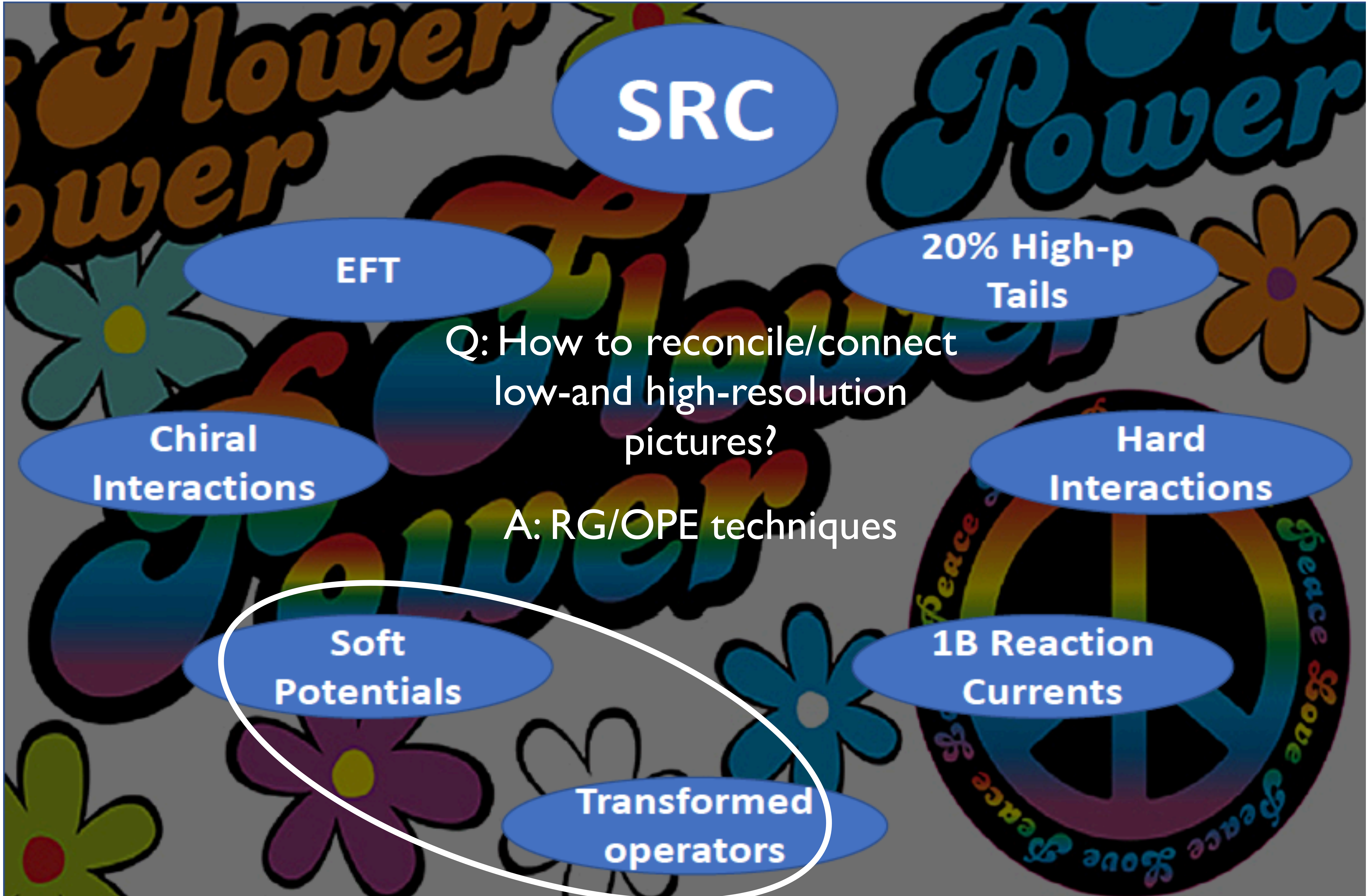
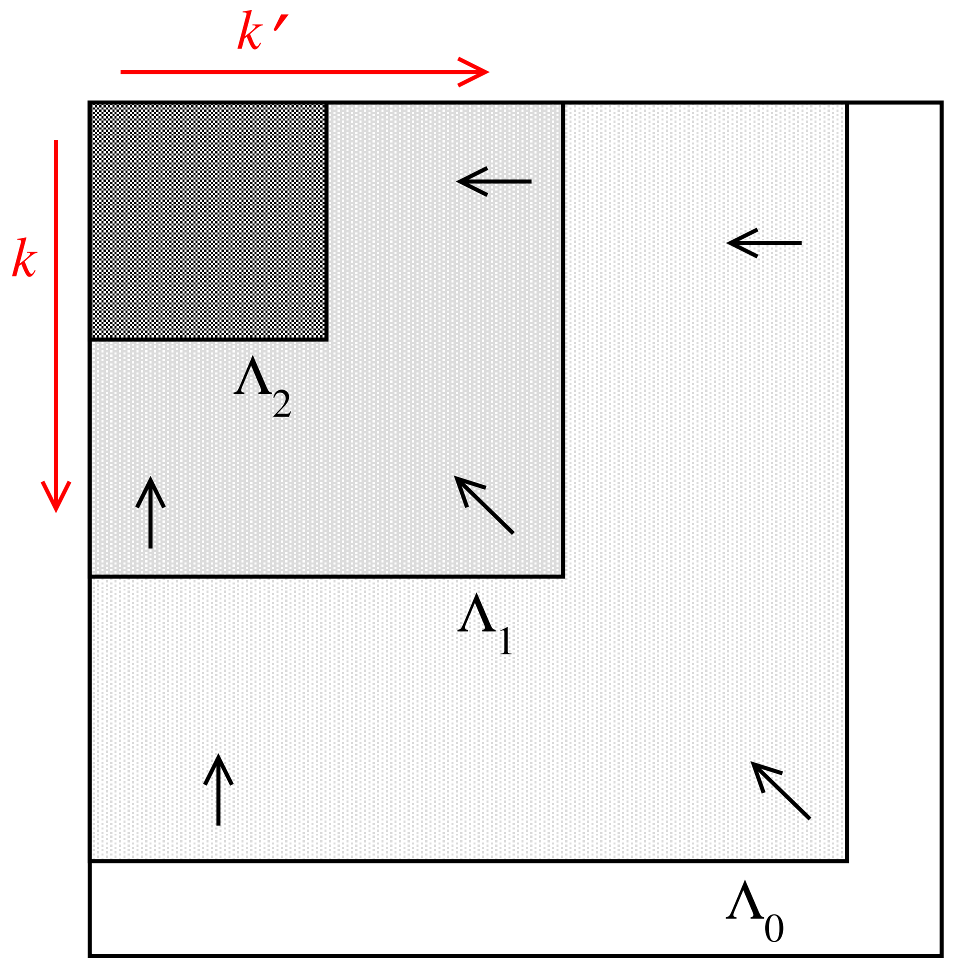





SRC
EFT
20% High-p Tails
Q: How to reconcile/connect low-and high-resolution pictures?
Chiral Interactions
Hard Interactions
A: RG/OPE techniques
Soft Potentials
1B Reaction Currents
Transformed operators

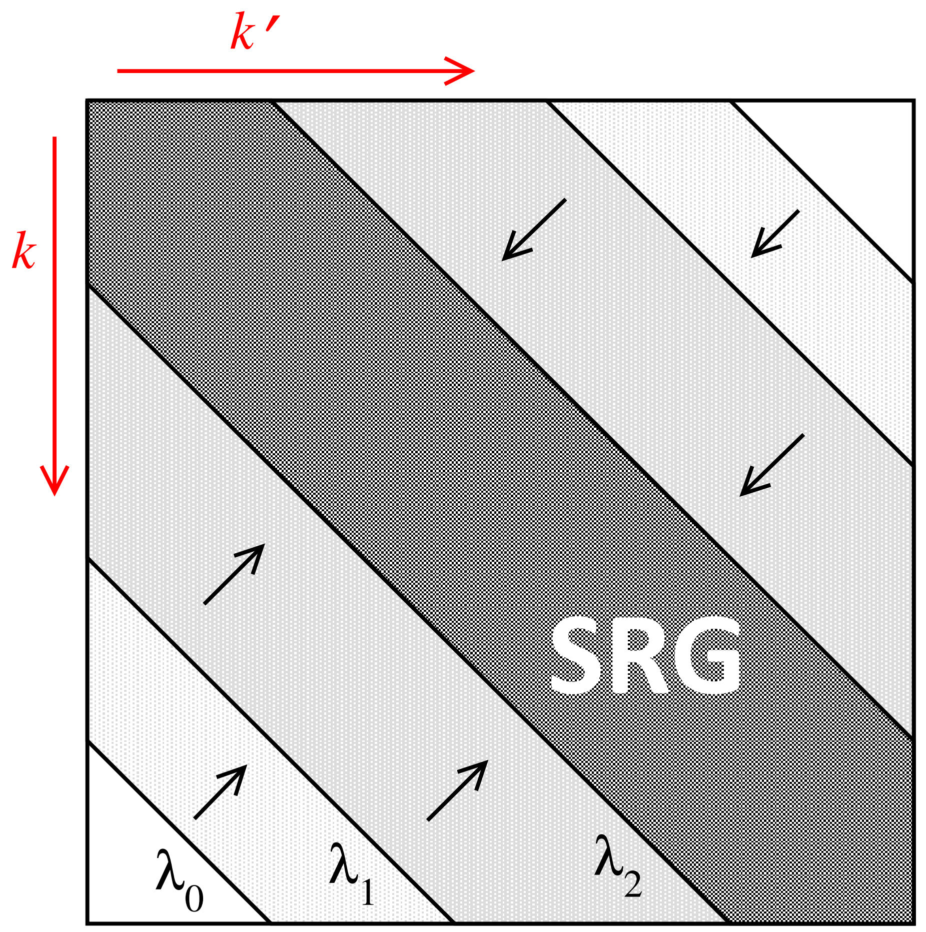
RG in low energy nuclear physics

Bogner, Furnstahl, Schwenk Prog. Part. Nucl. Phys. 2010



Integrate out momenta $k > \Lambda$

preserve physics **up to** Λ



Unitary RG ("**S**imilarity **R**enormalization **G**roup")

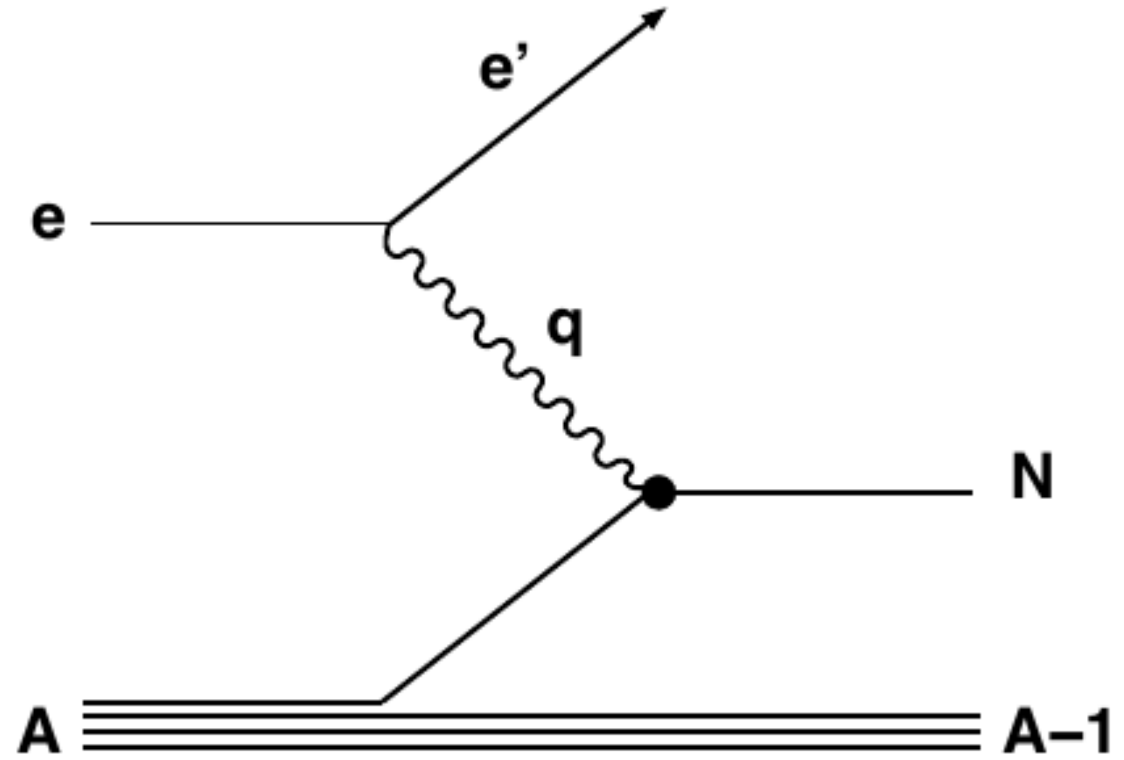
$H(\lambda) = U(\lambda) H U^{\dagger}(\lambda) \qquad O(\lambda) = U(\lambda) O U^{\dagger}(\lambda)$

preserves all physics (unitary) if no approximations

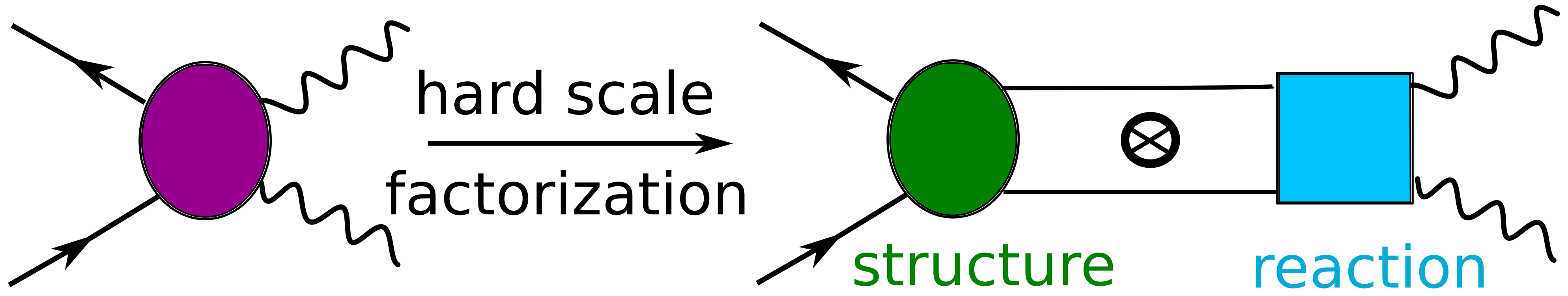
low E states => $k \gtrsim \lambda$ highly suppressed

Bridging structure and reactions

- Goal: Extract nuclear properties from experiments and predict them from theory
- $\frac{d\sigma}{d\Omega} \propto \left|\langle\psi_f|\widehat{O}(q)|\psi_i\rangle\right|^2$
- Factorization to isolate components and extract process-independent properties



e.g., nucleon knockout reaction



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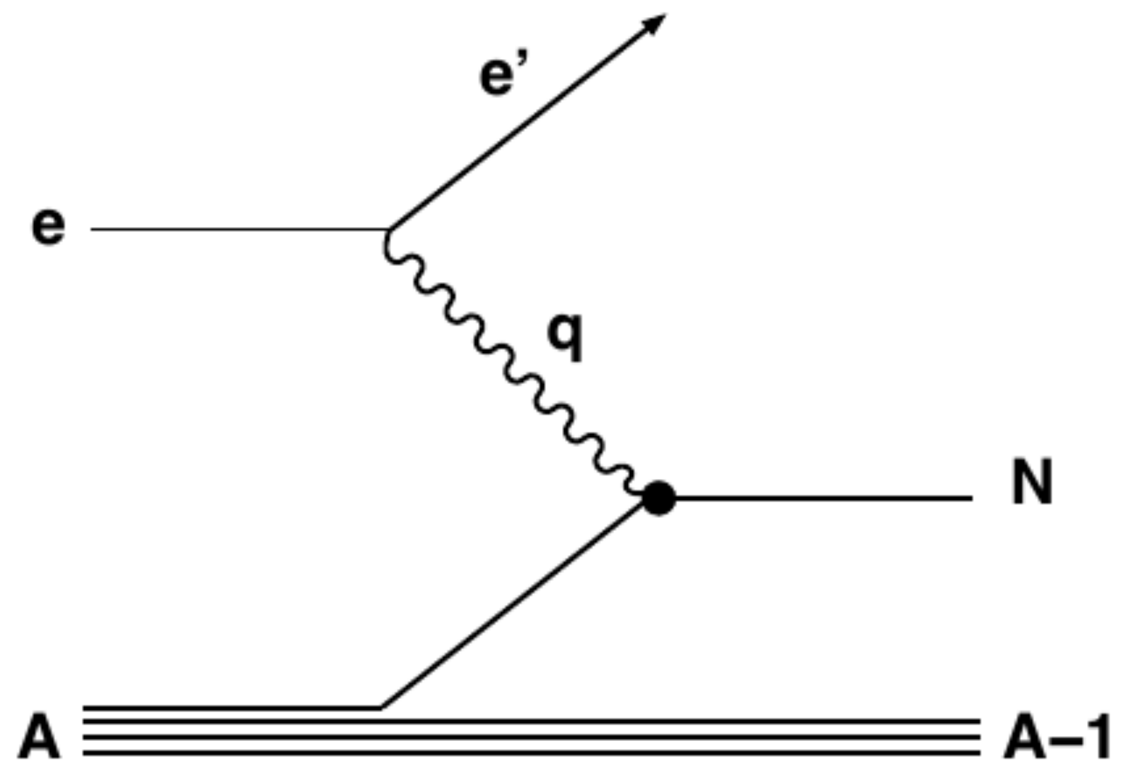

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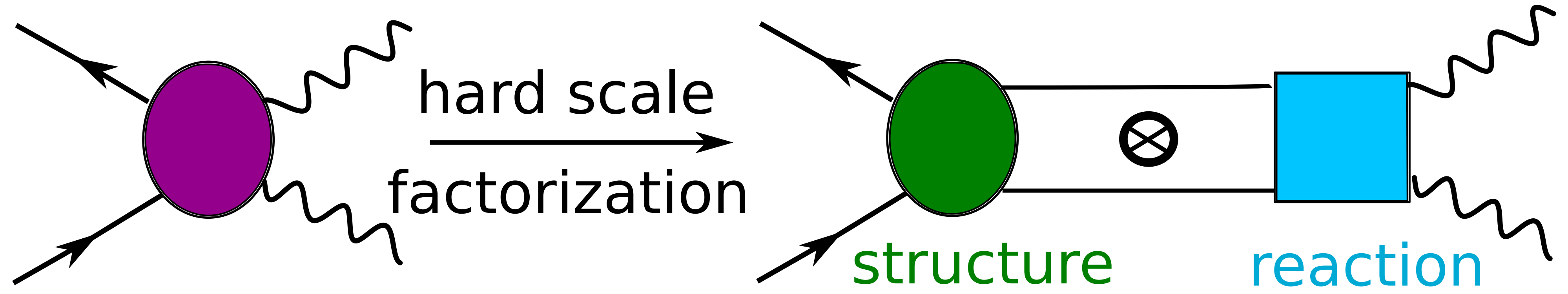
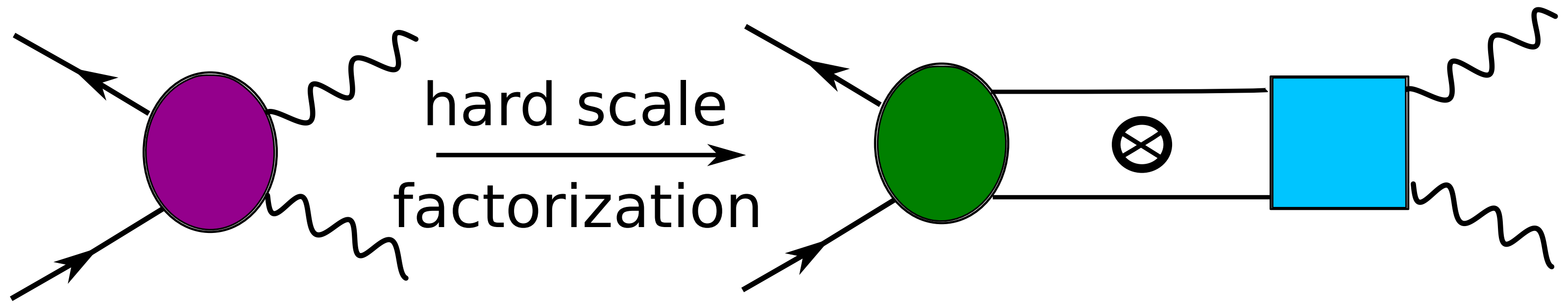


$$\underbrace{\langle\psi_f|}_{\text{structure}}\overbrace{\widehat{O}(q)}^{\text{reaction}}\underbrace{|\psi_i\rangle}_{\text{structure}} = \langle\psi_f|U_\lambda U_\lambda^\dagger \widehat{O}(q) U_\lambda U_\lambda^\dagger|\psi_i\rangle = \underbrace{\langle\psi_f^\lambda|}_{\text{structure}(\lambda)}\overbrace{\widehat{O}^\lambda(q)}^{\text{reaction}(\lambda)}\underbrace{|\psi_i^\lambda\rangle}_{\text{structure}(\lambda)}$$

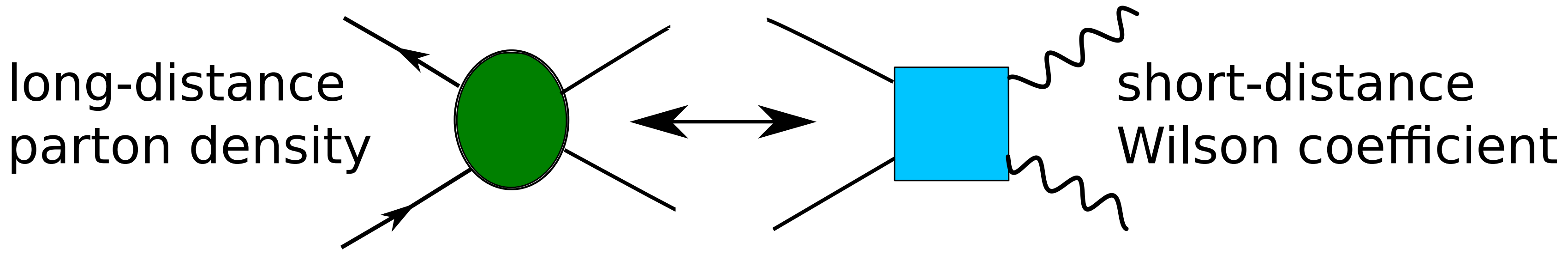
Factorization is scale-dependent (not unique)!!

Analogy with DIS in QCD

High-E QCD



$$F_2(x,Q^2) \sim \sum_a f_a(x,\mu_f) \otimes \widehat{F}_2^a(x,Q/\mu_f)$$



Low-E Nuclear

Observable: cross section

Structure model: spectroscopic factor

Reaction model: single-particle cross section

$$\sigma^{if} = \sum_{|J_i - J_f| \leq j \leq J_i + J_f} S_j^{if} \, \sigma_{sp}$$

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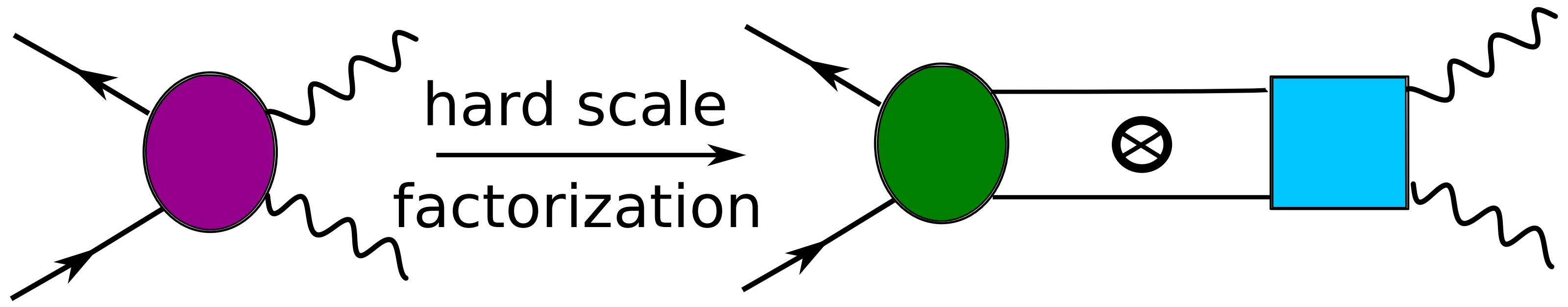

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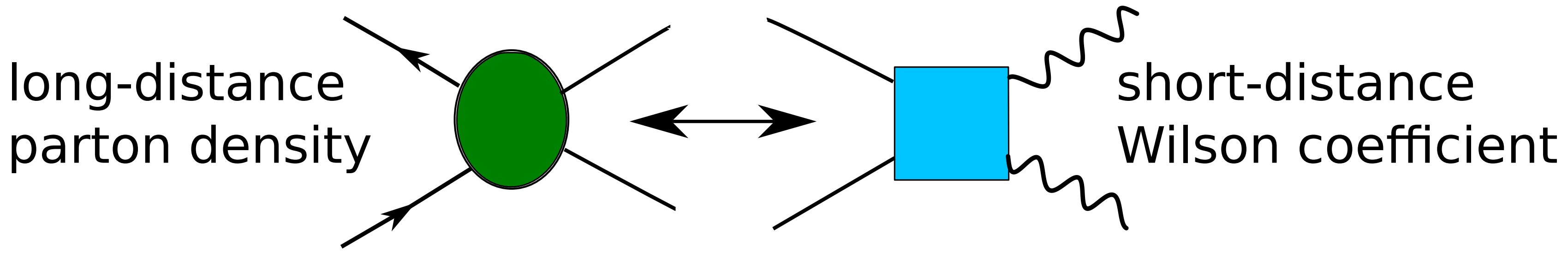

- Separation not unique, depends on the scale μ_f
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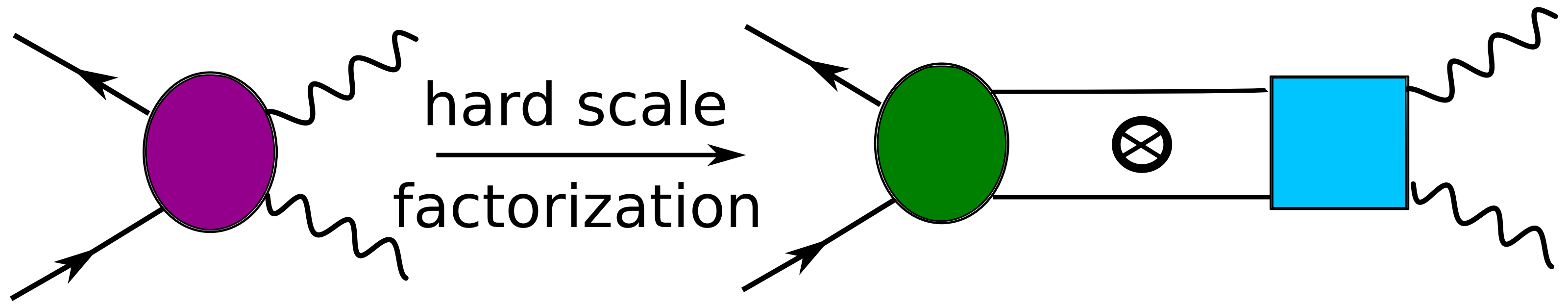

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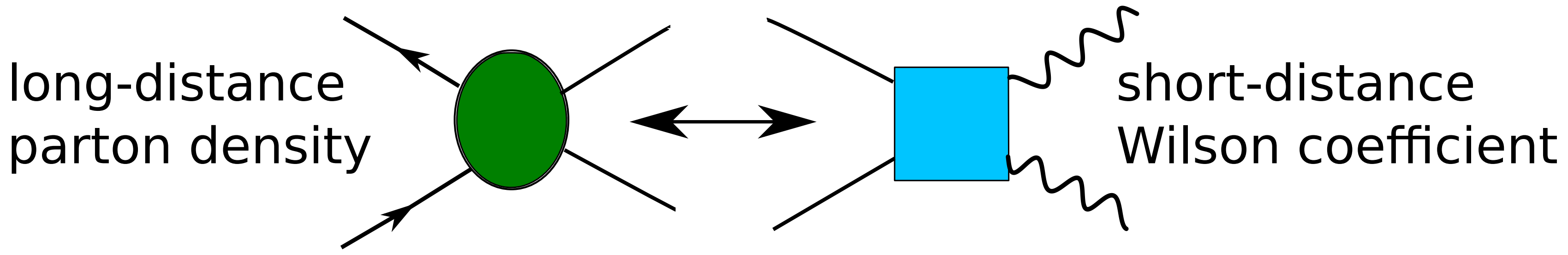

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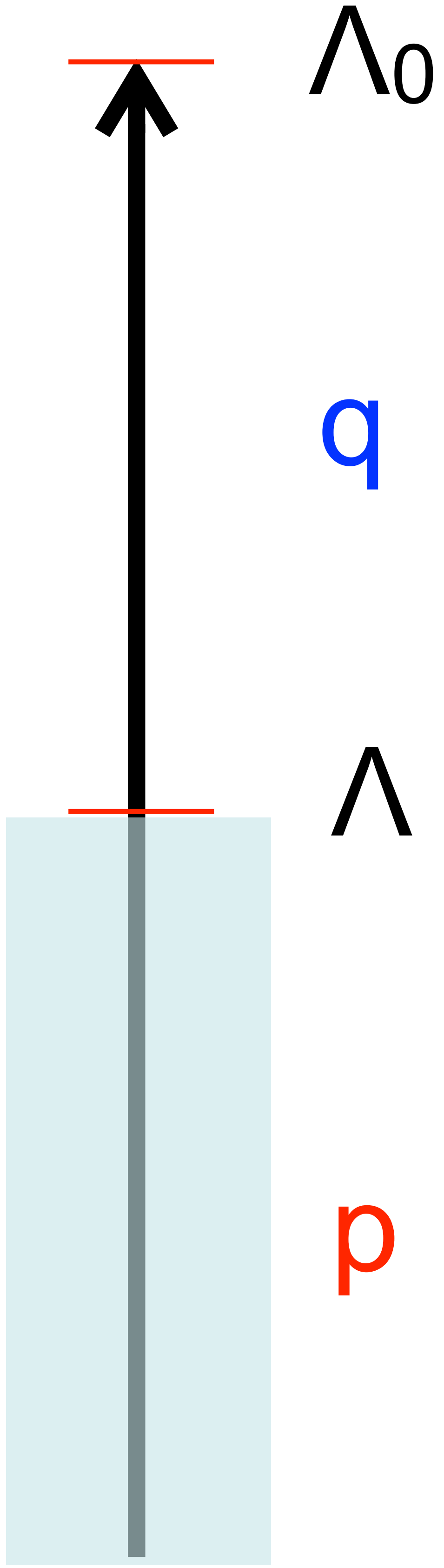
Open Questions

- What is the scale/scheme dependence of extracted props?
- Extract at one scale (e.g., to minimize FSI) and evolve to another?
- Scale/scheme dependence of interpretations? Are some better?
- **Structure of evolved operators?**

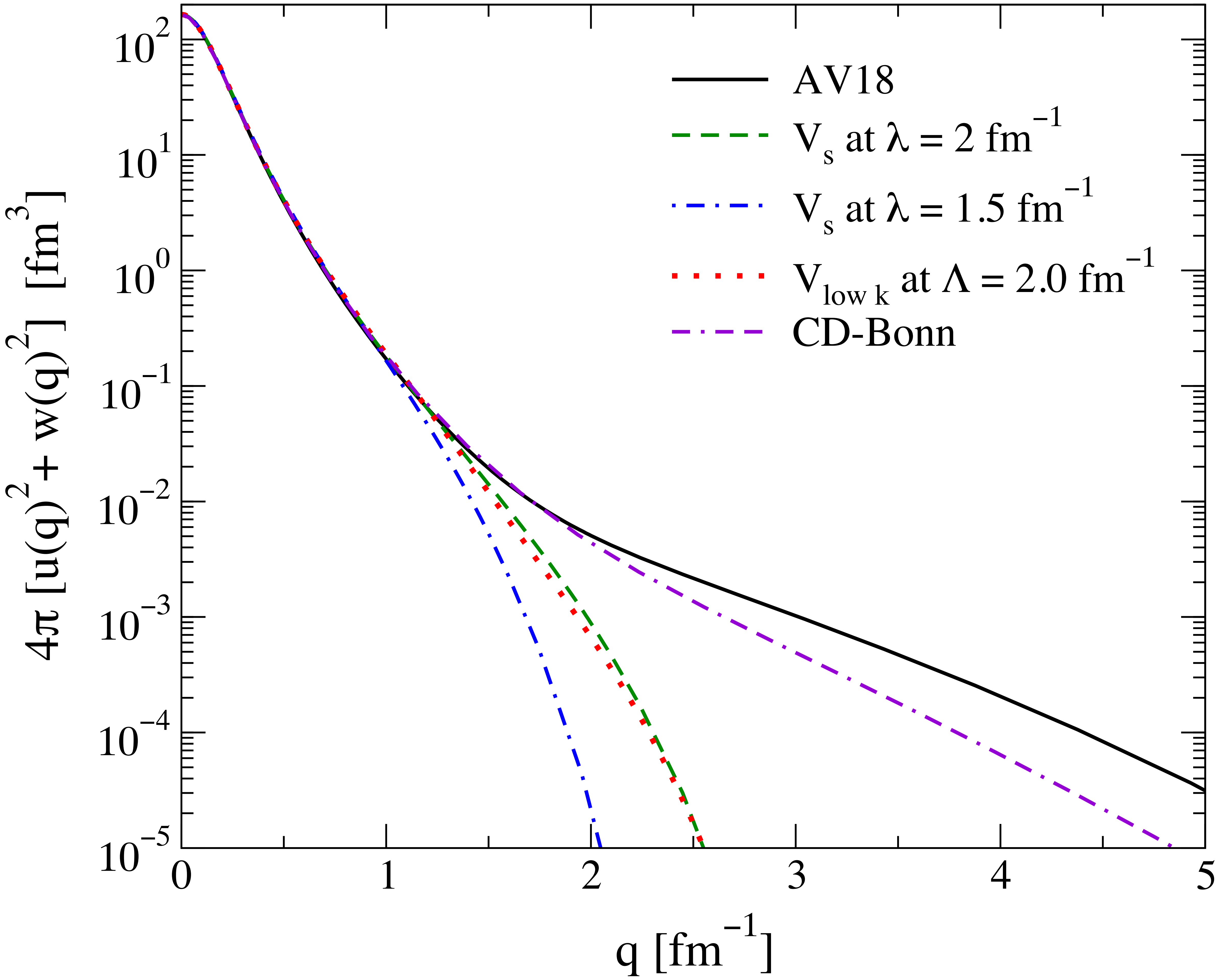
High-q operators evolved to low-resolution scales

Consider low-k components of low-E wf's for A=2.



RG preserves long distance structure

$$\psi_\alpha^{\Lambda_0}(\mathbf{p}) \approx Z_\Lambda \psi_\alpha^{\Lambda}(\mathbf{p})$$



Anderson et al., PRC **82** (2010)

SKB and Roscher, PRC **86** (2012)

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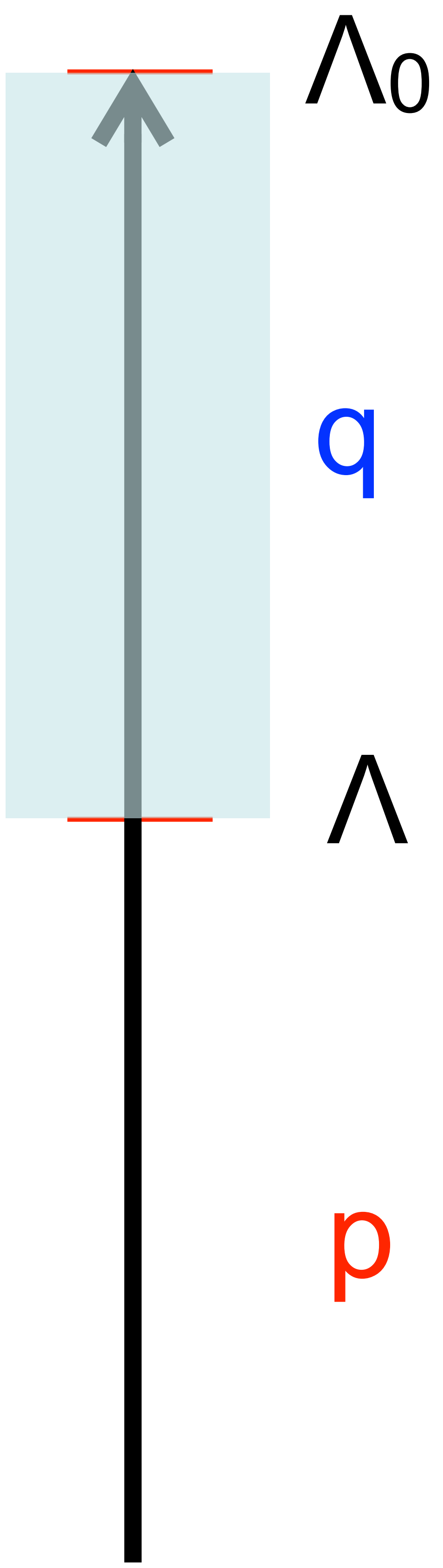

Scale separation ($E_\alpha << \Lambda^2 << q^2$)

$$\psi_\alpha^{\Lambda_0}(\mathbf{q}) \approx \gamma(\mathbf{q};\Lambda) \int_0^\Lambda d^3p \, Z_\Lambda \psi_\alpha^\Lambda(\mathbf{p}) + \eta(\mathbf{q};\Lambda) \int_0^\Lambda d^3p \, \mathbf{p}^2 Z_\Lambda \psi_\alpha^\Lambda(\mathbf{p}) \cdots$$

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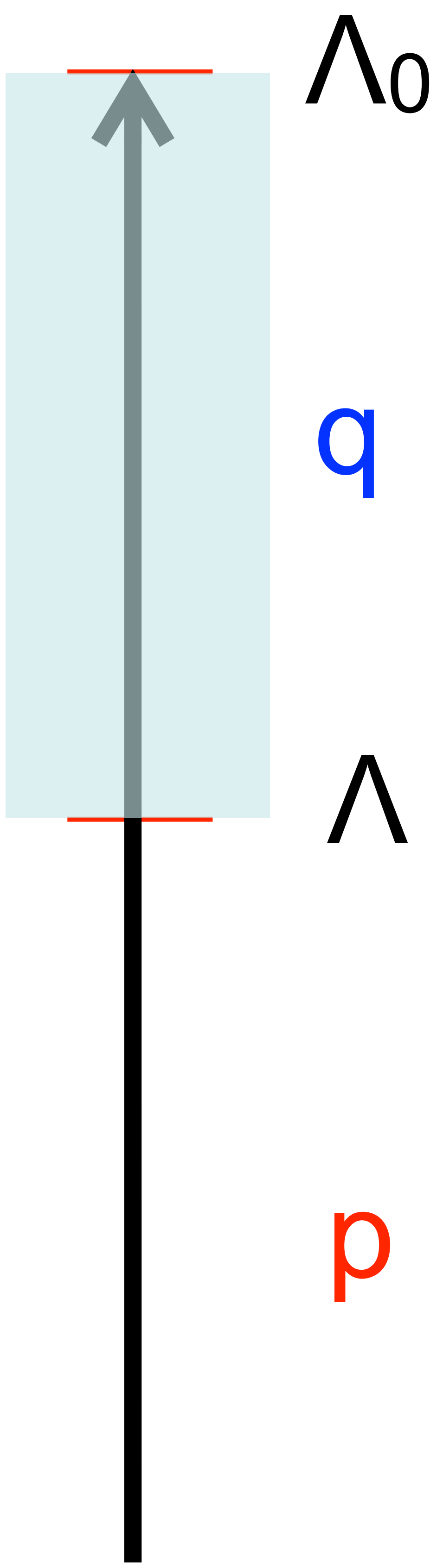

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Operator **P**roduct **E**xpansion
of wave function a-la Lepage

$$\gamma(\mathbf{q};\Lambda) = -\int_\Lambda^{\Lambda_0} d\mathbf{q}' \,\langle \mathbf{q}| \frac{1}{QH^{\Lambda_0}Q} |\mathbf{q}'\rangle V^{\Lambda_0}(\mathbf{q}',0)$$

$$\beta(\mathbf{q};\Lambda) = -\int_\Lambda^{\Lambda_0} d\mathbf{q}' \,\langle \mathbf{q}| \frac{1}{QH^{\Lambda_0}Q} |\mathbf{q}'\rangle \frac{\partial^2}{\partial p^2} V^{\Lambda_0}(\mathbf{q}',\mathbf{p})\Big|_{\mathbf{p}=0}$$

State-independent
Wilson Coefficients

Anderson et al., PRC **82** (2010)
SKB and Roscher, PRC **86** (2012)

High-q operators evolved to low-resolution scales

$$\begin{aligned}\langle\psi_\alpha^{\Lambda_0}|\widehat{O}_{\Lambda_0}|\psi_\alpha^{\Lambda_0}\rangle &= \int_0^\Lambda dp \int_0^\Lambda dp'\, \psi_\alpha^{\Lambda_0 *}(p) O(p,p') \psi_\alpha^{\Lambda_0}(p') + \int_0^\Lambda dp \int_\Lambda^{\Lambda_0} dq\, \psi_\alpha^{\Lambda_0 *}(p) O(p,q) \psi_\alpha^{\Lambda_0}(q) \\ &+ \int_\Lambda^{\Lambda_0} dq \int_0^\Lambda dp\, \psi_\alpha^{\Lambda_0 *}(q) O(q,p) \psi_\alpha^{\Lambda_0}(p) + \int_\Lambda^{\Lambda_0} dq \int_\Lambda^{\Lambda_0} dq'\, \psi_\alpha^{\Lambda_0 *}(q) O(q,q') \psi_\alpha^{\Lambda_0}(q')\end{aligned}$$

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Now use:

$$\psi_\alpha^{\Lambda_0}(\mathbf{q}) \approx \gamma(\mathbf{q};\Lambda) \int_0^\Lambda d^3p \, Z_\Lambda \psi_\alpha^\Lambda(\mathbf{p}) + \cdots$$ OPE for w.f.'s

$$\psi_\alpha^{\Lambda_0}(\mathbf{p}) \approx Z_\Lambda \psi_\alpha^\Lambda(\mathbf{p})$$ IR structure unaltered

$$O(q,p) \approx O(q,0) + \cdots$$ Scale separation

High-q operators evolved to low-resolution scales

NSCL FRIB

$$\langle \psi_\alpha^{\Lambda_0} | \widehat{O}_{\Lambda_0} | \psi_\alpha^{\Lambda_0} \rangle \approx Z_\Lambda^2 \langle \psi_\alpha^{\Lambda} | \widehat{O}_{\Lambda_0} | \psi_\alpha^{\Lambda} \rangle + g^{(0)}(\Lambda) \, \langle \psi_\alpha^{\Lambda} | \delta^{(3)}(\mathbf{r}) | \psi_\alpha^{\Lambda} \rangle + \cdots$$

state-independent
high-q physics
depends on operator

state dependent
soft m.e. (low-k)
same for all high-q operators

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E.g.,

$$g^{(0)}(\Lambda) \equiv 2Z_\Lambda^2 \int_\Lambda^{\Lambda_0} d\tilde{q}\, O(0,q)\gamma(q;\Lambda) + Z_\Lambda^2 \int_\Lambda^{\Lambda_0} d\tilde{q} \int_\Lambda^{\Lambda_0} d\tilde{q}'\, \gamma^*(q;\Lambda) O(q,q')\gamma(q';\Lambda)$$

Generically:

$$\widehat{O}_\Lambda = Z_\Lambda^2\, \widehat{O}_{\Lambda_0} + g^{(0)}(\Lambda)\,\delta(\mathbf{r}) + g^{(2)}(\Lambda)\,\nabla^2\delta(\mathbf{r}) + \cdots$$

High-q operators evolved to low-resolution scales

How does an operator that probes high-momentum w.f. components look in a low-momentum effective theory?

$$\langle \psi_\alpha^{\Lambda_0} | \widehat{O}_{\Lambda_0} | \psi_\alpha^{\Lambda_0} \rangle \approx Z_\Lambda^2 \langle \psi_\alpha^\Lambda | \widehat{O}_{\Lambda_0} | \psi_\alpha^\Lambda \rangle + g^{(0)}(\Lambda) \, \langle \psi_\alpha^\Lambda | \delta^{(3)}(\mathbf{r}) | \psi_\alpha^\Lambda \rangle + \cdots$$

= 0 since $P_\Lambda O_{\Lambda_0} P_\Lambda = 0$

SKB and Roscher, PRC **86** (2012)
Tropiano, SKB, Furnstahl (in progress)

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E.g., momentum distribution for q >> Λ

$$\langle\psi_\alpha^{\Lambda_0}|a_\mathbf{q}^\dagger a_\mathbf{q}|\psi_\alpha^{\Lambda_0}\rangle \approx \gamma^2(\mathbf{q};\Lambda)\,Z_\Lambda^2|\langle\psi_\alpha^\Lambda|\delta(\mathbf{r})|\psi_\alpha^\Lambda\rangle|^2$$

low-E states have the same large-q tails if leading OPE term dominates

Generalize to arbitrary **A-body** states

SKB and Roscher, PRC **86** (2012)
Tropiano, SKB, Furnstahl (in progress)

High-q operators evolved to low-resolution scales

NSCL FRIB

Ex1: momentum distribution (Λ << q < Λ_0):

$$\langle \psi^{\Lambda_0}_{\alpha,A} | a^\dagger_{\mathbf{q}} a_{\mathbf{q}} | \psi^{\Lambda_0}_{\alpha,A} \rangle \approx \gamma^2(\mathbf{q};\Lambda) \times \sum_{\mathbf{k},\mathbf{k}',\mathbf{K}}^{\Lambda} Z^2_{\Lambda} \langle \psi^{\Lambda}_{\alpha,A} | a^\dagger_{\frac{\mathbf{K}}{2}+\mathbf{k}} a^\dagger_{\frac{\mathbf{K}}{2}-\mathbf{k}} a_{\frac{\mathbf{K}}{2}-\mathbf{k}'} a_{\frac{\mathbf{K}}{2}+\mathbf{k}'} | \psi^{\Lambda}_{\alpha,A} \rangle$$

High-q operators evolved to low-resolution scales

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Ex2: static structure factor (Λ << q < Λ_0):

$$\langle \psi^{\Lambda_0}_{\alpha,A} | \widehat{S}(\mathbf{q}) | \psi^{\Lambda_0}_{\alpha,A} \rangle \approx \left\{ 2\gamma(\mathbf{q};\Lambda) + \sum_{\mathbf{P}} \gamma(\mathbf{P}+\mathbf{q};\Lambda)\gamma(\mathbf{P};\Lambda) \right\}$$

$$\times \sum^{\Lambda}_{\mathbf{K},\mathbf{k},\mathbf{k}'} Z^2_\Lambda \langle \psi^{\Lambda}_{\alpha,A} | a^\dagger_{\frac{\mathbf{K}}{2}+\mathbf{k}} a^\dagger_{\frac{\mathbf{K}}{2}-\mathbf{k}} a_{\frac{\mathbf{K}}{2}-\mathbf{k}'} a_{\frac{\mathbf{K}}{2}+\mathbf{k}'} | \psi^{\Lambda}_{\alpha,A} \rangle$$

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- hard (high q) physics
- Universal (state-indep)
- depends on probe
- fixed from few-body

X

- soft (low-k) m.e.
- same for all high-q probes
- A-dependent scale factor

High-q operators evolved to low-resolution scales

links few- and A-body systems ("derives" the GCF)

Correlations/scaling for 2 observables w/same leading OPE

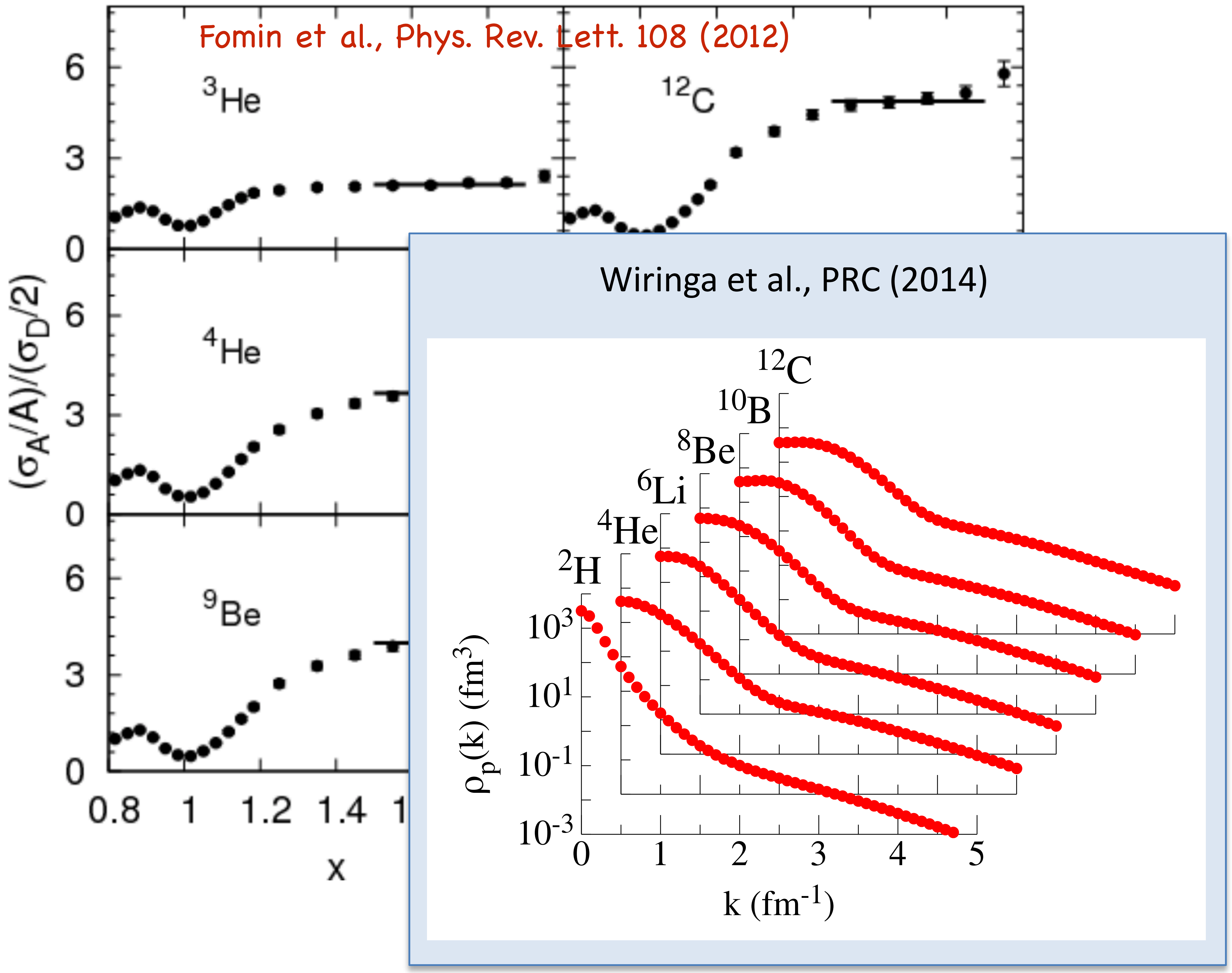
Subleading OPE ==> deviations from scaling calculable in principle?

- fixed from few-body

Tropiano, SKB, Furnstahl (in progress)

1) Universal high-momentum tails

inclusive ratios



SRC phenomenology revisited (low-res picture)

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Tropiano, SKB, Furnstahl (in progress)

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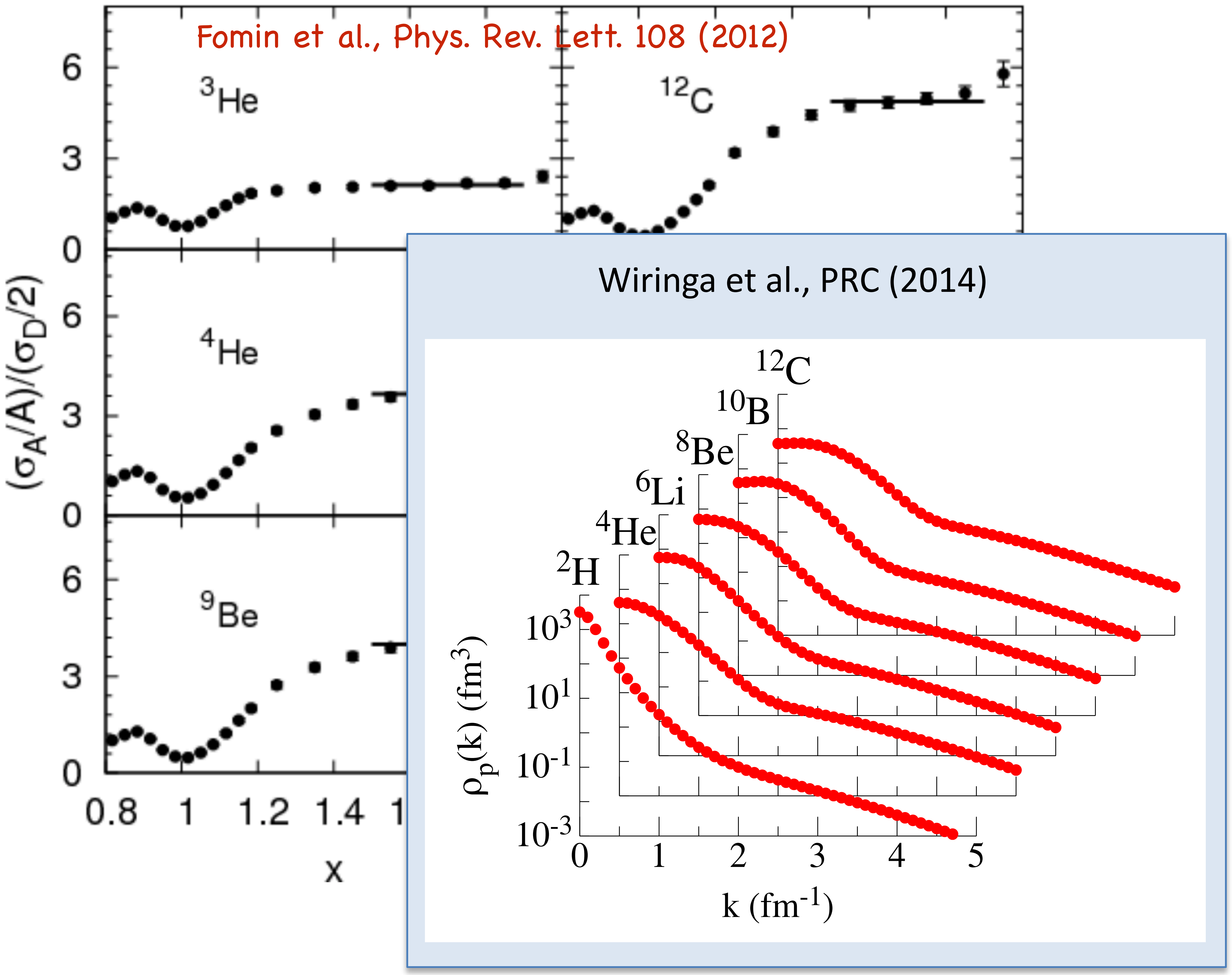

$$\langle \psi^{\Lambda_0}_{\alpha,A} | a^\dagger_{\mathbf{q}} a_{\mathbf{q}} | \psi^{\Lambda_0}_{\alpha,A}\rangle \approx \gamma^2(\mathbf{q};\Lambda) \times \sum^{\Lambda}_{\mathbf{k},\mathbf{k}',\mathbf{K}} Z^2_\Lambda \langle \psi^\Lambda_{\alpha,A}| a^\dagger_{\frac{\mathbf{K}}{2}+\mathbf{k}} a^\dagger_{\frac{\mathbf{K}}{2}-\mathbf{k}} a_{\frac{\mathbf{K}}{2}-\mathbf{k}'} a_{\frac{\mathbf{K}}{2}+\mathbf{k}'} |\psi^\Lambda_{\alpha,A}\rangle$$

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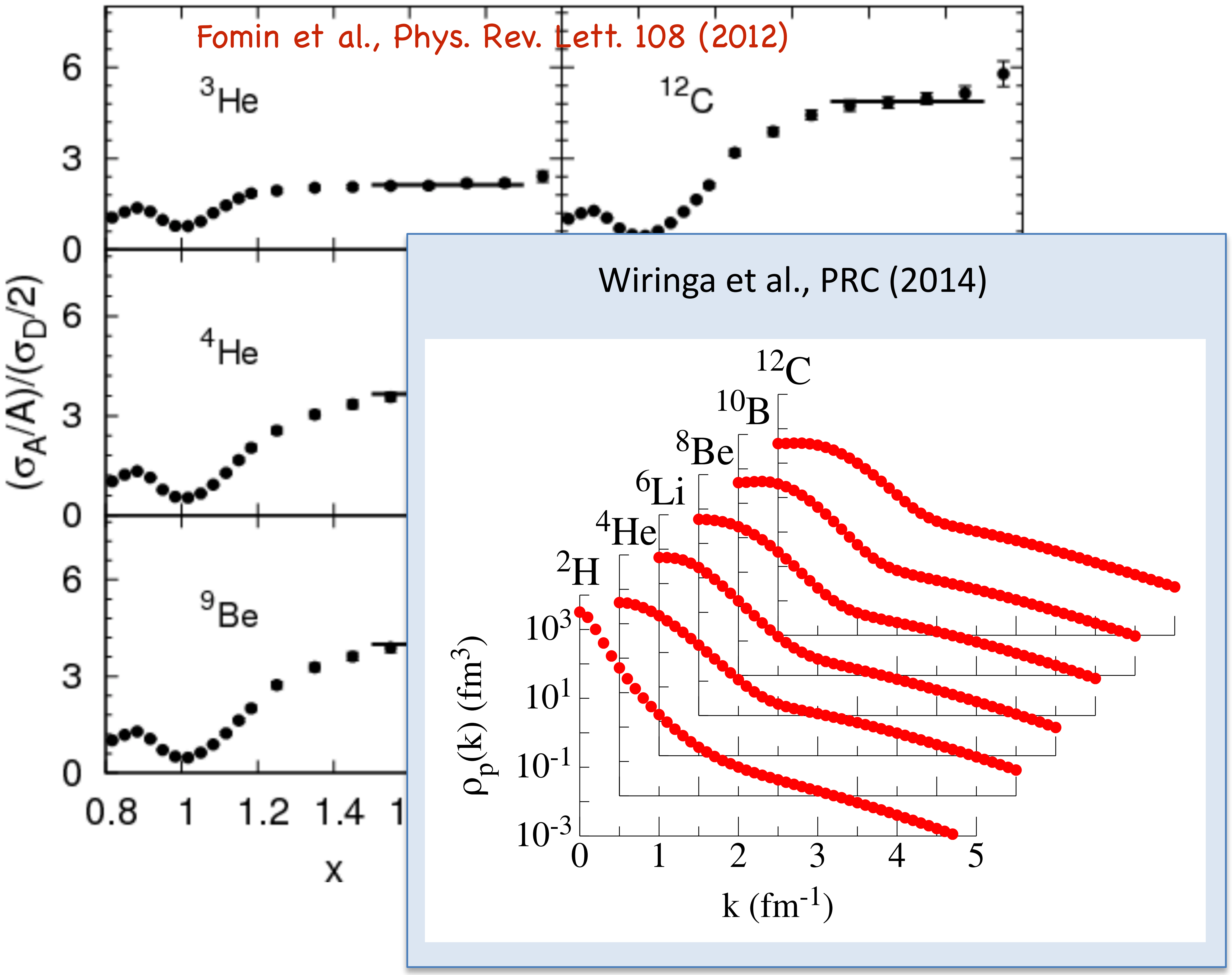

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$$C(A,2)\equiv\frac{n_A(\mathbf{q})}{n_D(\mathbf{q})}\sim\frac{\sum_{\mathbf{k},\mathbf{k}',\mathbf{K}}\langle\psi^\Lambda_{\alpha,A}|a^\dagger_{\frac{\mathbf{K}}{2}+\mathbf{k}}a^\dagger_{\frac{\mathbf{K}}{2}-\mathbf{k}}a_{\frac{\mathbf{K}}{2}-\mathbf{k}'}a_{\frac{\mathbf{K}}{2}+\mathbf{k}'}|\psi^\Lambda_{\alpha,A}\rangle}{\sum_{\mathbf{k},\mathbf{k}',\mathbf{K}}\langle\psi^\Lambda_{\alpha,D}|a^\dagger_{\frac{\mathbf{K}}{2}+\mathbf{k}}a^\dagger_{\frac{\mathbf{K}}{2}-\mathbf{k}}a_{\frac{\mathbf{K}}{2}-\mathbf{k}'}a_{\frac{\mathbf{K}}{2}+\mathbf{k}'}|\psi^\Lambda_{\alpha,D}\rangle}$$

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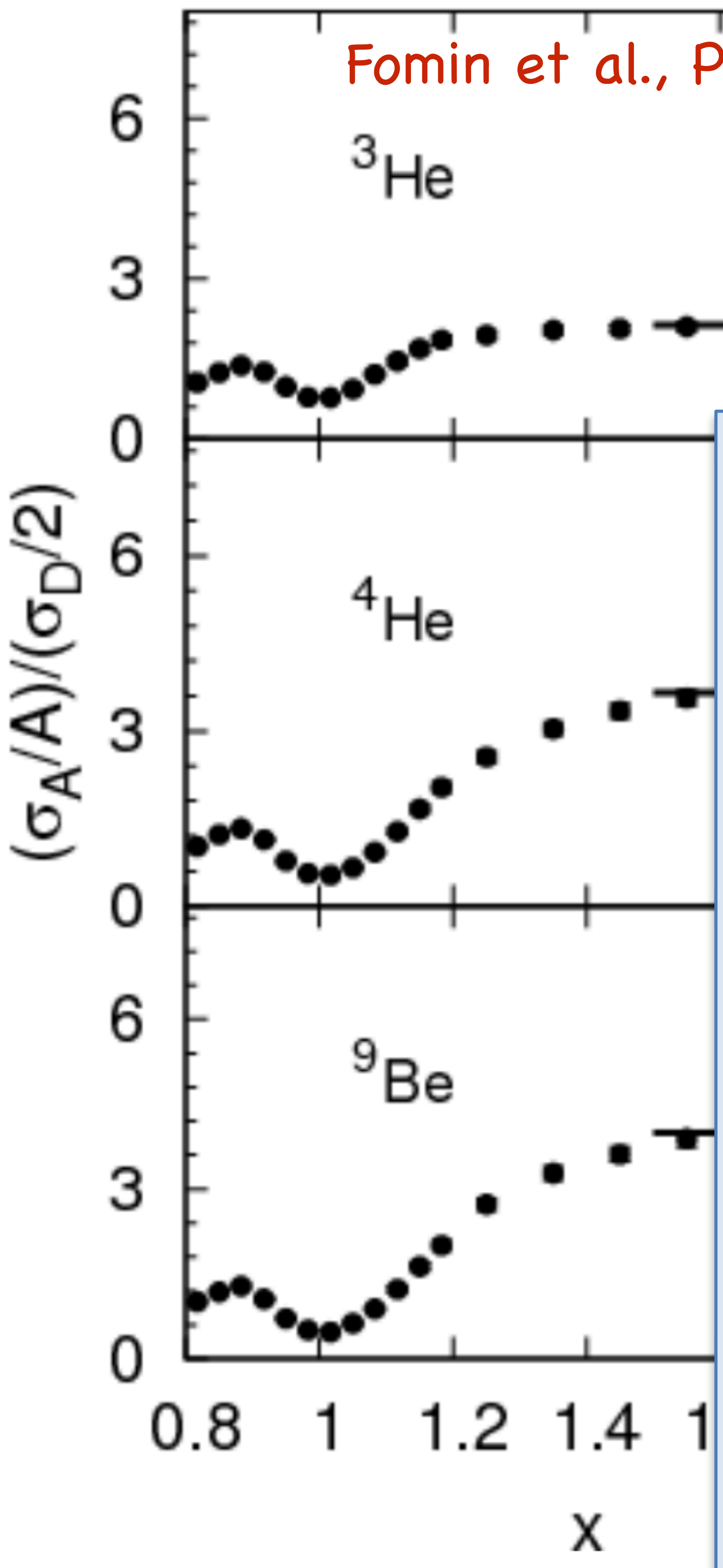

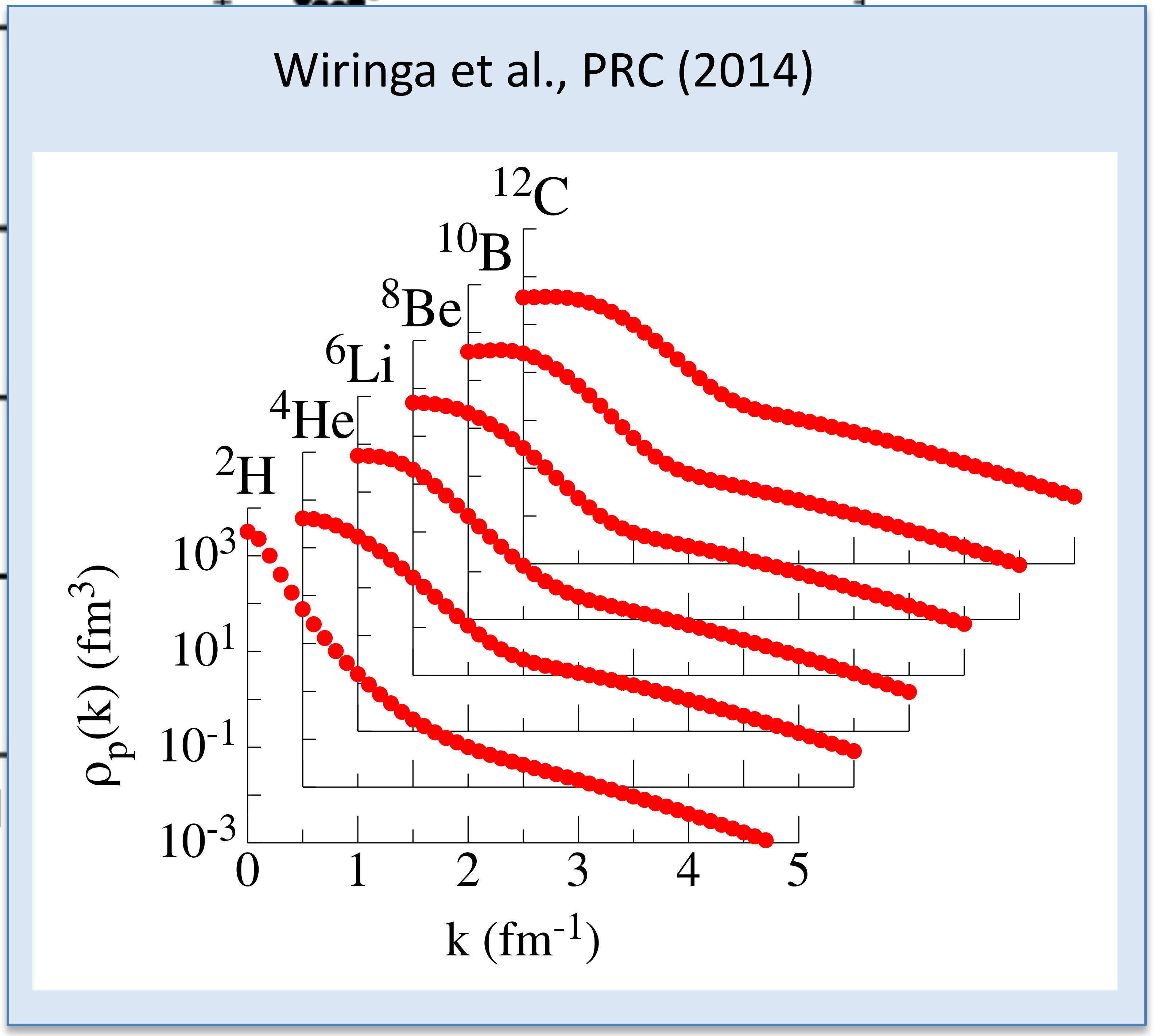

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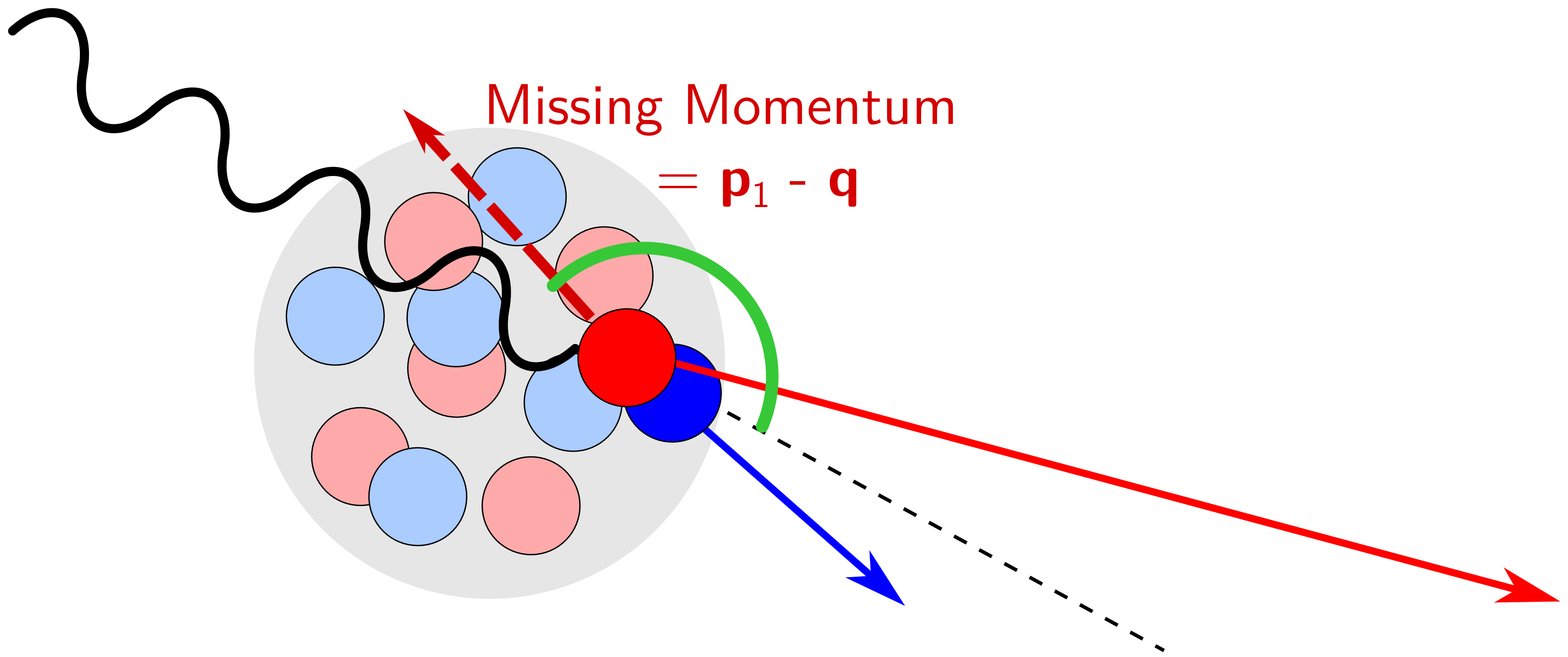
low-momentum/mean field physics => scale/scheme indeed to leading order

supports/explains GCF conclusions about inclusive ratios

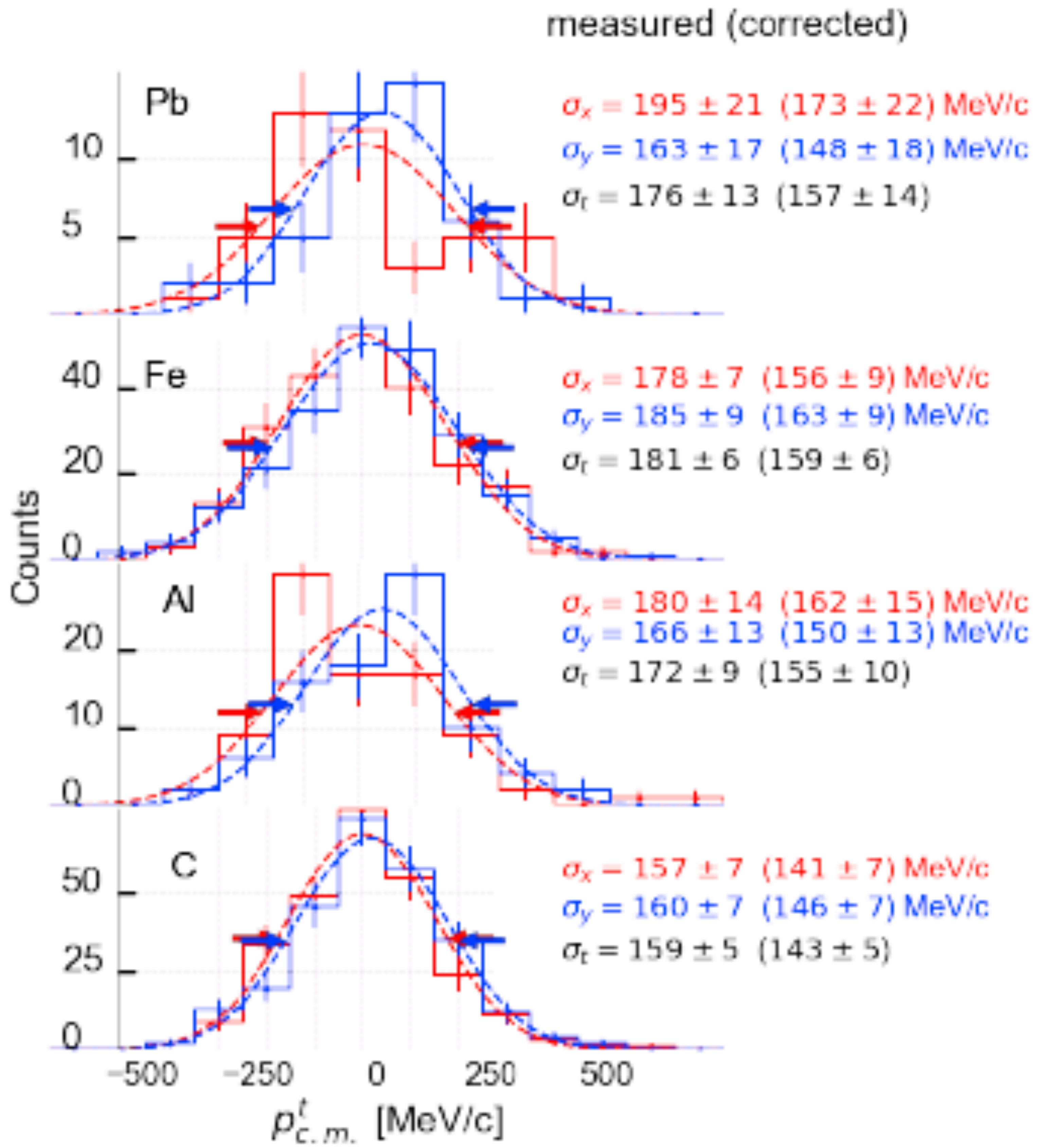
SRC phenomenology revisited (low-res picture)

Tropiano, SKB, Furnstahl (in progress)

2) Kinematics of knocked-out nucleons



knocked out SRC nucleons fly out almost back-to-back
(relative s-wave pairs)



pair CM momentum distribution gaussian of width ~ k_F

SRC phenomenology revisited (low-res picture)

Tropiano, SKB, Furnstahl (in progress)

2) Kin

evolved pair momentum distribution $(\lambda \sim k_F < < q)$

$$\rho_{NN,\alpha}(Q,q) \sim \gamma_\alpha^2(q;\Lambda) \sum_{k,k'} |\langle\psi^A(\Lambda)|\left[a^\dagger_{\frac{Q}{2}+k} a^\dagger_{\frac{Q}{2}-k} a_{\frac{Q}{2}-k'} a_{\frac{Q}{2}+k'}\right]_\alpha |\psi^A(\Lambda)\rangle$$

kn

alm

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SRC phenomenology revisited (low-res picture)

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m.e. of smeared contact operator ==>

high q pairs dominated relative s-waves

evolved $\psi(\Lambda)$ "soft", dominated by MFT configs ==>

CM Q distribution smooth/gaussian with width ~ k_F

kn

alm

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ution

SRC phenomenology revisited (low-res picture)

Tropiano, SKB, Furnstahl (in progress)

3) np dominance at intermediate (300-500 MeV) relative momenta

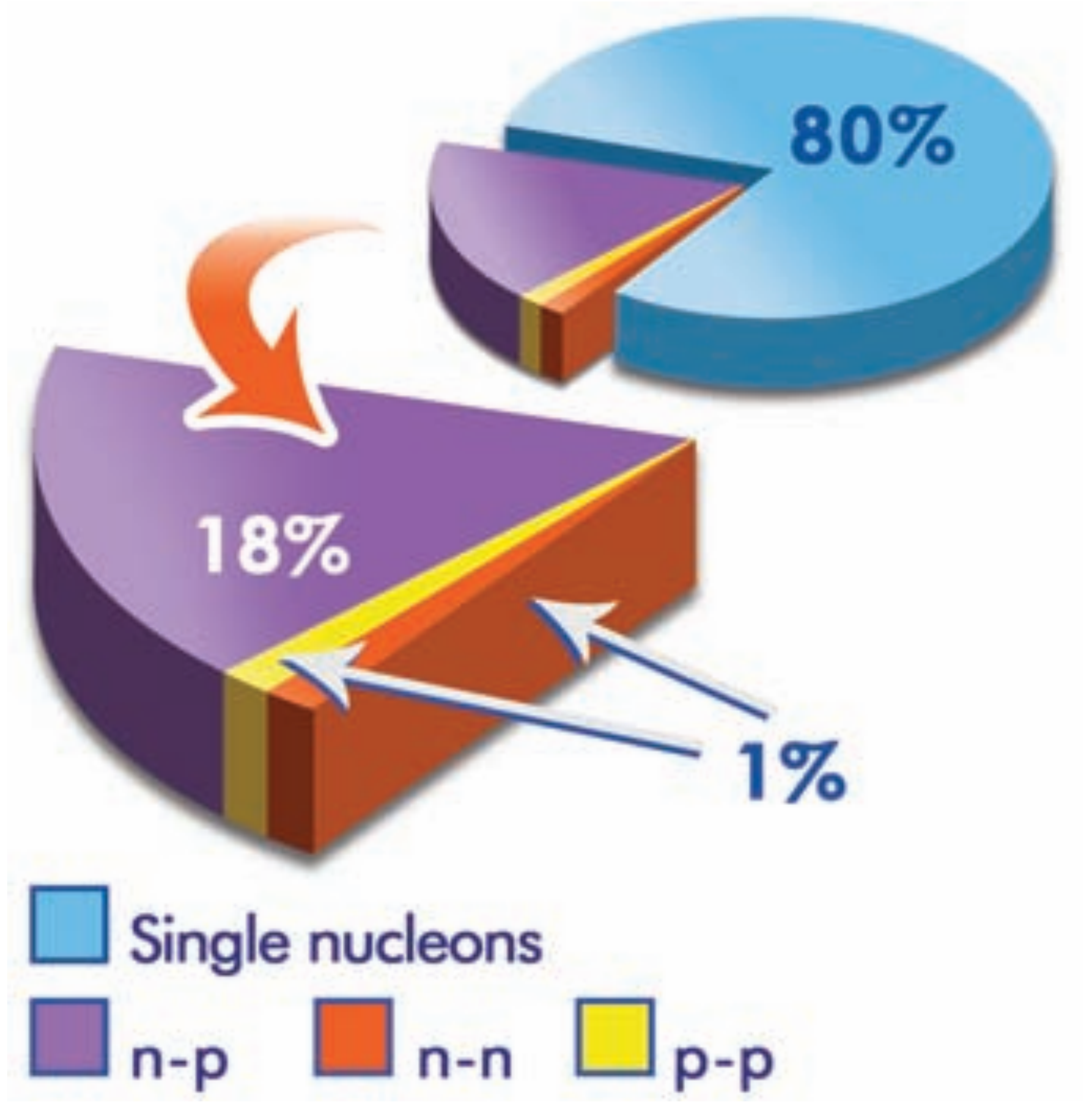

Fig. 3. The average fraction of nucleons in the various initial-state configurations of ^{12}C.

R. Subedi et al., Science (2008)

20% of nucleons in SRC pairs
but mostly neutron-proton

SRC phenomenology revisited (low-res picture)

Tropiano, SKB, Furnstahl (in progress)

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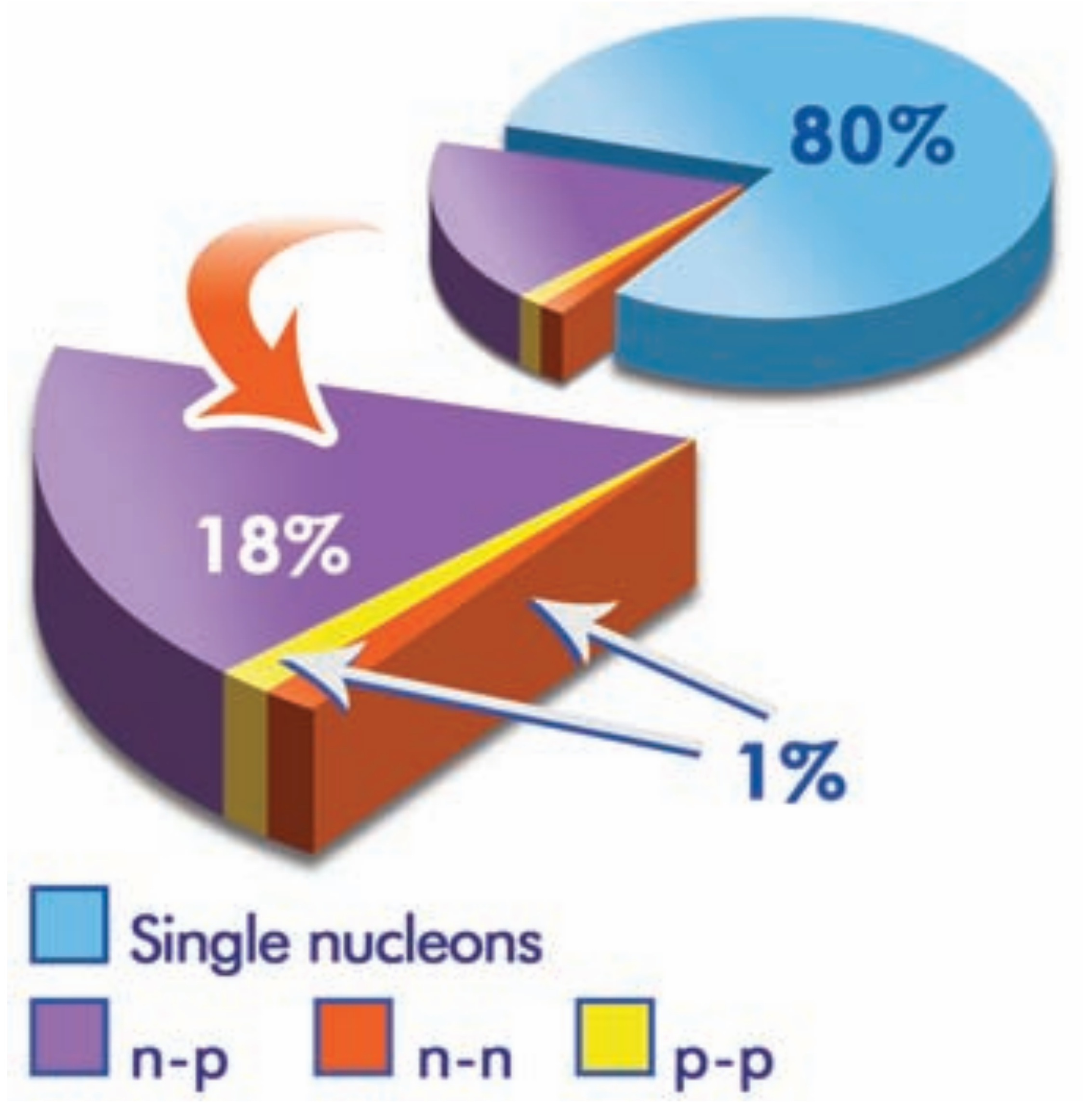

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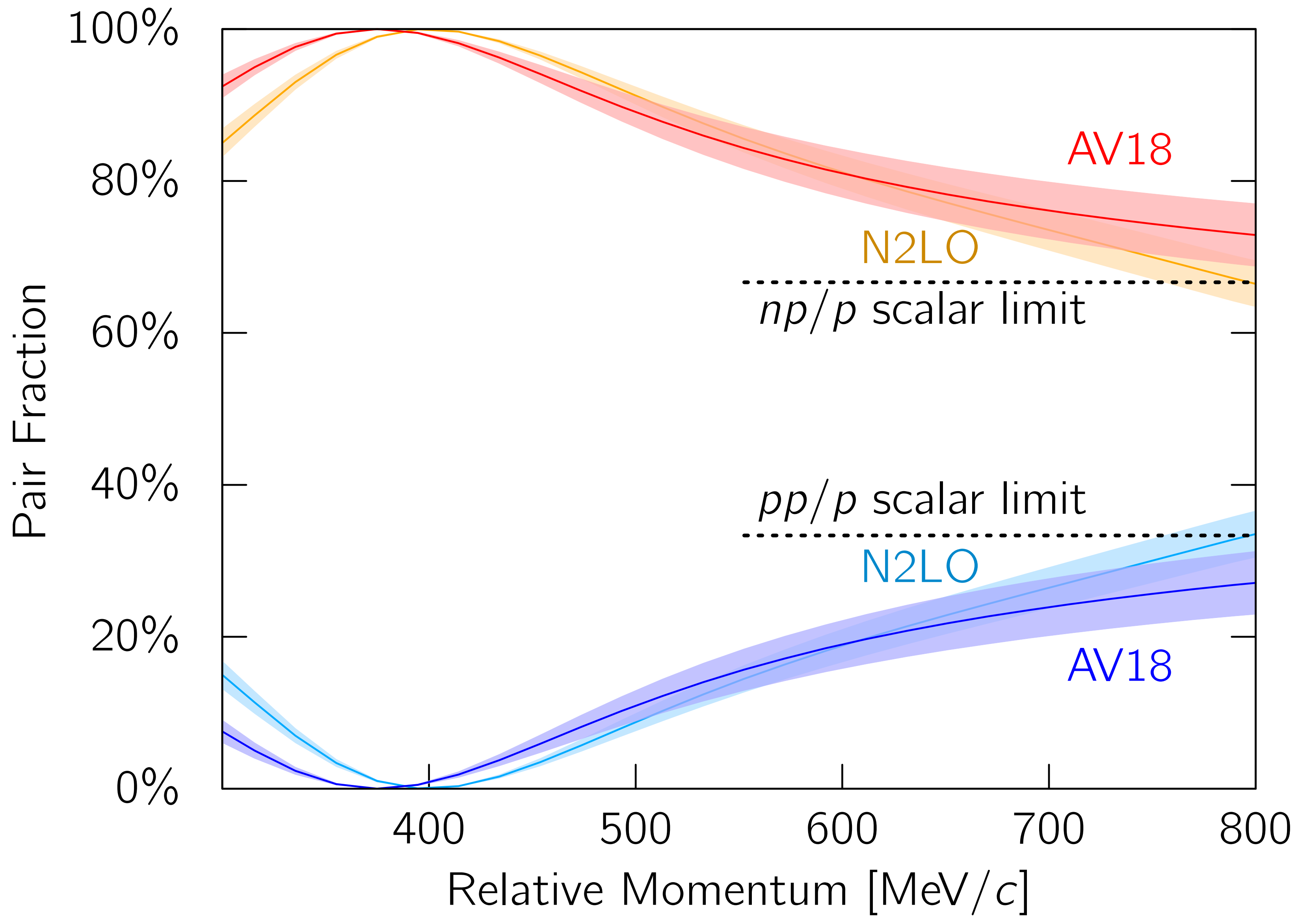

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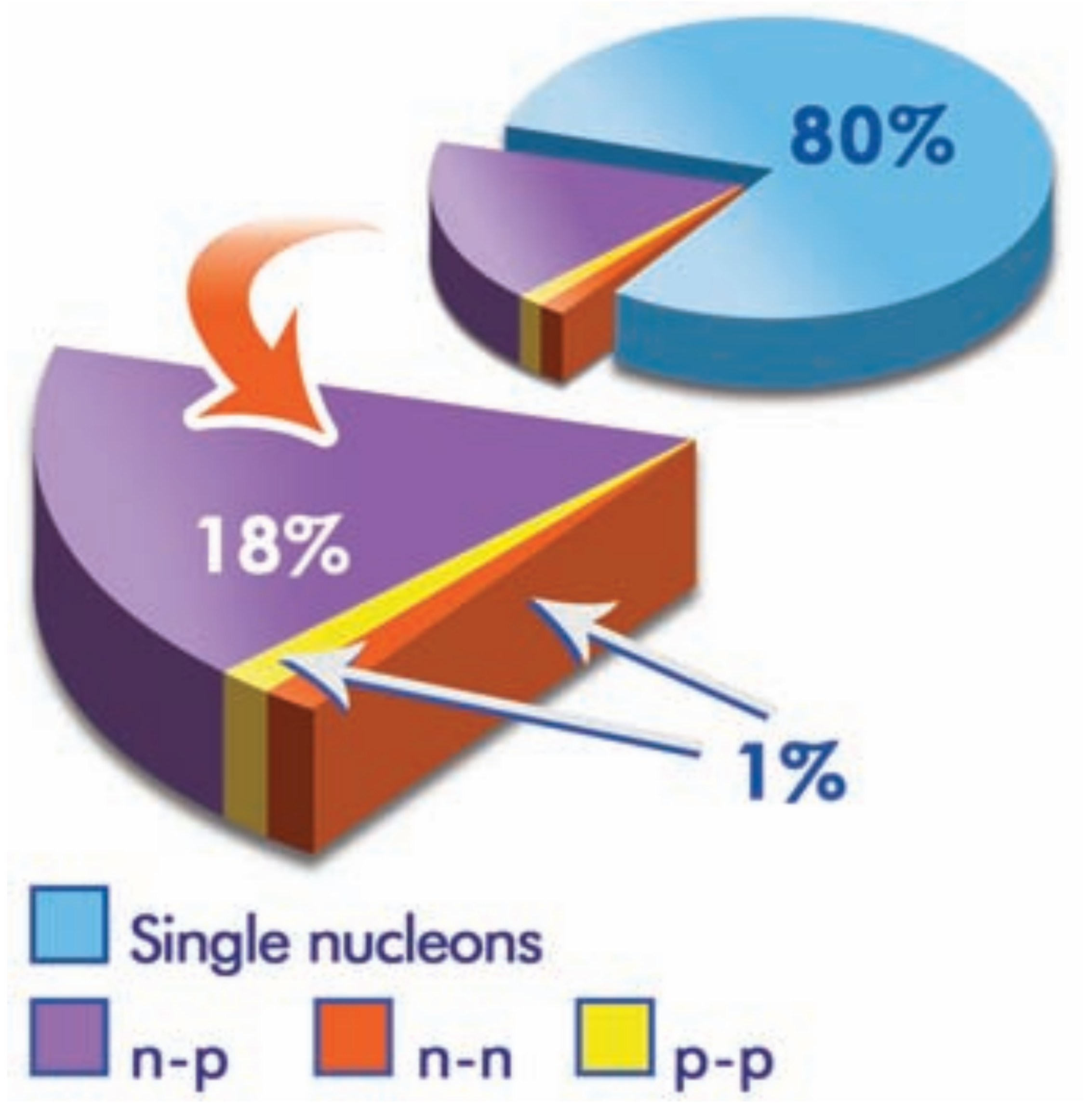

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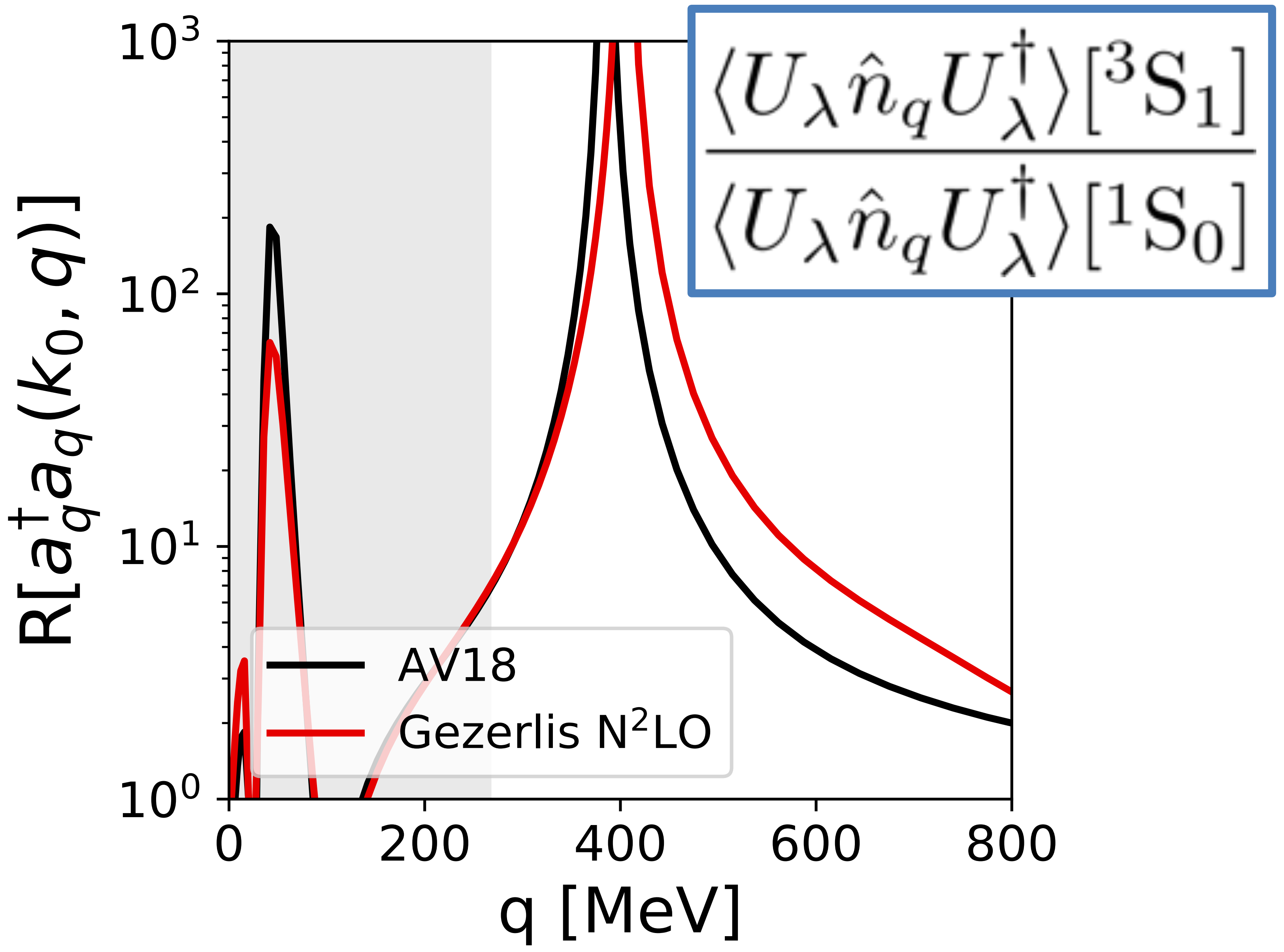
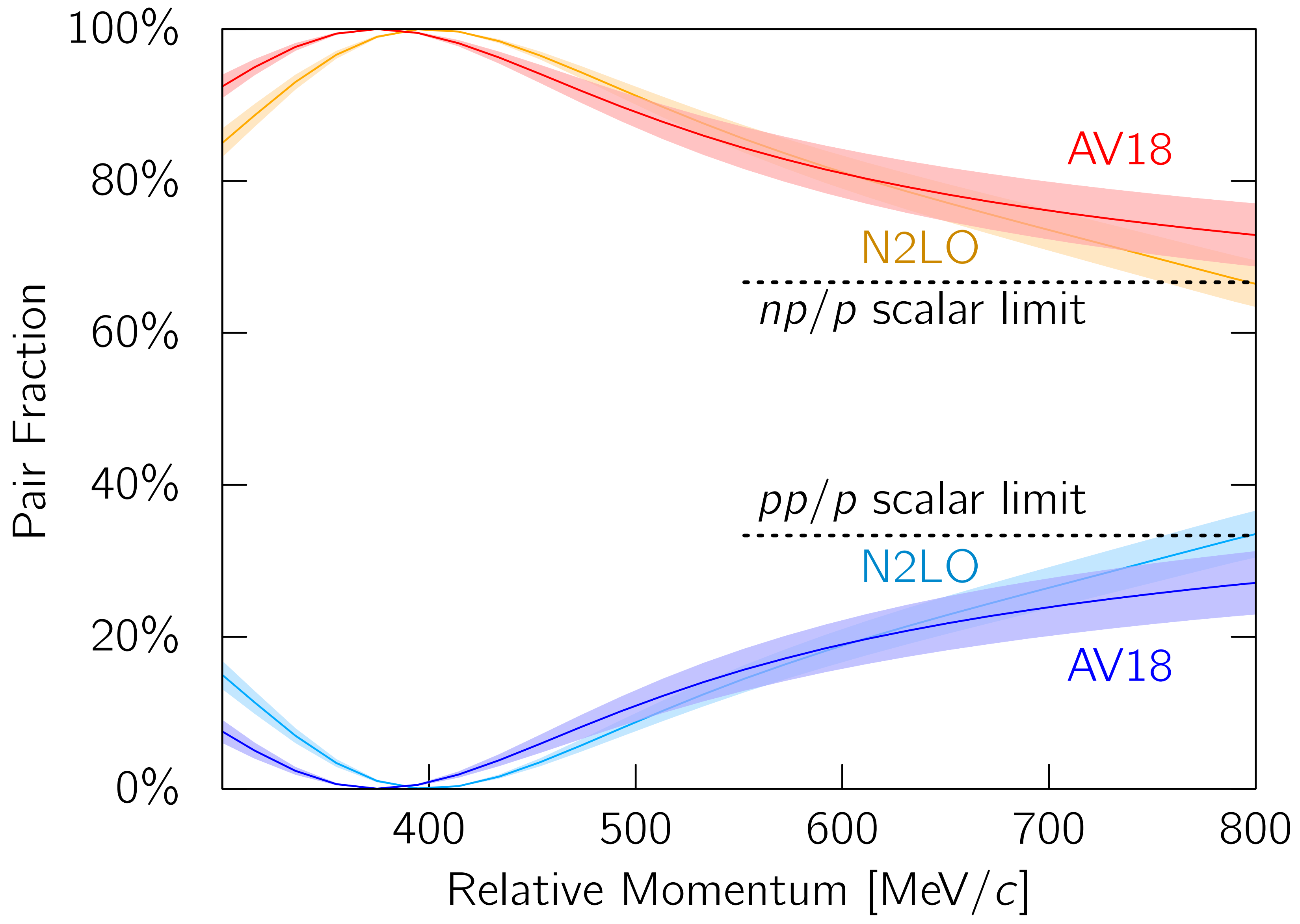
Ratio of *evolved* high-mom. distributions in a low-mom. state (insensitive to details!)





SRC phenomenology revisited (low-res picture)

Tropiano, SKB, Furnstahl (in progress)

6) Generalized Contact Formalism (GCF)

SRC phenomenology revisited (low-res picture)

Tropiano, SKB, Furnstahl (in progress)

6) Generalized Contact Formalism (GCF)

$$\rho_A^{NN,\alpha}(r) = C_A^{NN,\alpha} \times |\varphi_{NN}^{\alpha}(r)|^2$$
$$n_A^{NN,\alpha}(q) = C_A^{NN,\alpha} \times |\varphi_{NN}^{\alpha}(q)|^2$$

A-dep scale factors ("nuclear contacts") $C_A \sim < \chi | \chi >$

Universal (same all A, **not** V_{NN}) shape from two-body zero energy wf ϕ

SRC phenomenology revisited (low-res picture)

Tropiano, SKB, Furnstahl (in progress)

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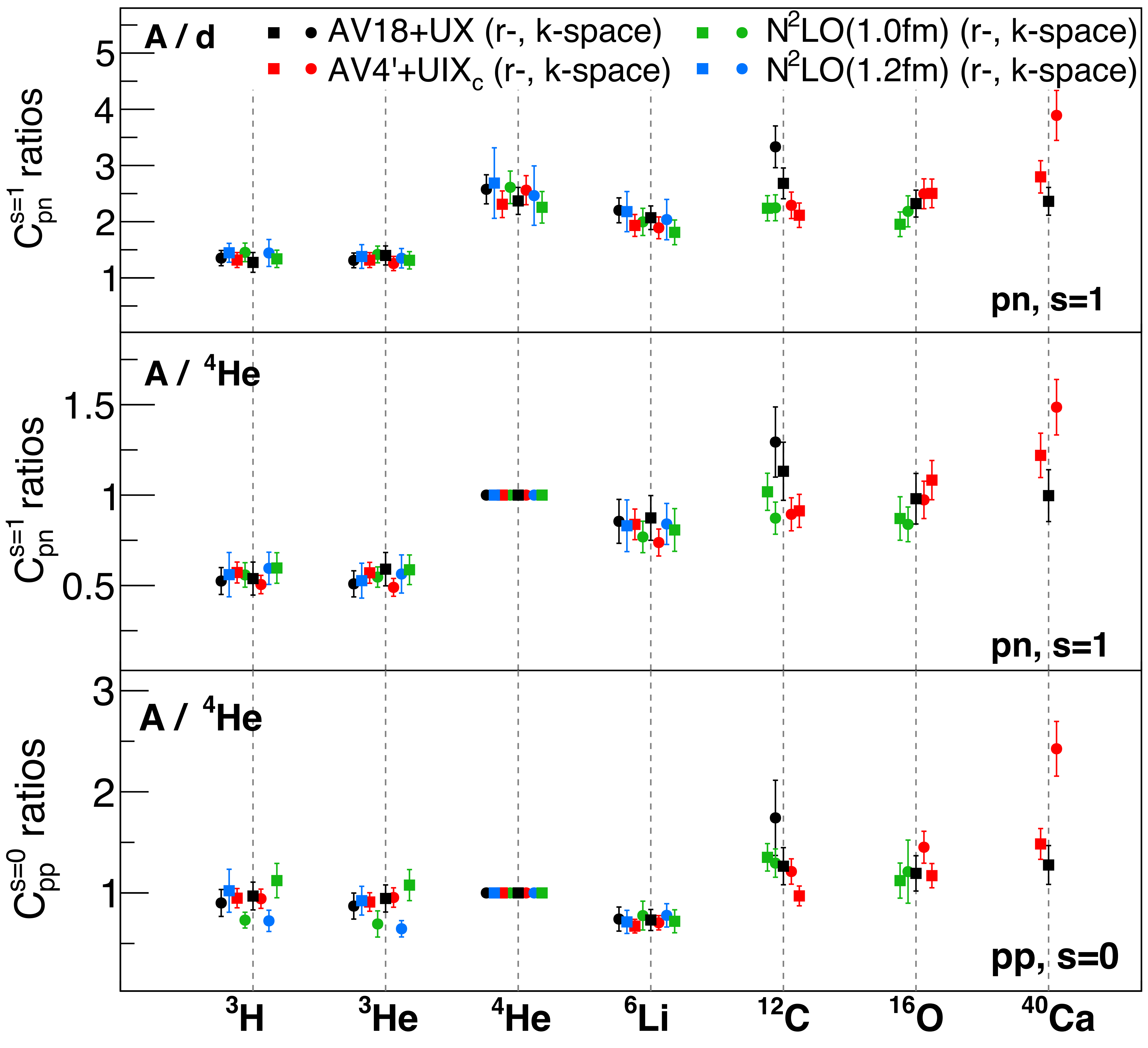
A-dep scale factors ("nuclear contacts") $C_A \sim <\chi|\chi>$

Universal (same all A, **not** V_{NN}) shape from two-body zero energy wf ϕ

But φ_{NN} is scale and scheme dependent. Ratios are independent but only probe "mean field" part



SRC phenomenology revisited (low-res picture)

Tropiano, SKB, Furnstahl (in progress)

6) Gene

$\rho_A^{NN,\alpha}($

$n_A^{NN,\alpha}($

A-dep scale

Universal (s
two-body z

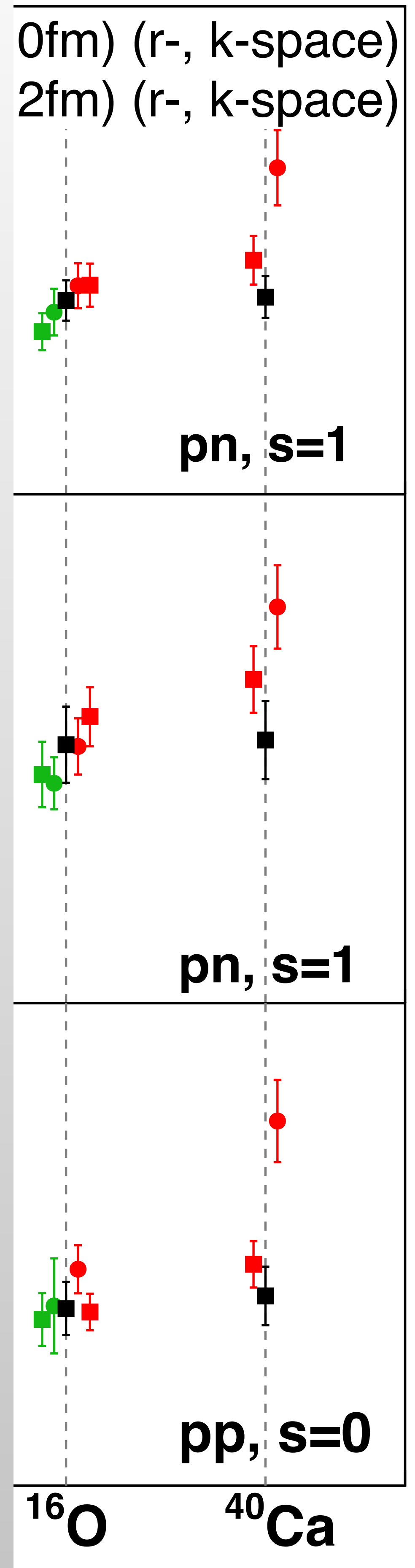
But
schem
are in
probe

Contacts **not** RG invariant

$$C_A = \sum_{K,k',k}^{\Lambda_0} \langle \psi^A_{\Lambda_0} | a^\dagger_{\frac{K}{2}+k} a^\dagger_{\frac{K}{2}-k} a_{\frac{K}{2}-k'} a_{\frac{K}{2}+k'} | \psi^A_{\Lambda_0} \rangle \quad \Rightarrow \quad f(\Lambda) \sum_{K,k',k}^{\Lambda} \langle \psi^A_{\Lambda} | a^\dagger_{\frac{K}{2}+k} a^\dagger_{\frac{K}{2}-k} a_{\frac{K}{2}-k'} a_{\frac{K}{2}+k'} | \psi^A_{\Lambda} \rangle$$

A-independent

…But ratios in different A approx. RG invariant



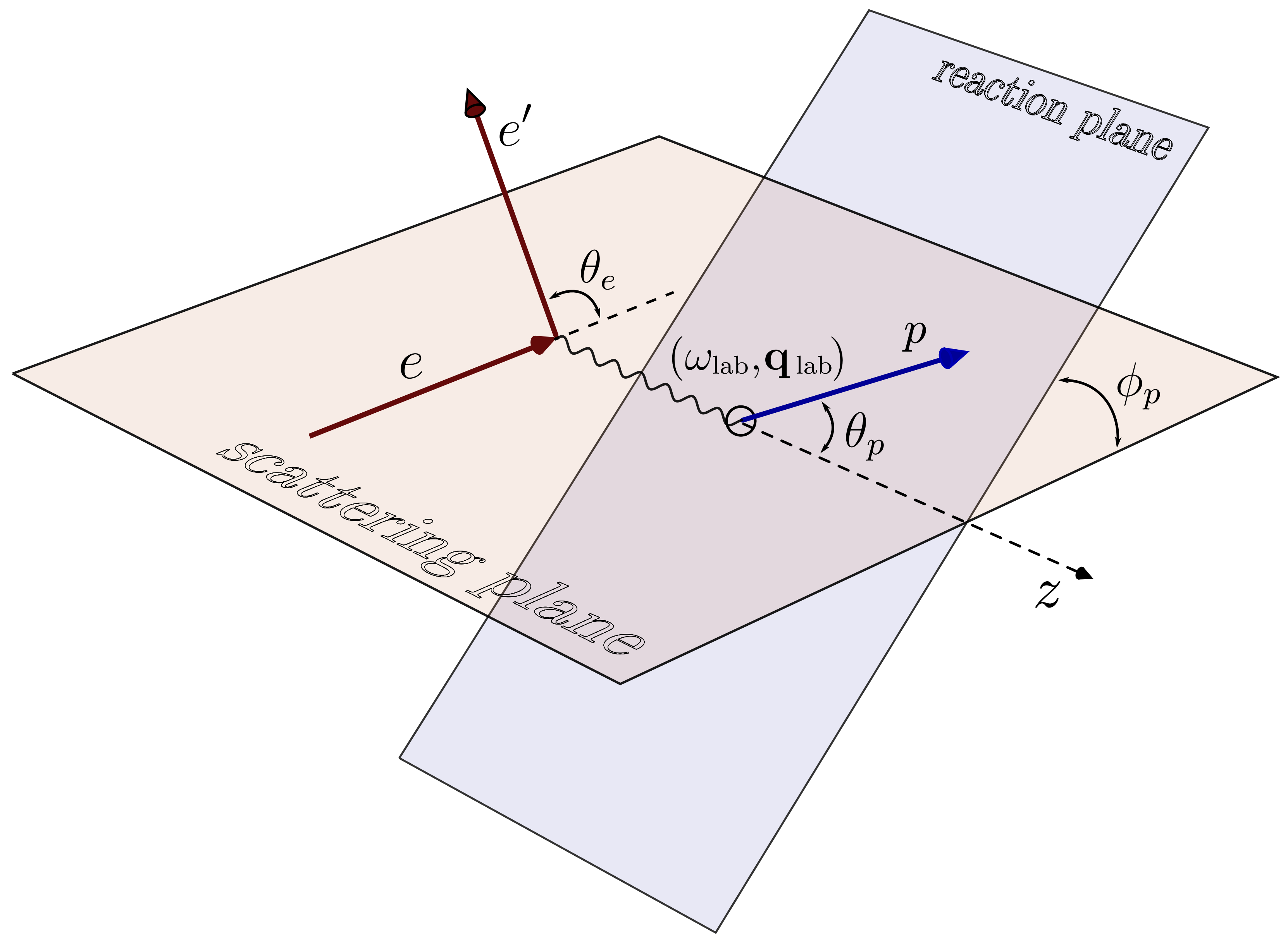
Scale dependence of deuteron electrodisintegration

S. More, SKB, R.J. Furnstahl, Phys. Rev. C **96** 054004, (2017)

Test ground: ^{2}H(e,e′p)n

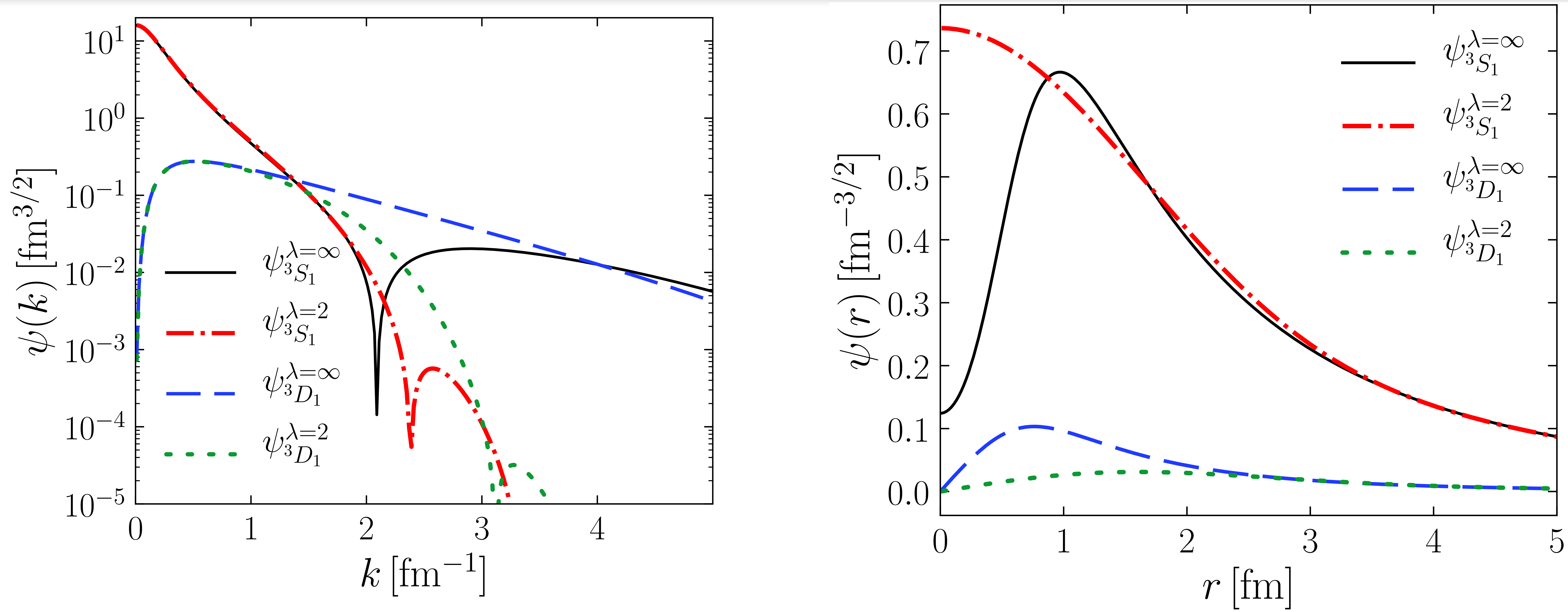
- Simplest knockout process (no induced 3N forces/currents)
- Focus on longitudinal structure function f_L

$$f_L \sim \sum_{m_s, m_J} \left| \langle \psi_f | J_0 | \psi_i \rangle \right|^2$$



- $$f_L^\lambda \sim \left| \underbrace{\langle \psi_f | U_\lambda^\dagger}_{\psi_f^\lambda} \underbrace{U_\lambda J_0 U_\lambda^\dagger}_{J_0^\lambda} \underbrace{U_\lambda | \psi_i \rangle}_{\psi_i^\lambda} \right|^2; \quad U_\lambda^\dagger U_\lambda = I; \quad f_L^\lambda = f_L$$
- Components (deuteron wf, transition operator, FSI) scale-dependent, total is not.
- Are some resolutions "better" than others? E.g., in a given kinematics, can FSI be minimized with different choices of λ?? How do interpretations change with scale?

Deuteron wave function evolution

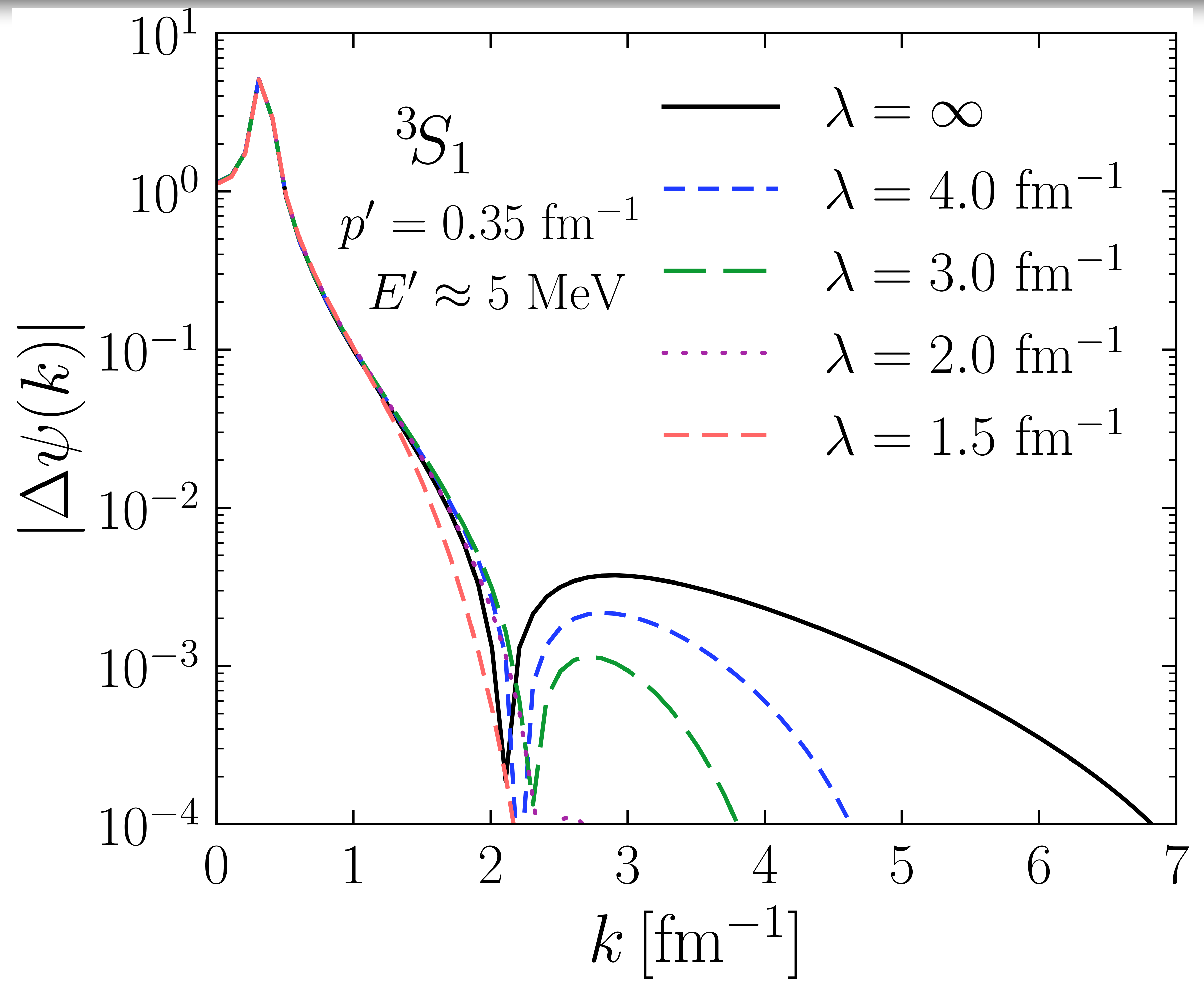


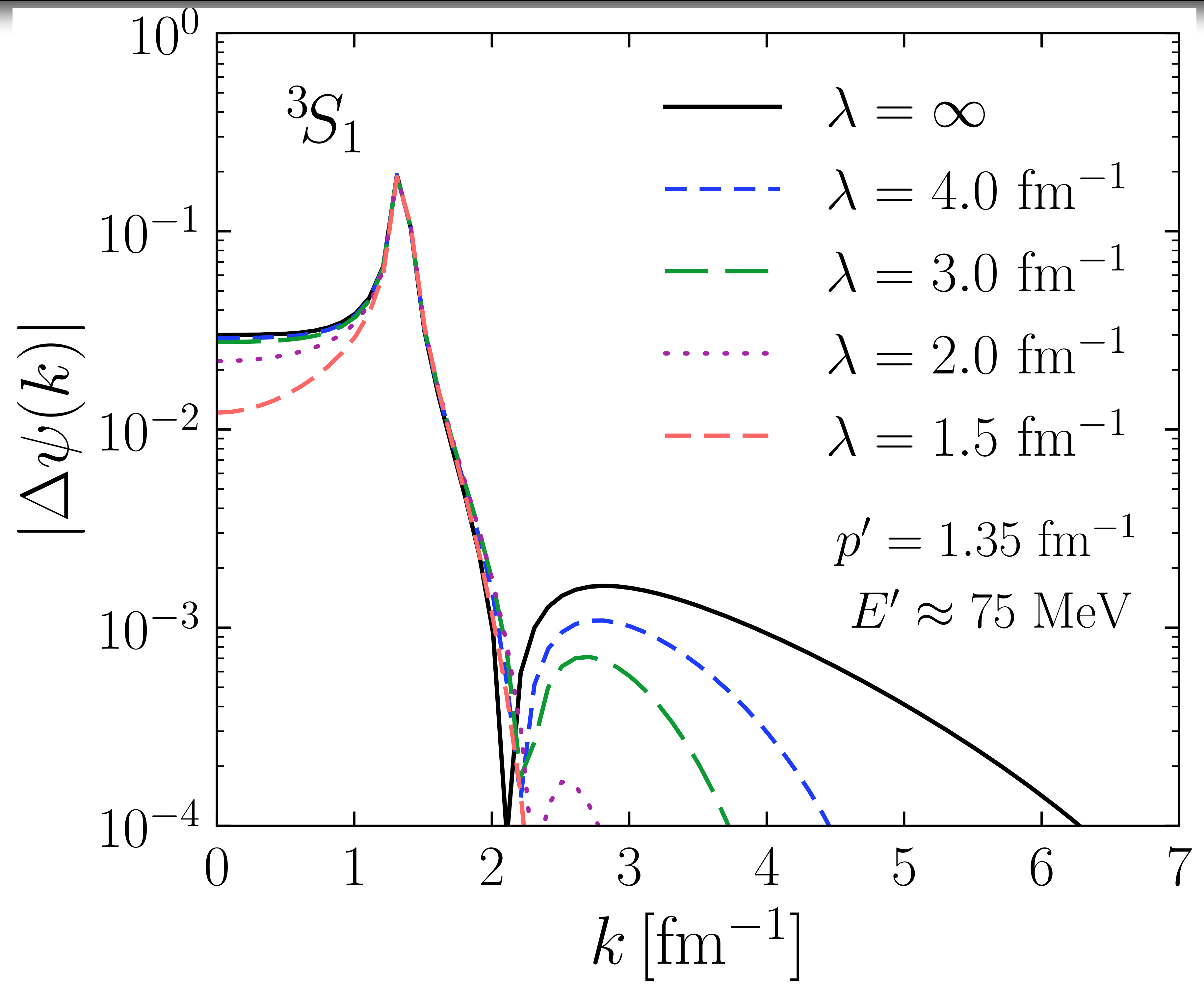
k < λ components invariant <==> RG preserves long-distance physics

k > λ components suppressed <==> short-range correlations blurred out

Folklore: Simple wave functions at low λ <==> more complicated operators?
especially for high-q processes?

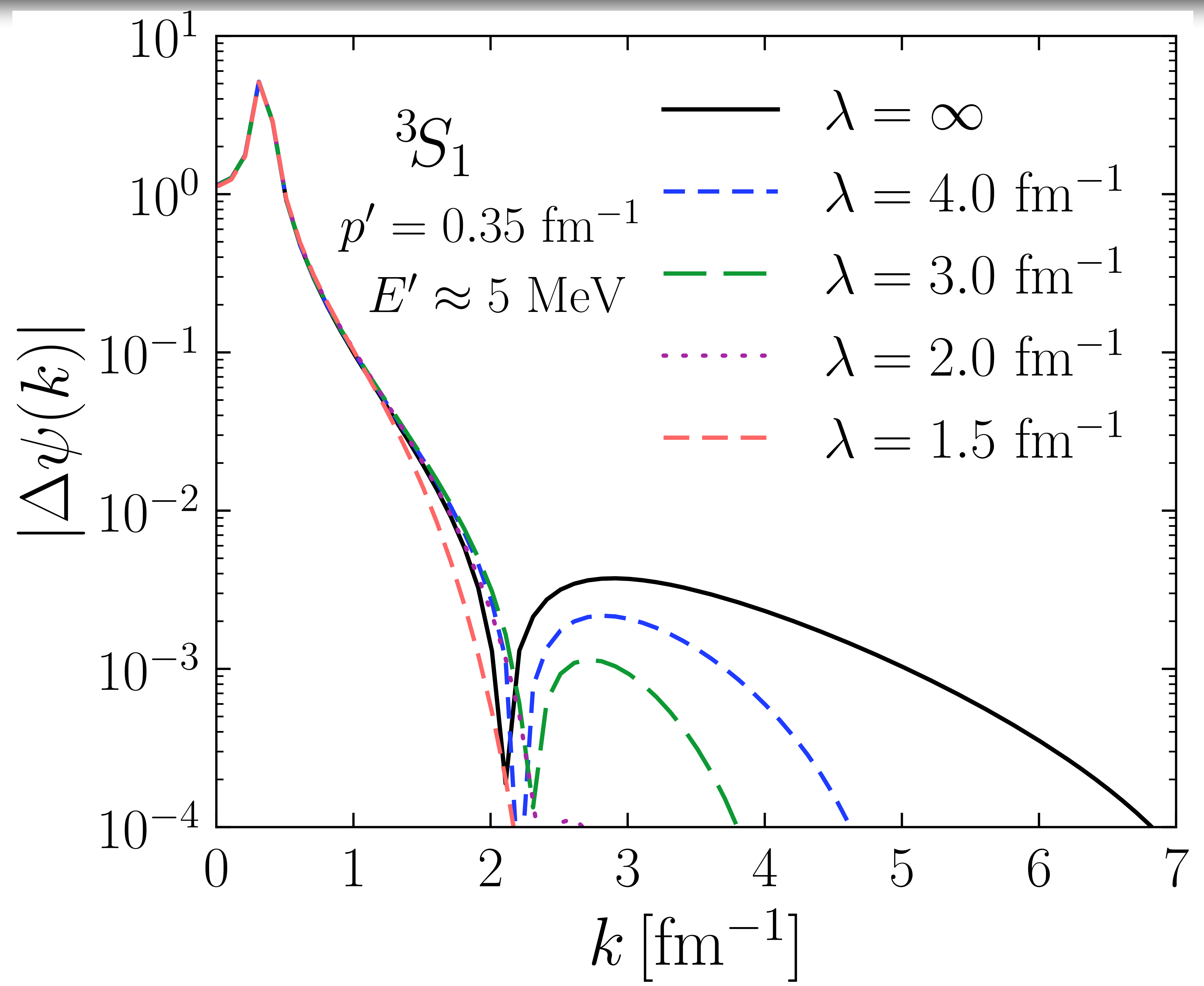
Final-state wave function evolution

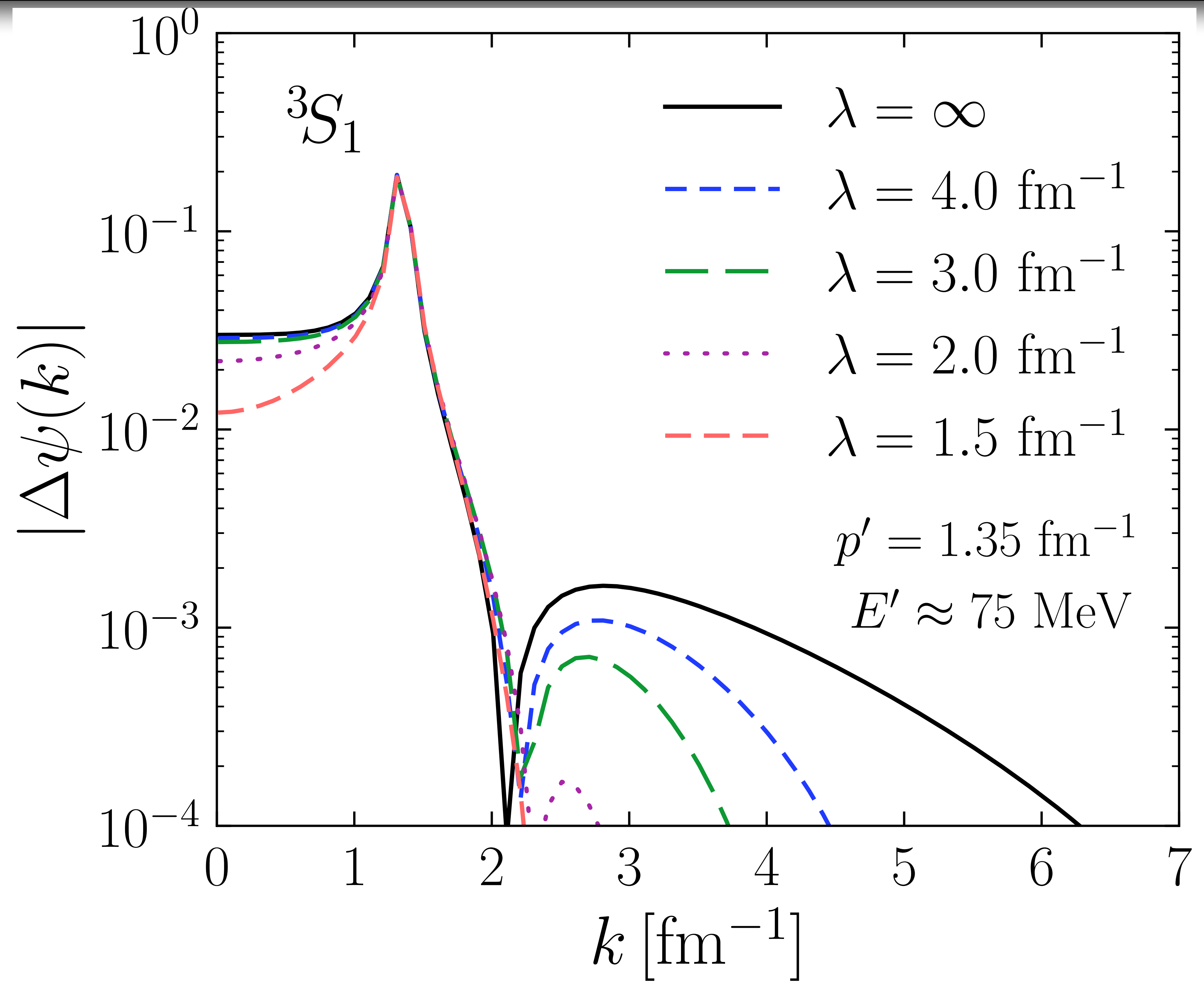




$$\psi_f^\lambda(p';k) = \underbrace{\phi_{p'}}_{\text{IA}} + \underbrace{\Delta\psi_\lambda(p';k)}_{\text{FSI}}$$

Final-state wave function evolution



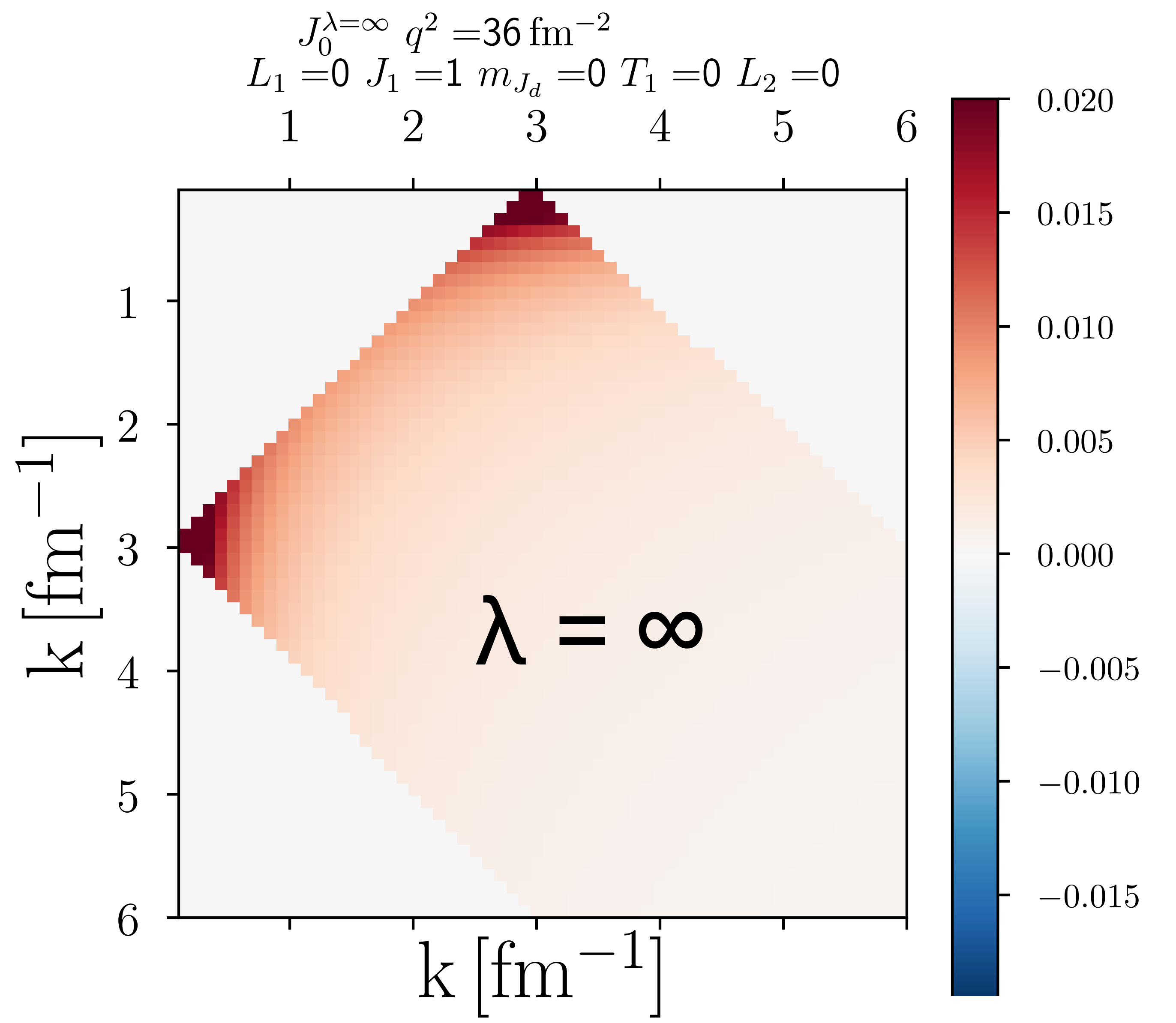


$$\psi_f^\lambda(p';k) = \underbrace{\phi_{p'}}_{\text{IA}} + \underbrace{\Delta\psi_\lambda(p';k)}_{\text{FSI}}$$

- High-k tail suppressed with evolution
- For $p' \gtrsim \lambda,\ \Delta\psi_f^\lambda(p';k)$ localized around outgoing p′

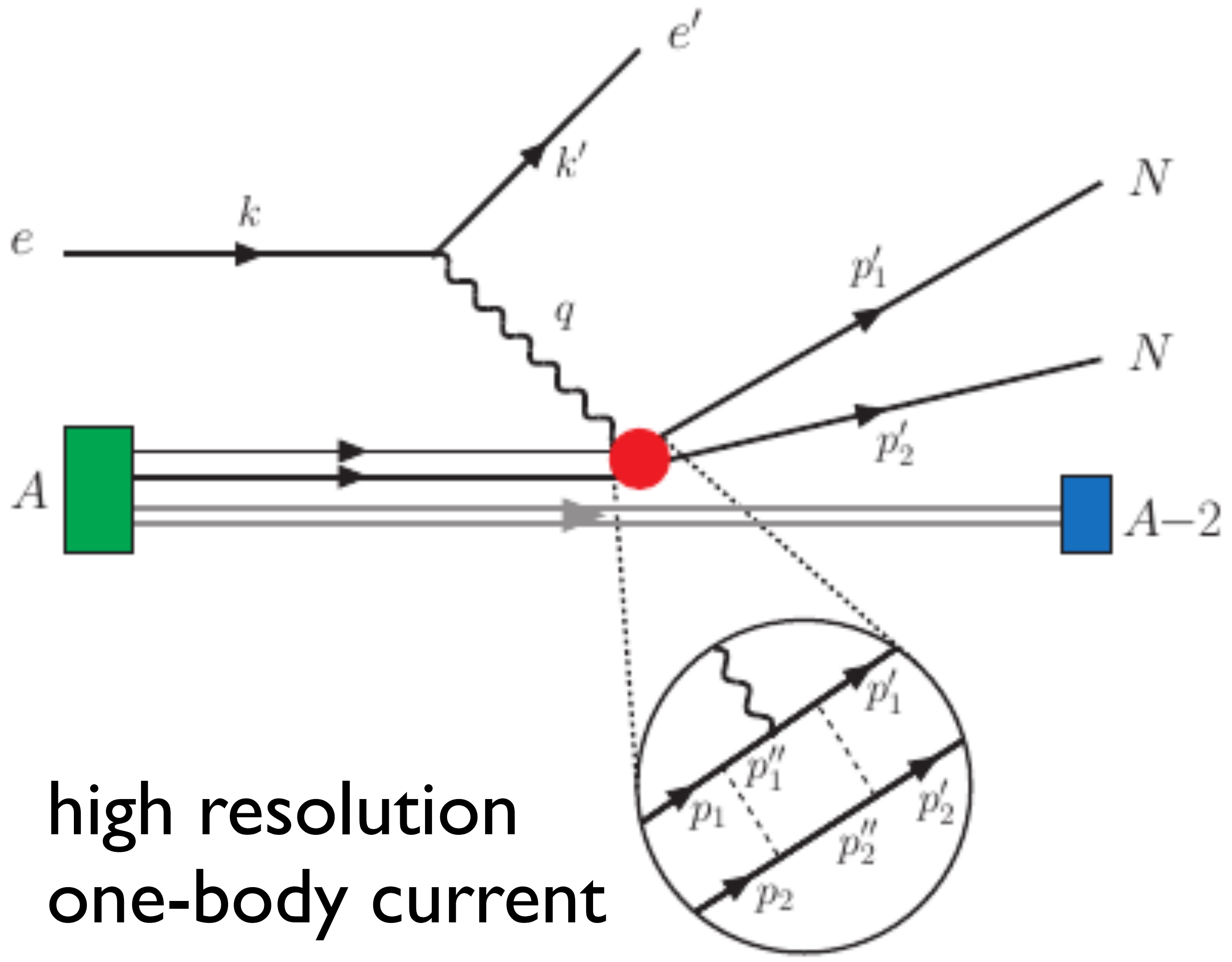
"local decoupling" Dainton et al. PRC 89 (2014)

Current operator evolution



$^3S_1$ channel

$q^2 = 36\ \mathrm{fm}^{-2}$



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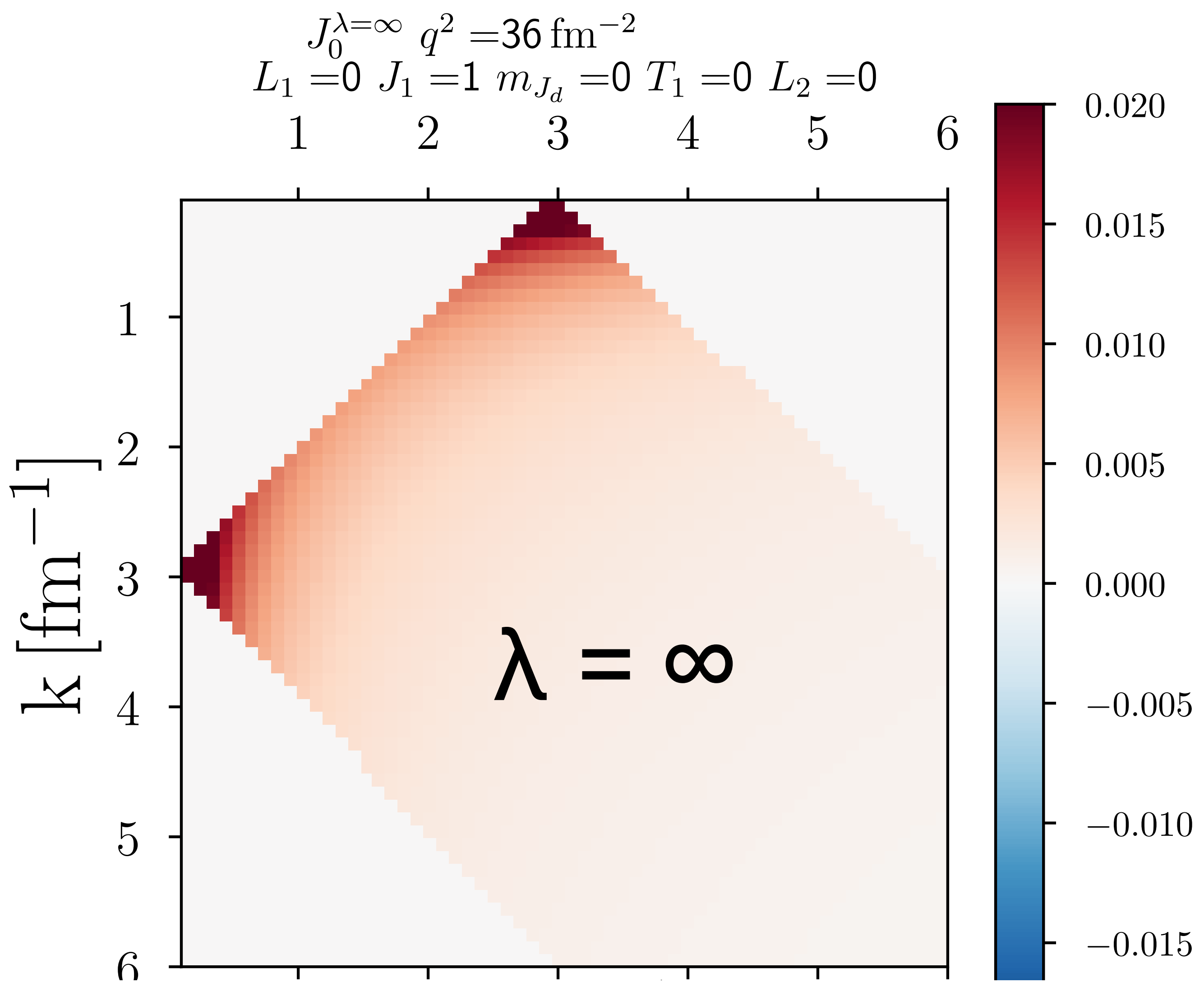

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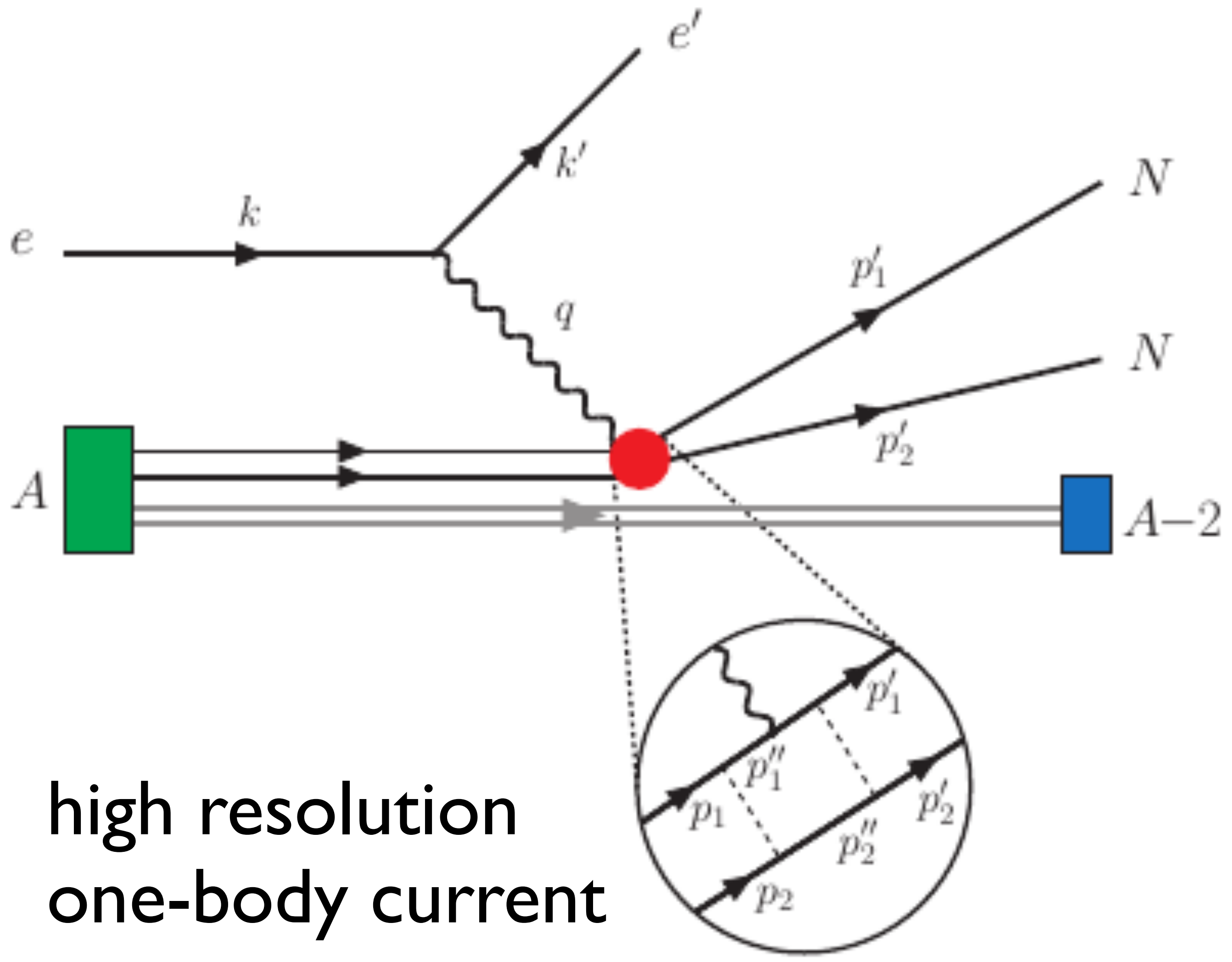
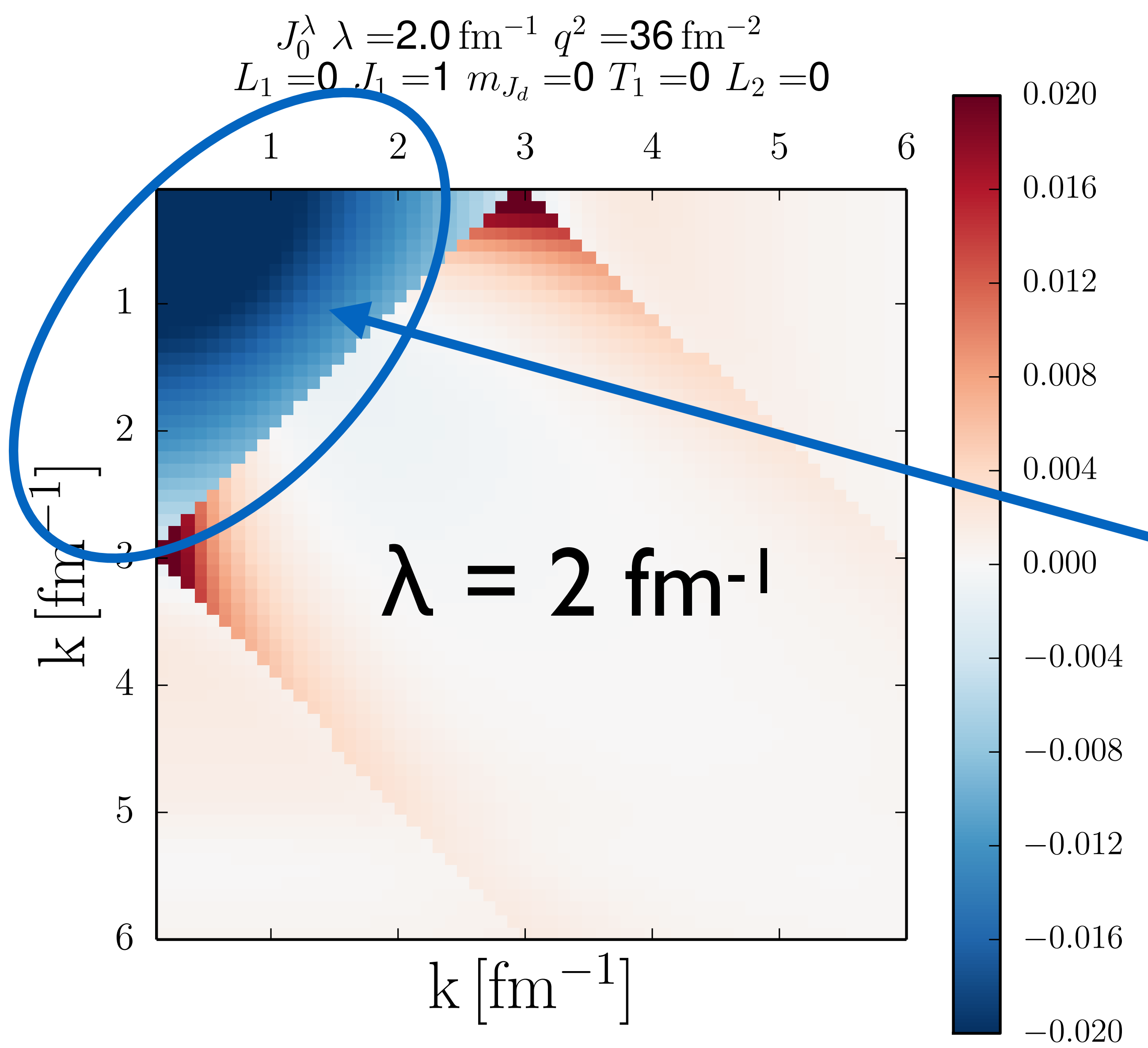
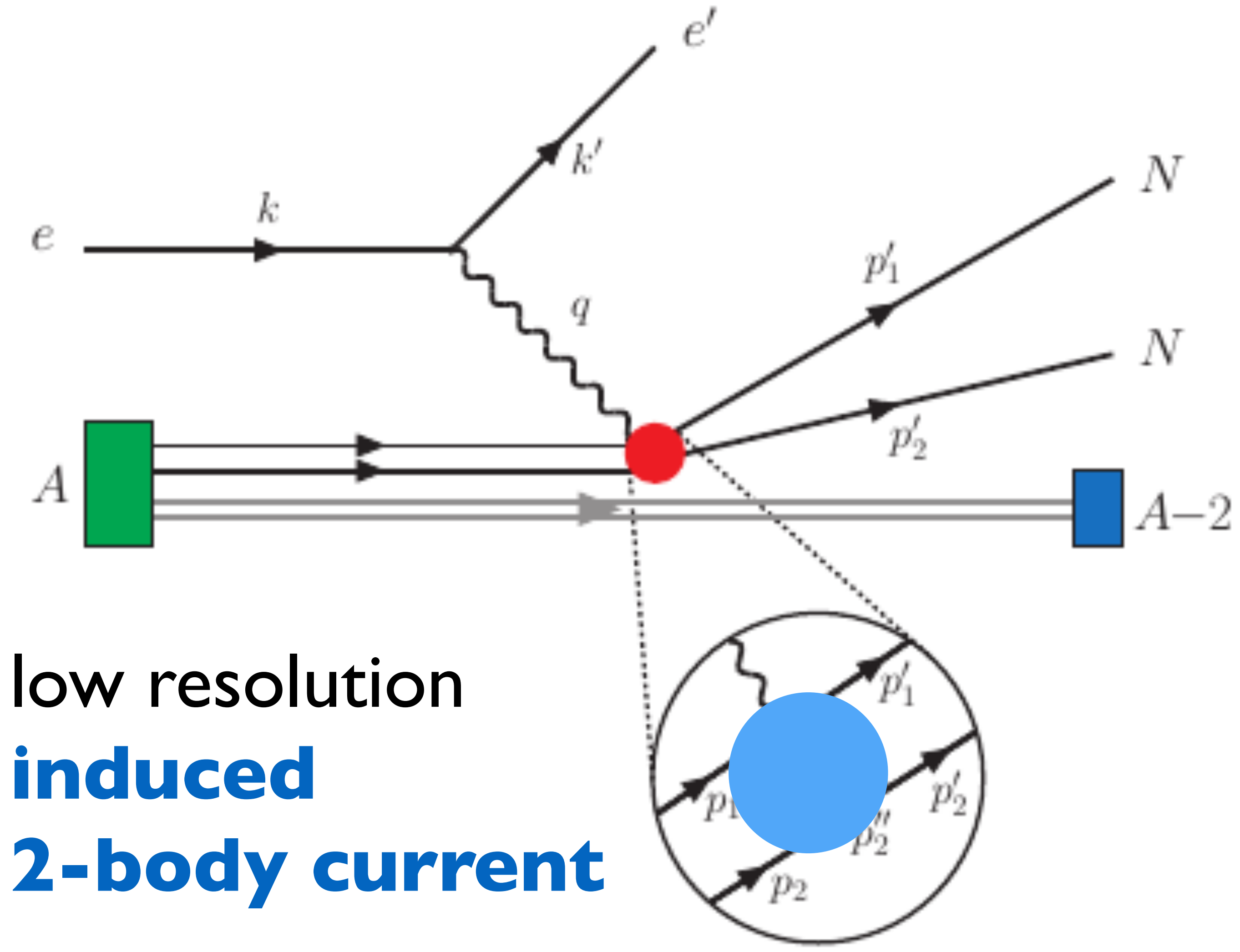
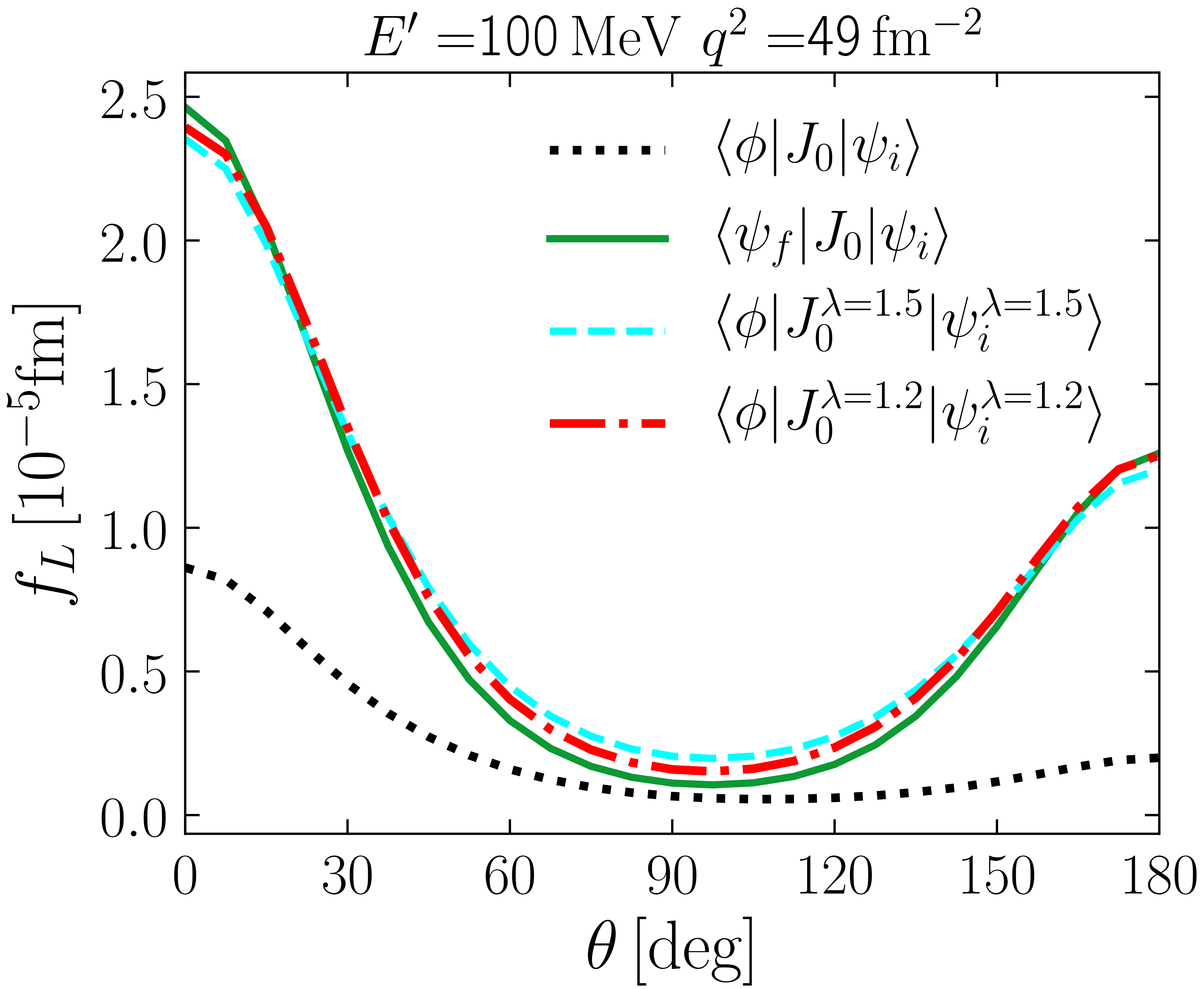






Scale Dependence of Final State Interactions

Look at kinematics relevant to SRC studies



x_B=1.64, Q^2= 1.78 GeV^2

Scale Dependence of Final State Interactions

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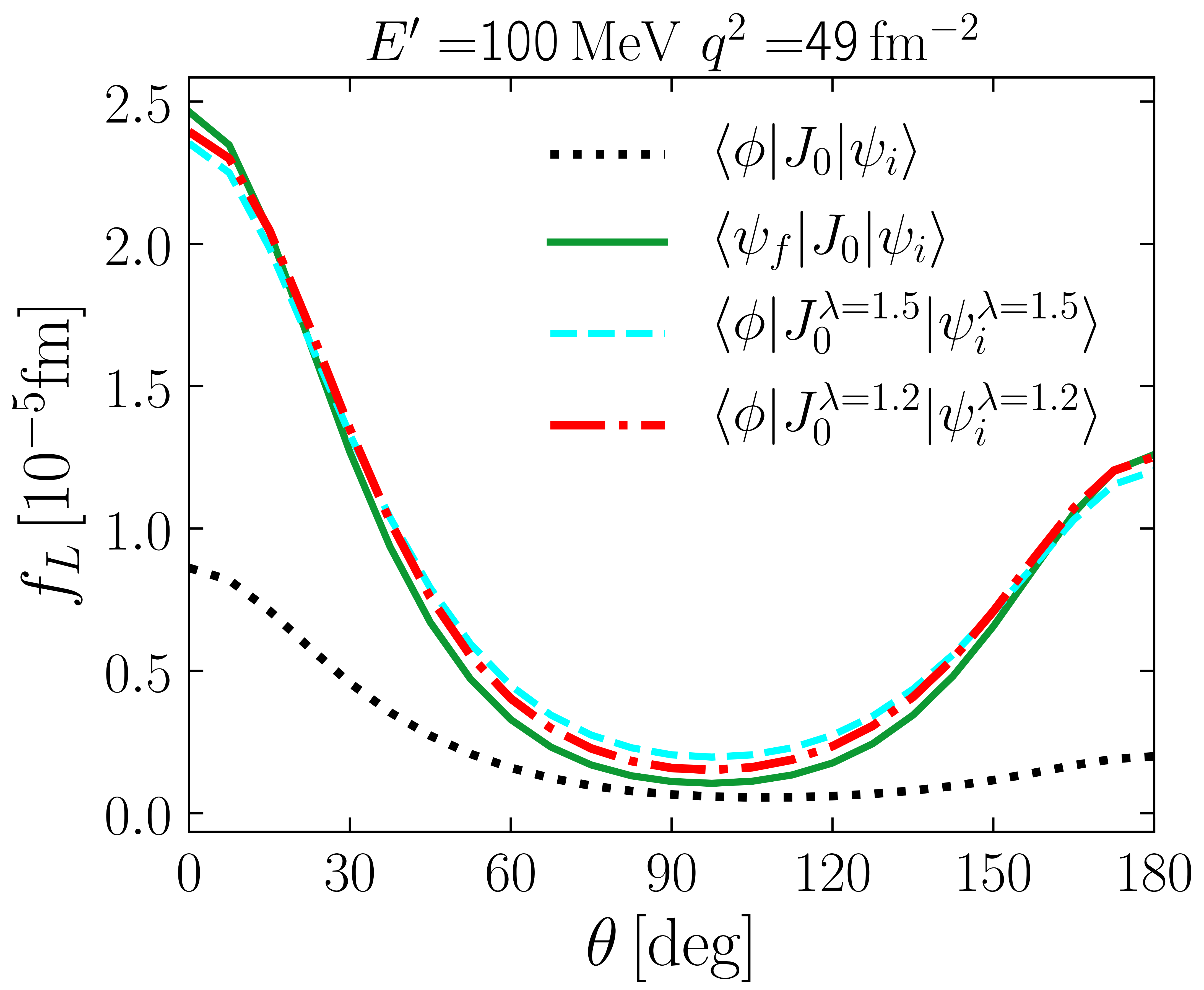


x_B=1.64, Q^2= 1.78 GeV^2

FSI sizable at large λ
but negligible at low-resolution!

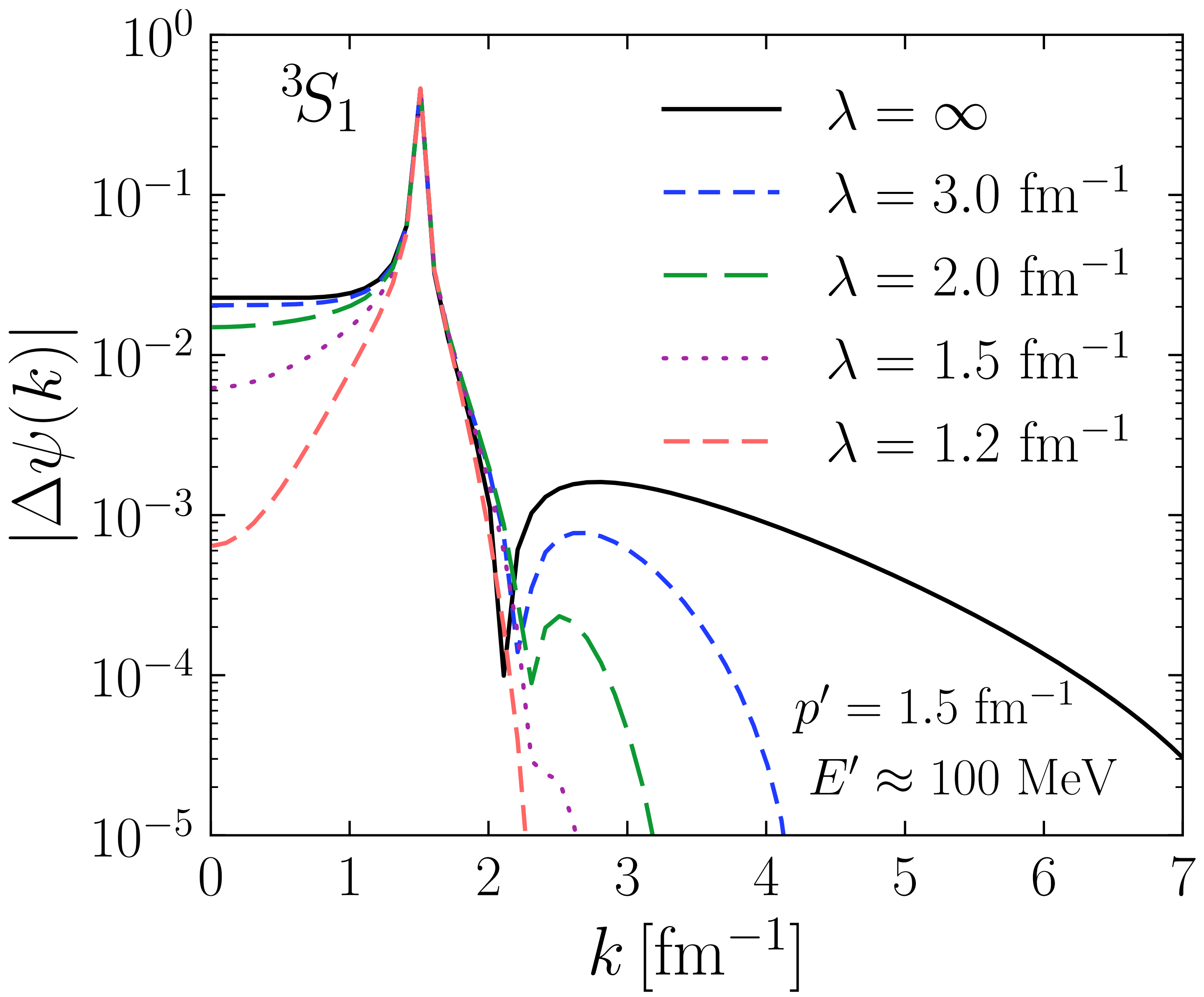
Folklore:

shouldn't hard processes
be complicated in low resolution
(λ << q) pictures?

Why are FSI so small at low λ in these kinematics ?

Scale Dependence of Final State Interactions

final state wf (FSI piece)



For p′ ≳ λ, interacting part of final state wf localized at k ≈ p′

Scale Dependence of Final State Interactions

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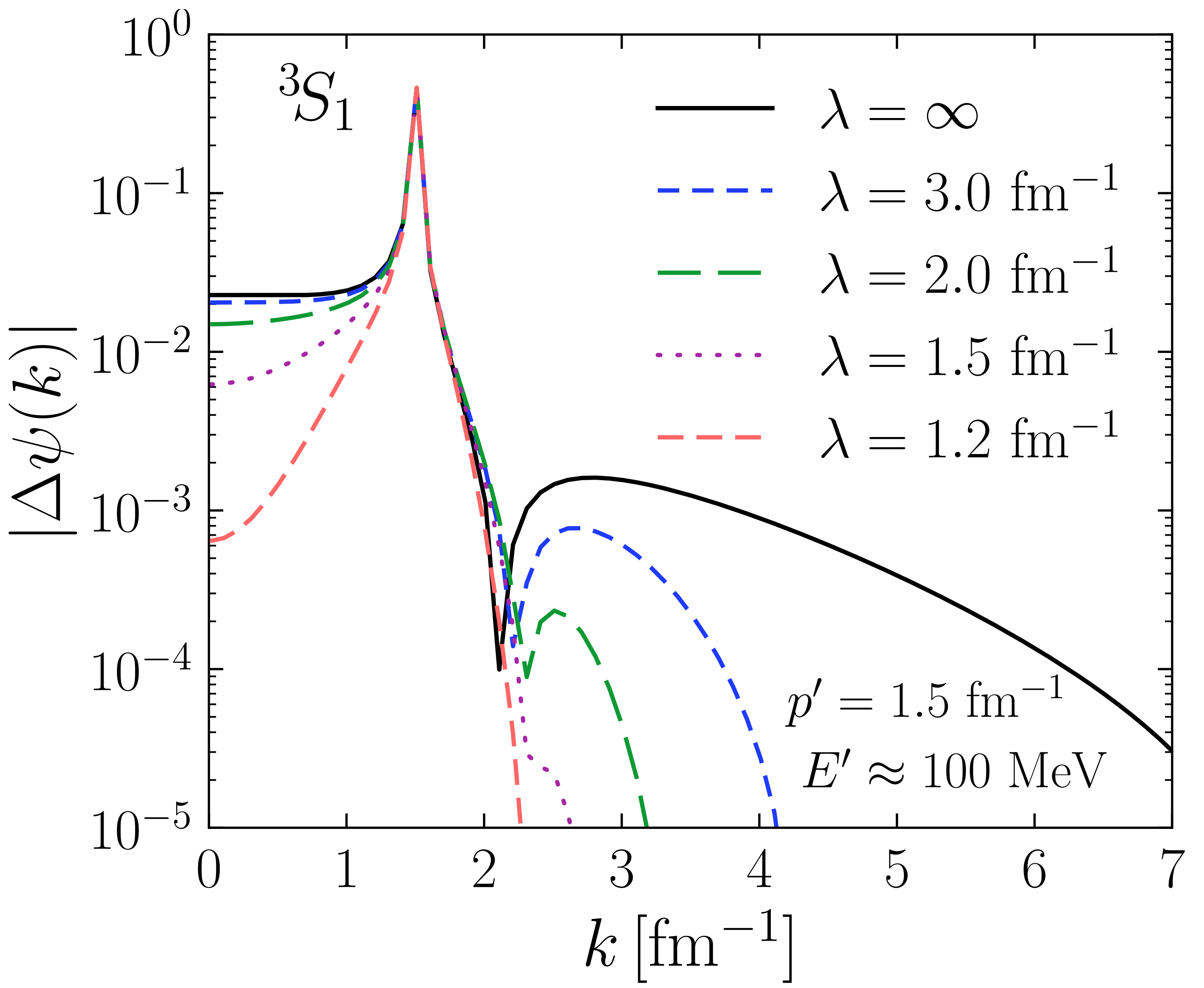
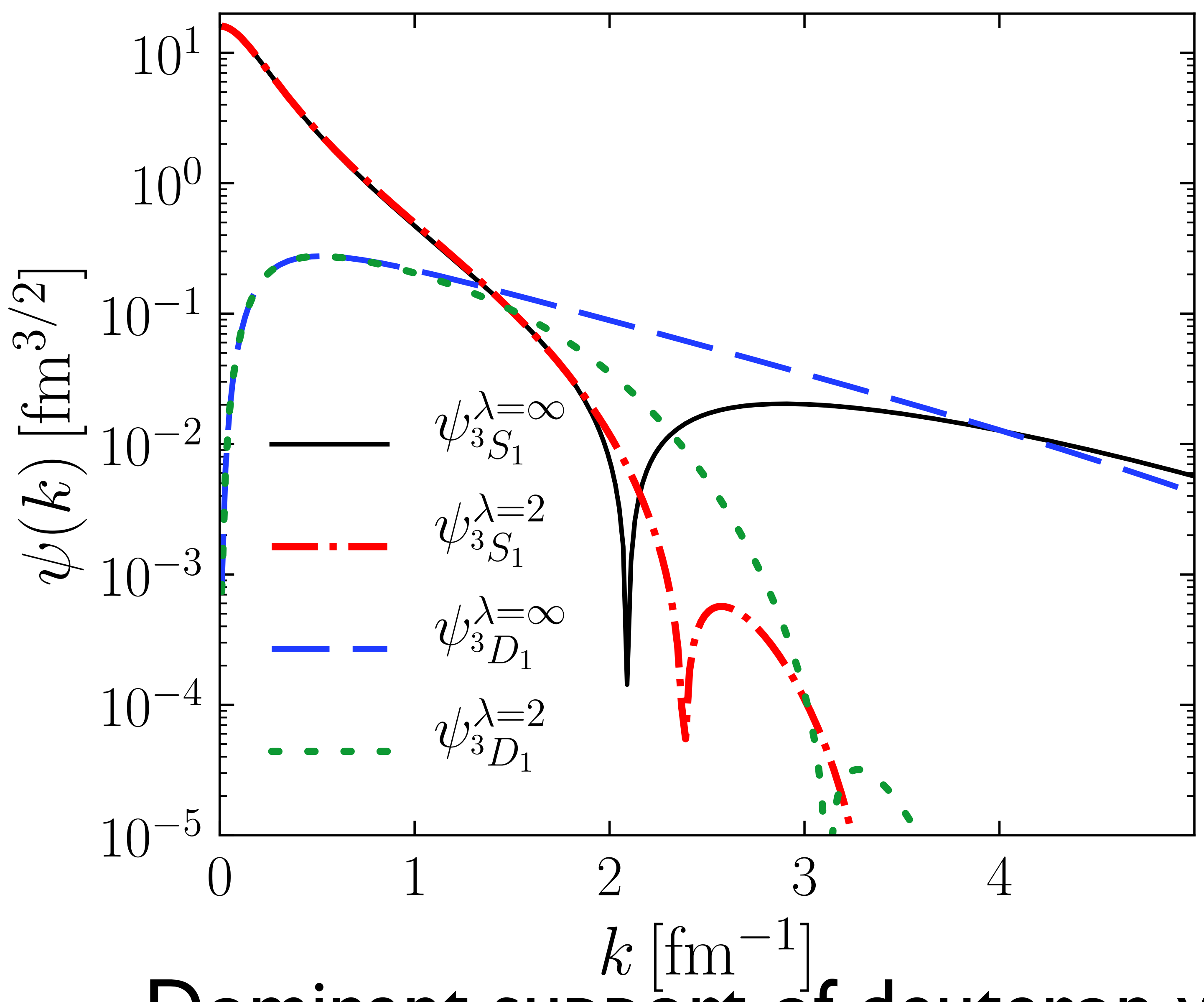


For p′ ≳ λ, interacting part of final state wf localized at k ≈ p′

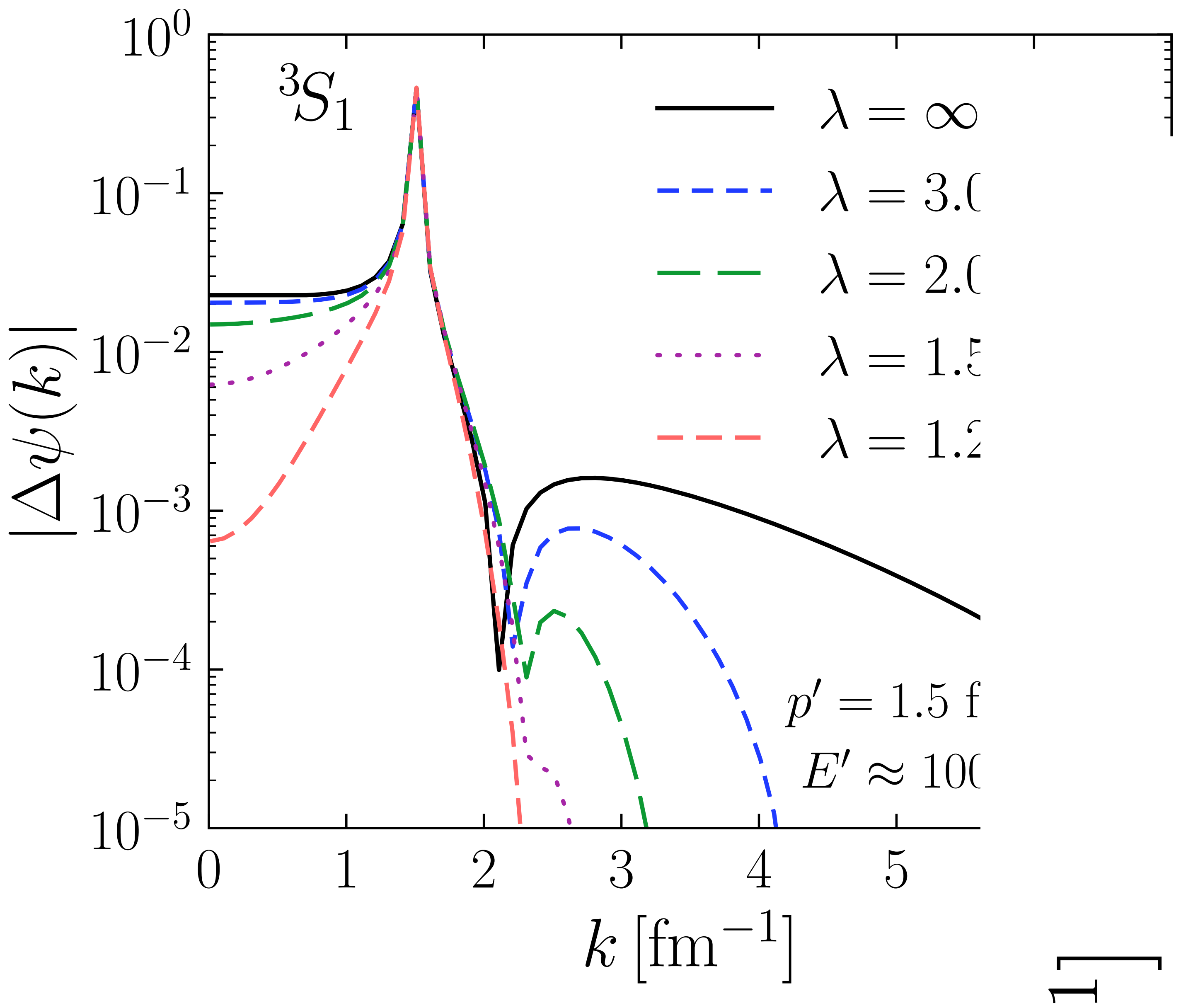
initial state (deuteron) wf

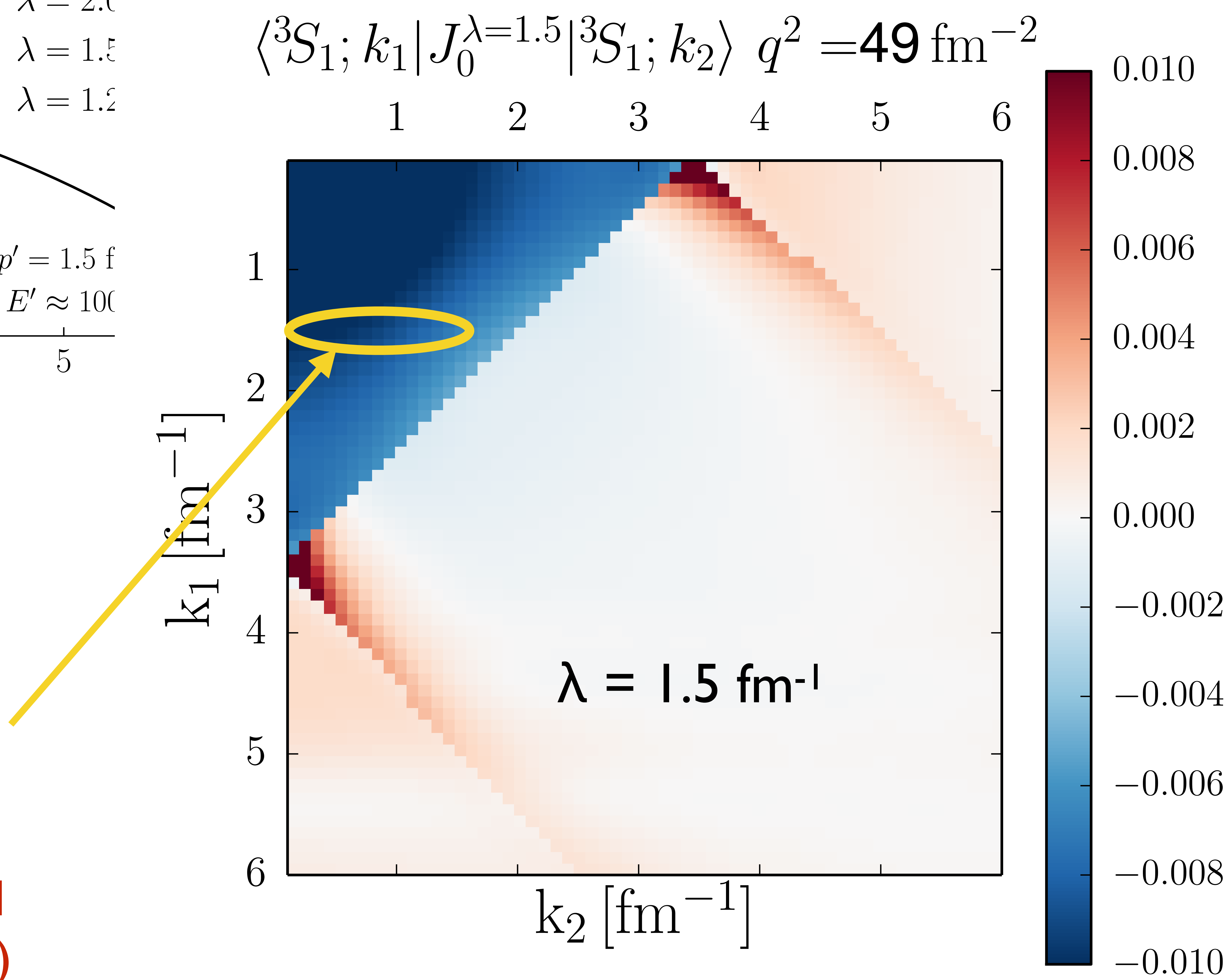


Dominant support of deuteron wf at k ≲ λ

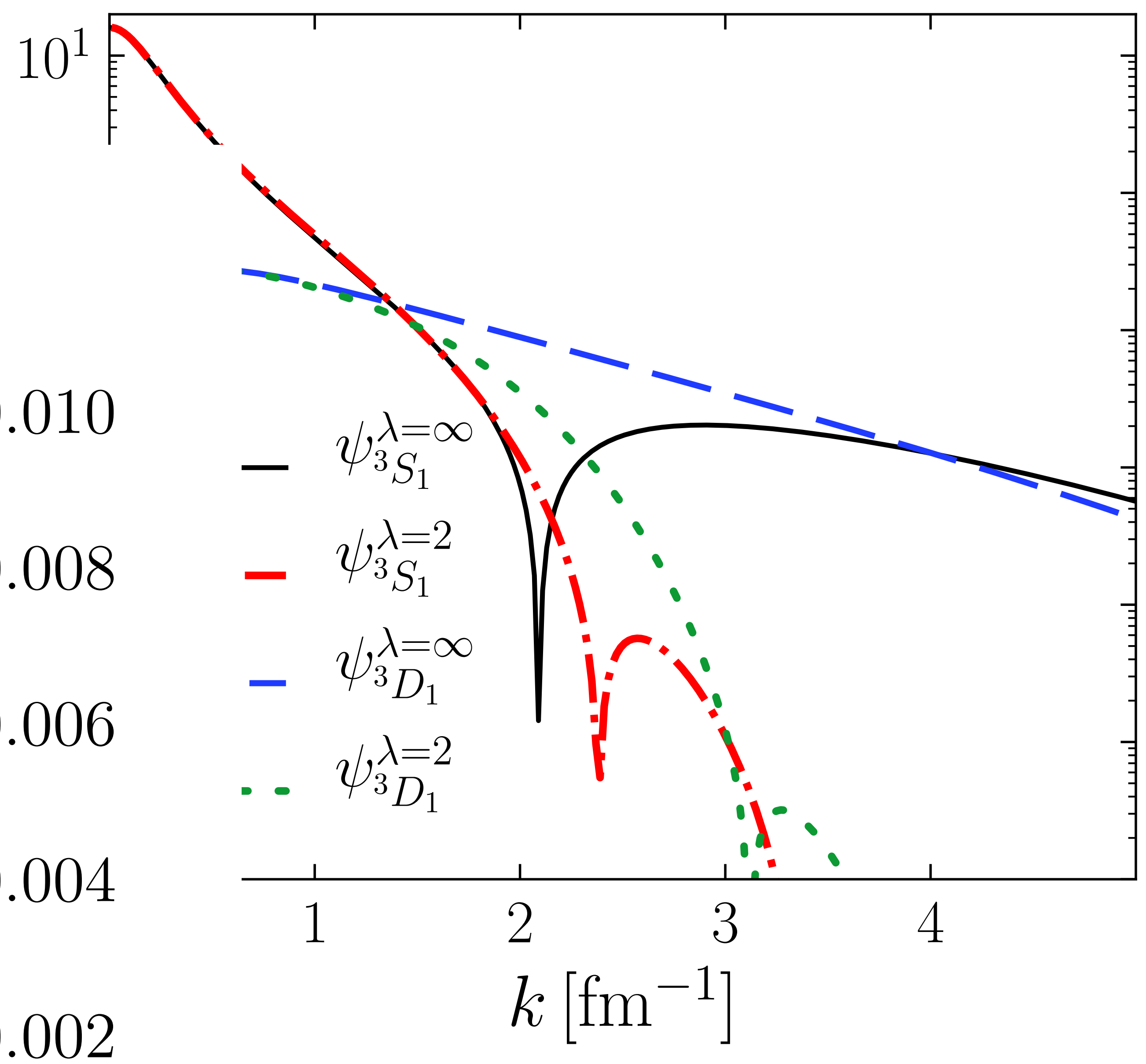
Scale Dependence of Final State Interactions

final state wf (FSI piece)





initial state (deuteron) wf



$J^{\lambda}_{\mathbf{q}}(k', k)$ probed by transition

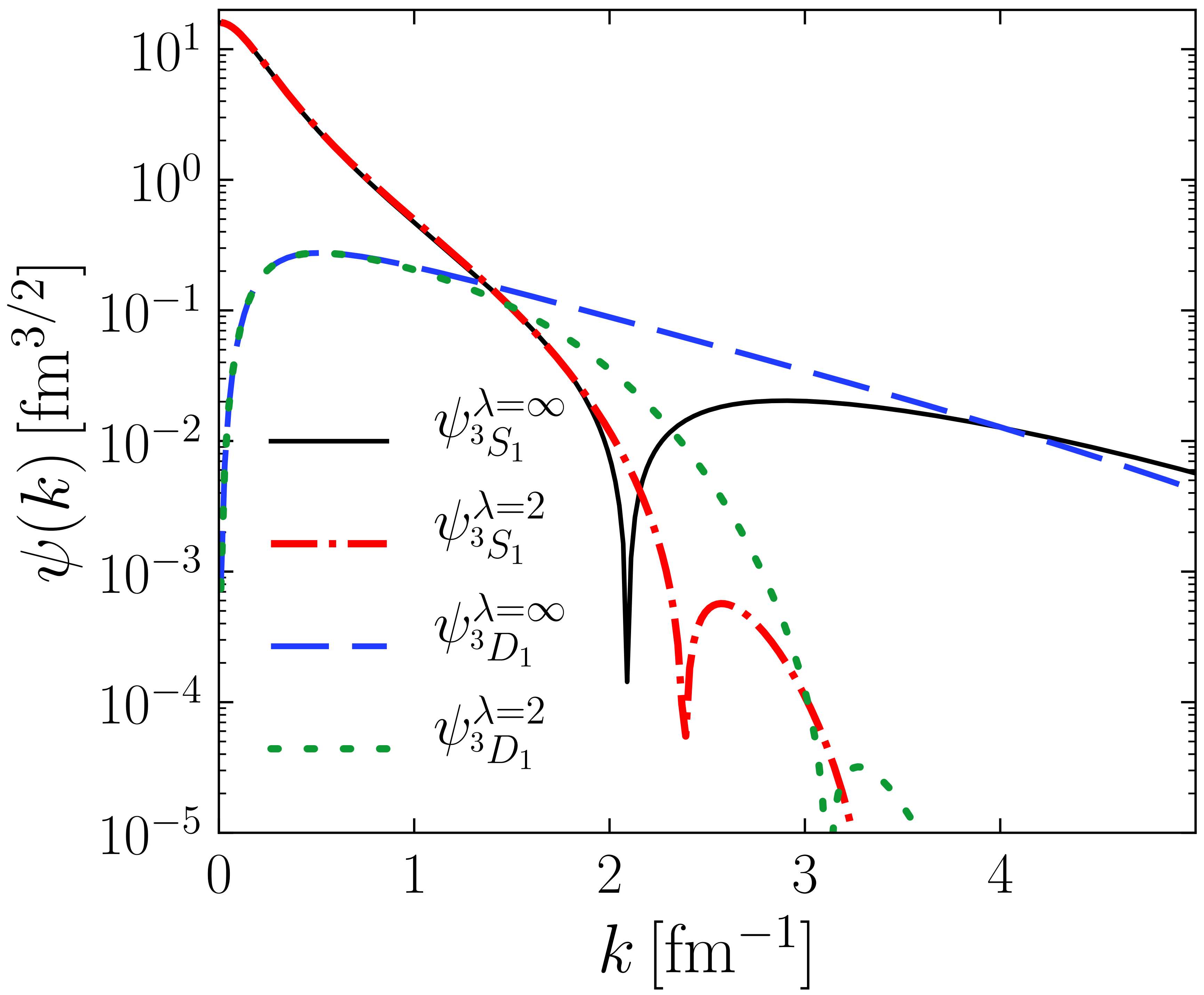
(smooth and non-singular)

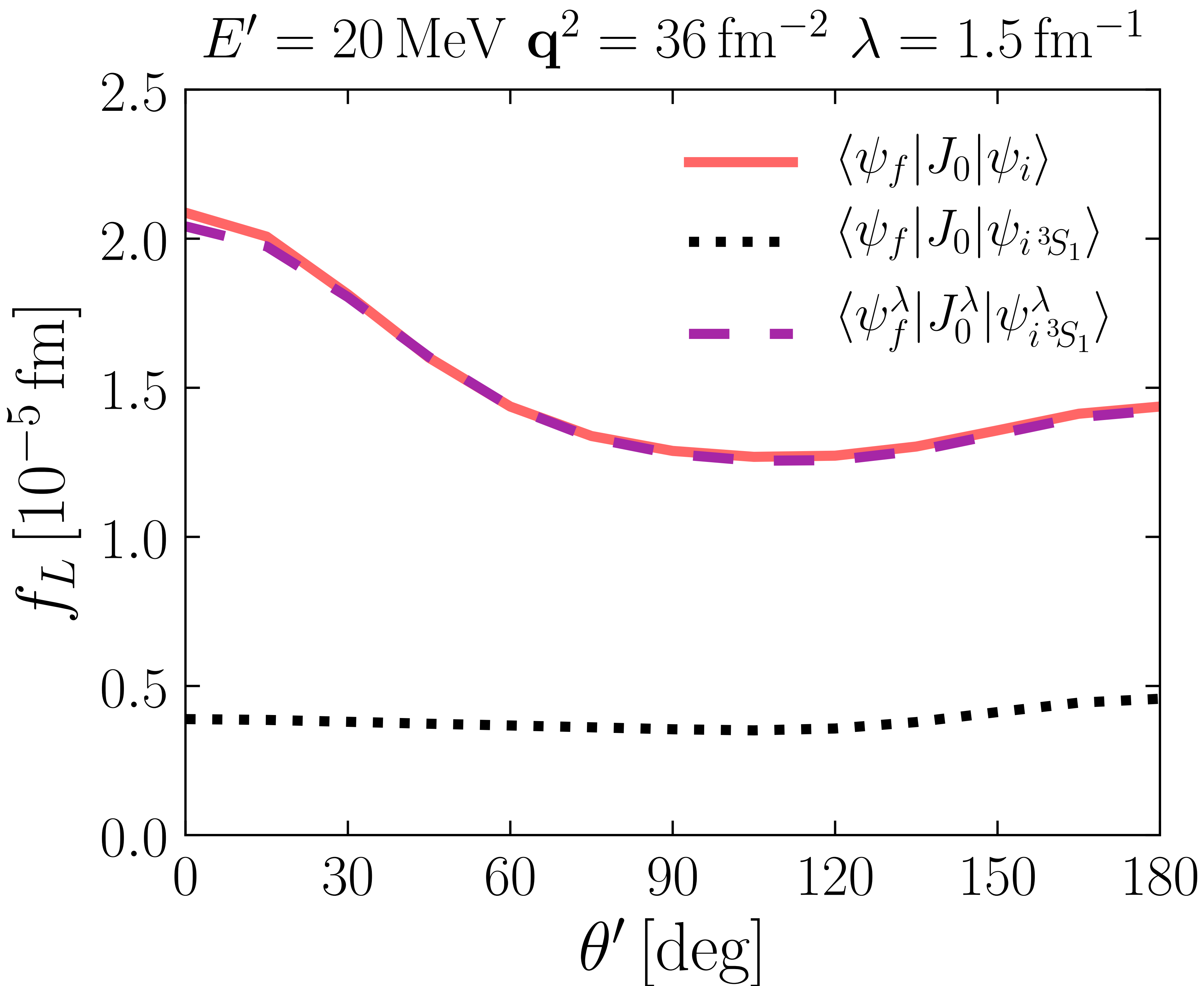
∴ FSI ~ T(p',p')

(small!)

Scale Dependence of Interpretations

- Analysis/interpretation of a reaction involves understanding which part of wave functions probed (highly scale dependent!)
- E.g., sensitivity to D-state w.f. in large $\mathbf{q}^2$ processes





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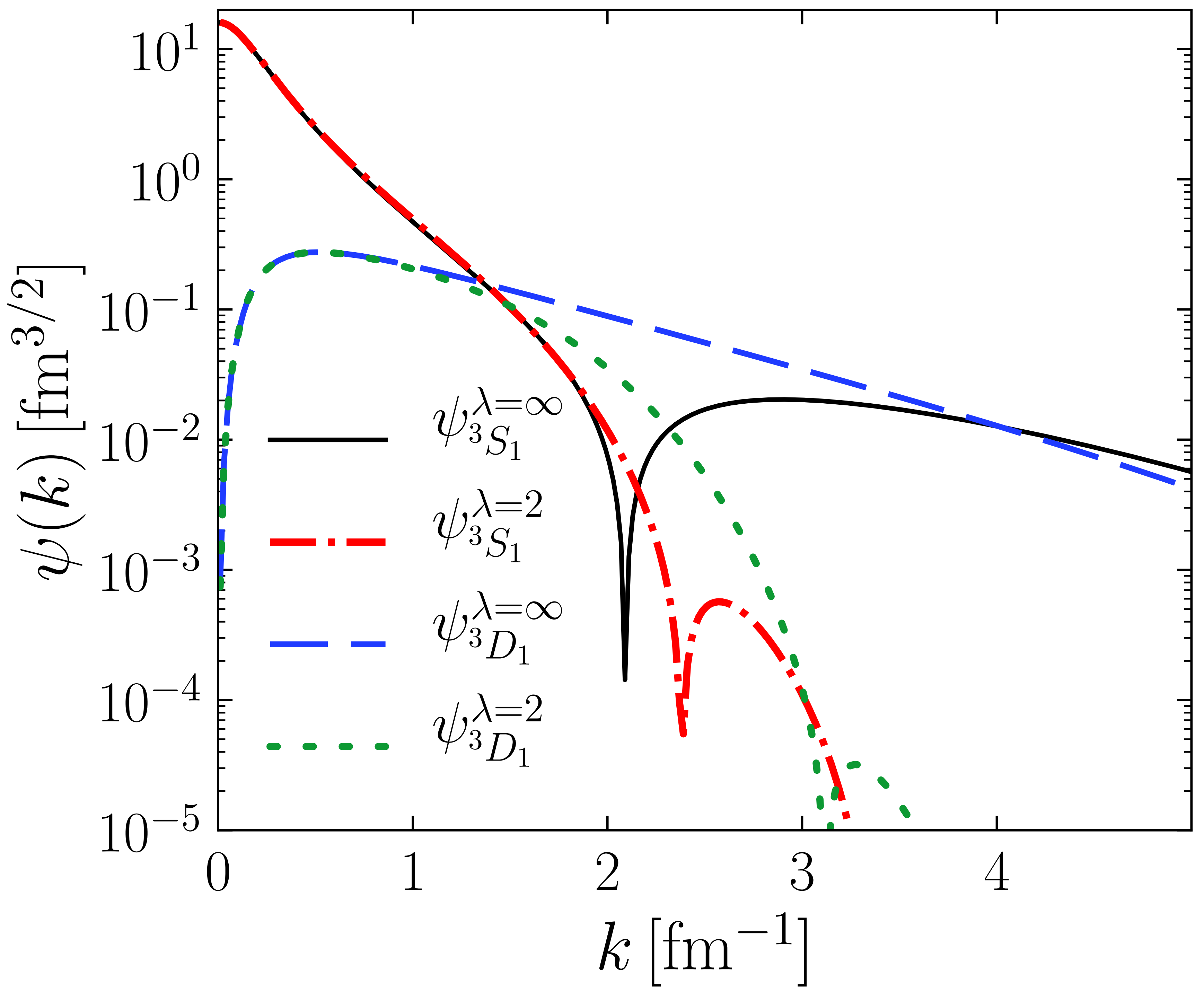
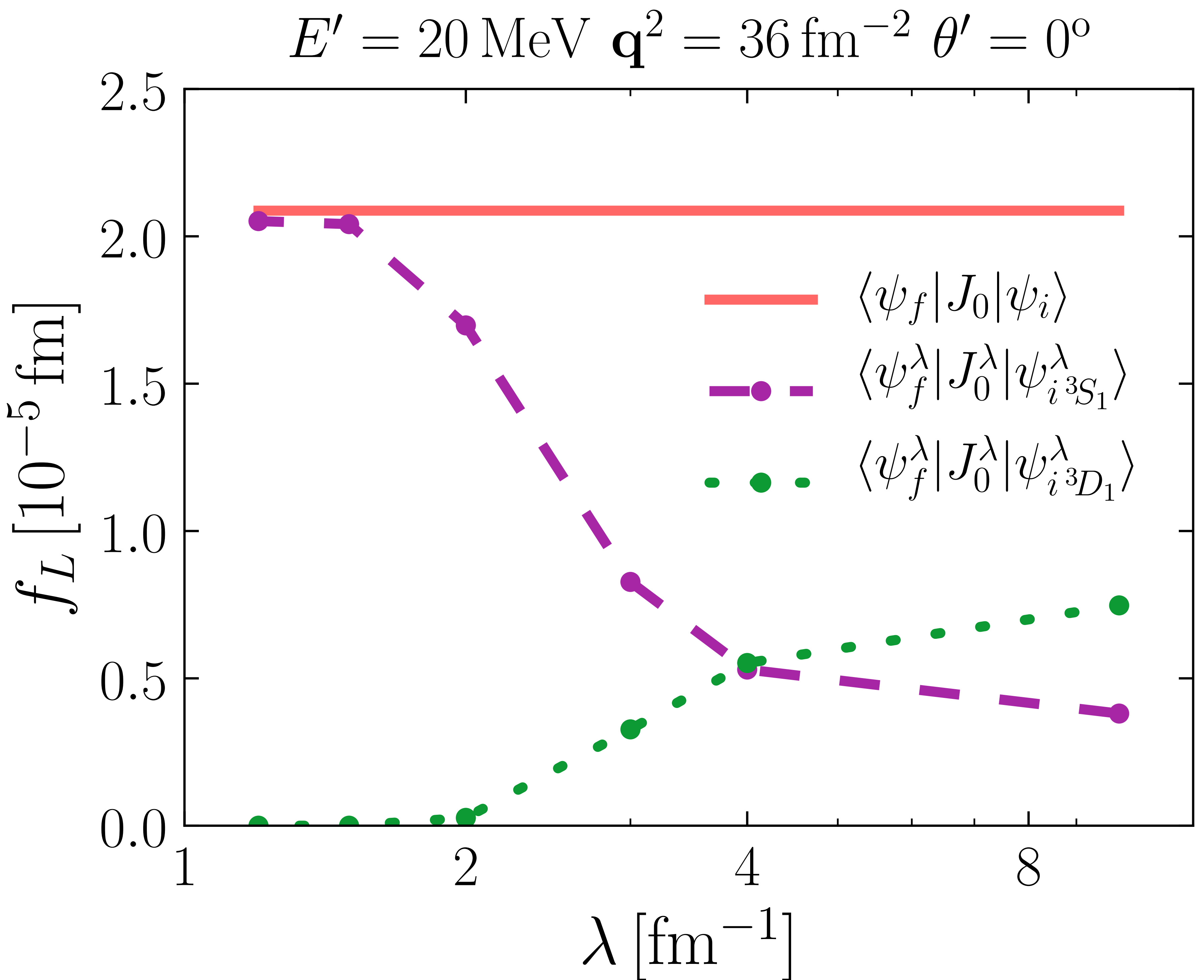
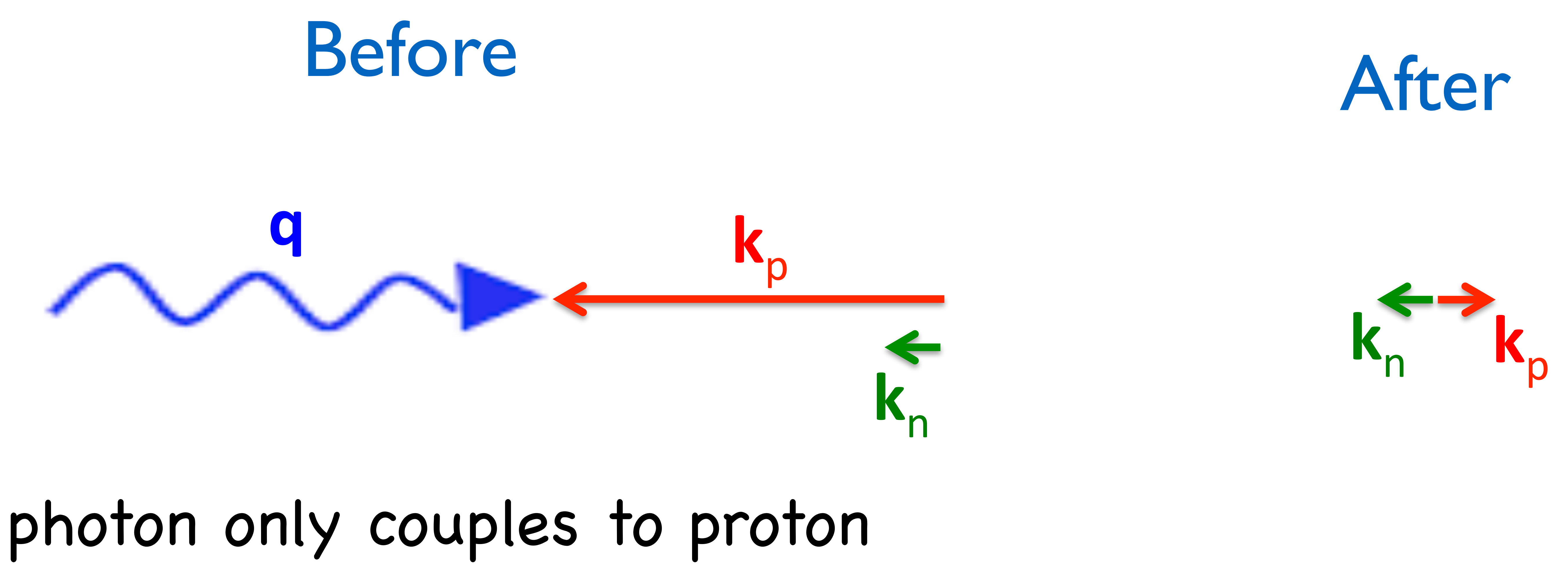




Scale Dependence of SRC Interpretation

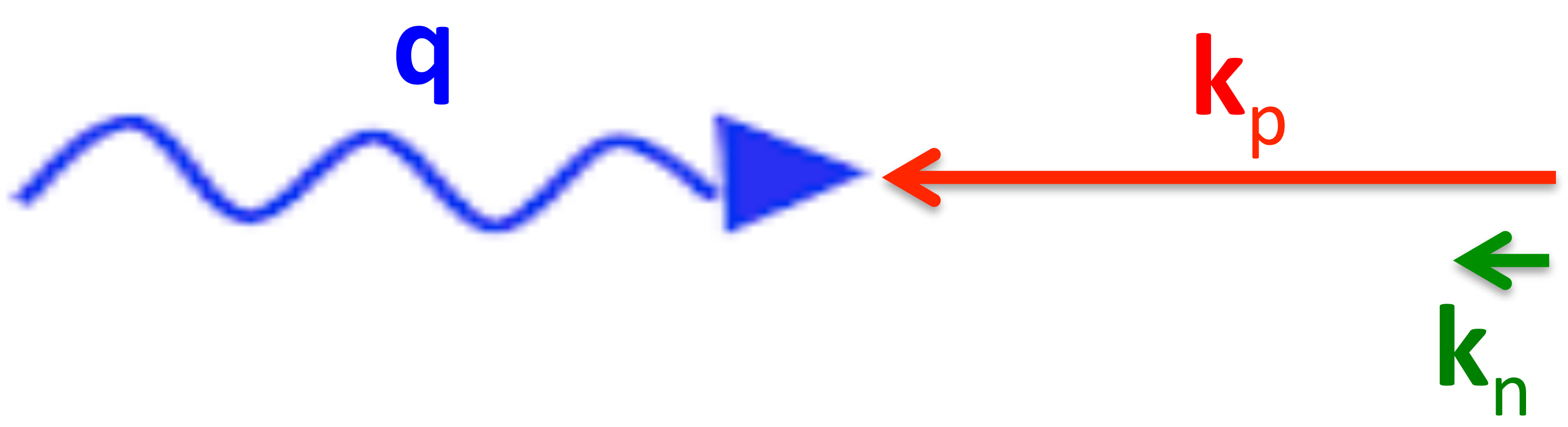
- Consider large q^2 near threshold (small p′) for θ=0 in **high-resolution** picture (COM frame of outgoing np)



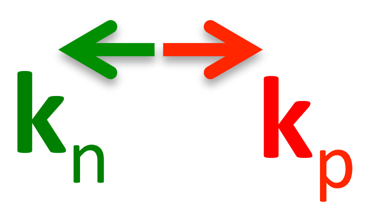
Scale Dependence of SRC Interpretation

- Consider large q^2 near threshold (small p′) for θ=0 in **high-resolution** picture (COM frame of outgoing np)

Before



After



photon only couples to proton

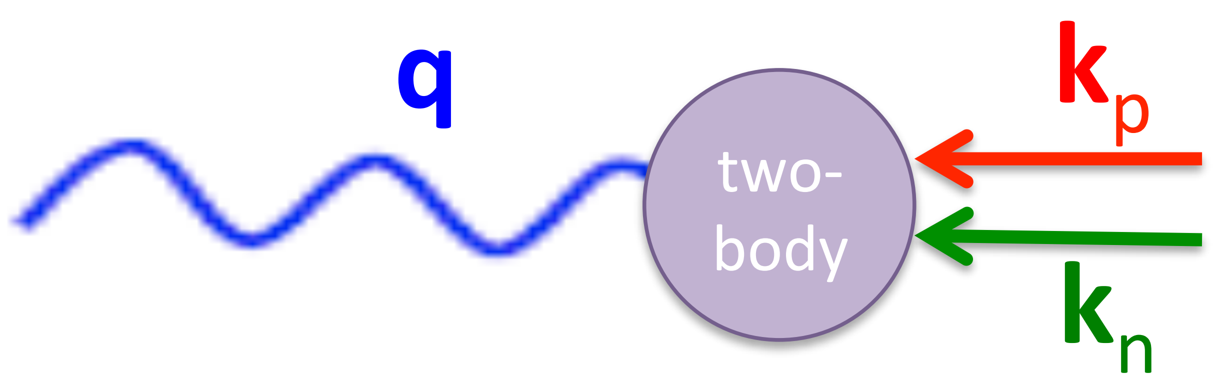
∴ proton has large momentum => initial large relative momentum

(i.e., SRC pair)

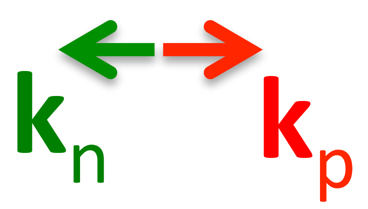
Scale Dependence of SRC Interpretation

- Consider large q^2 near threshold (small p') for θ=0 in **low-resolution** picture (COM frame of outgoing np)

Before



After



Scale Dependence of SRC Interpretation

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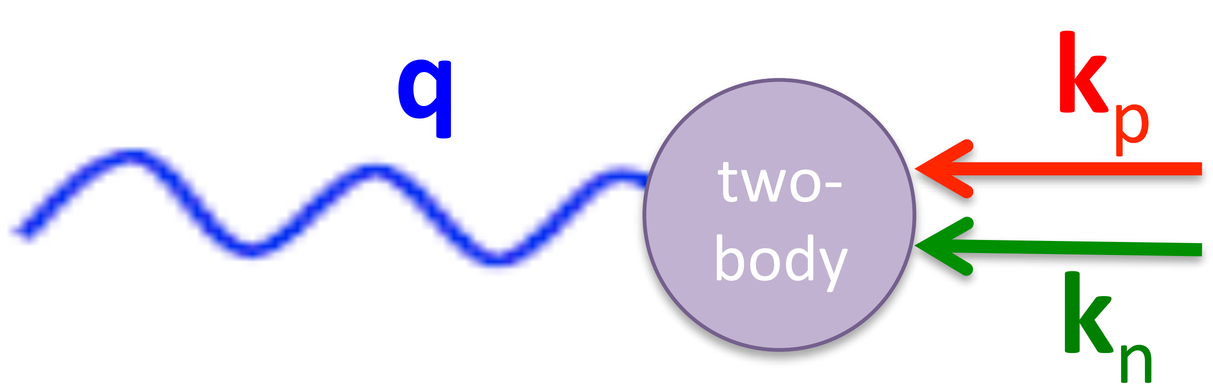
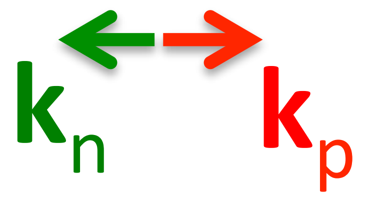
Before



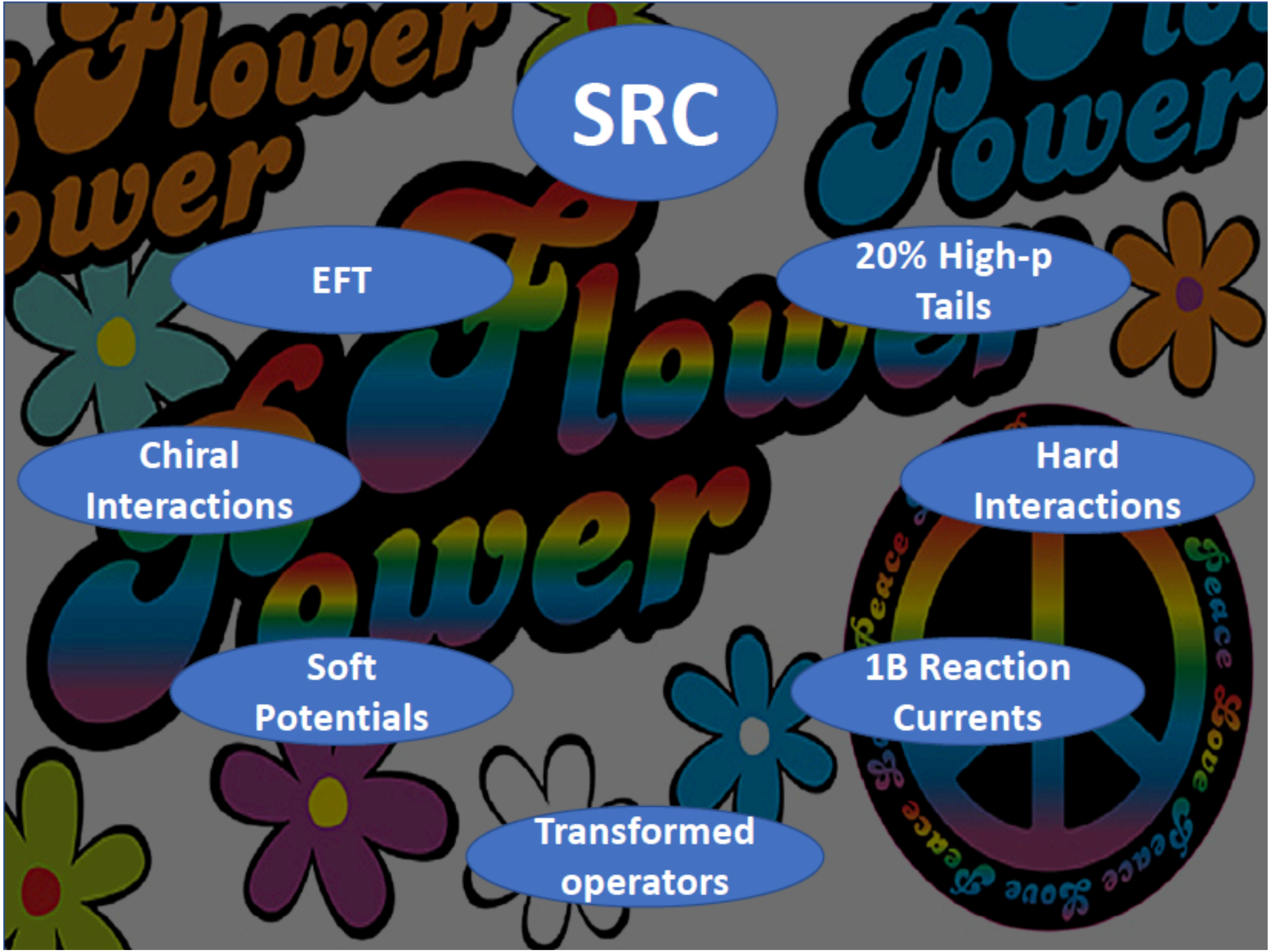
After



no large relative momentum in evolved deuteron wf

1-body current makes no contribution

∴ 2-body current mostly stops the low-relative momentum np pair


SRC
EFT
20% High-p Tails
Chiral Interactions
Hard Interactions
Soft Potentials
1B Reaction Currents
Transformed operators
Flower Power
Flower Power
Flower Power
Peace
Love

Summary/Questions

RG methods smoothly connect high- and low-resolution pictures. There is no "correct" picture (e.g., can reproduce SRC phenom. in a low resolution picture)

Interpretations vary with resolution scale (FSI, etc.), as do ease of calculations (simple wf's + complicated currents vs. complicated wf's + simple currents). Can we exploit this?

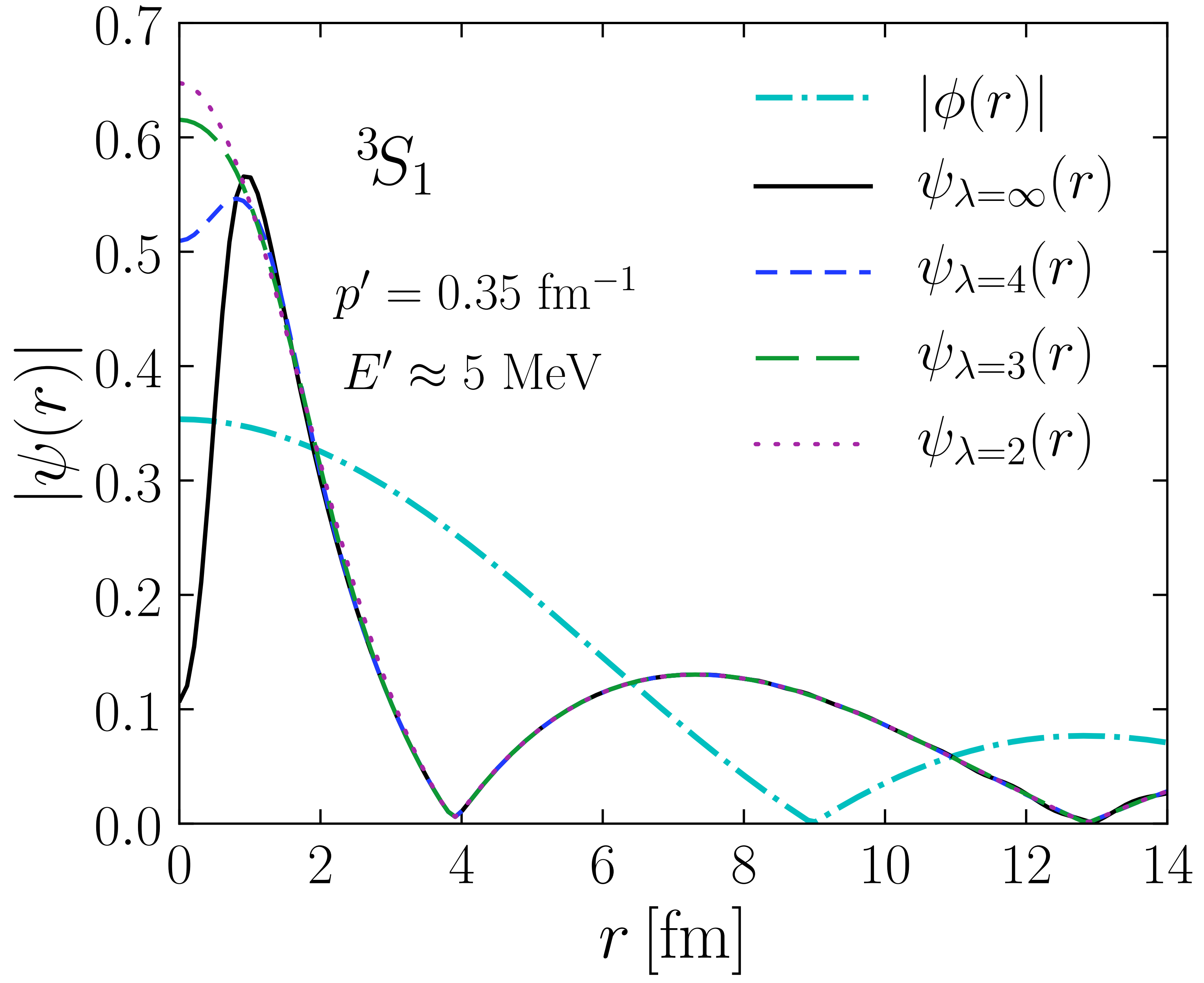
Can we use RG methods to connect SF's in low-resolution shell model picture and SRCs in high-resolution picture?

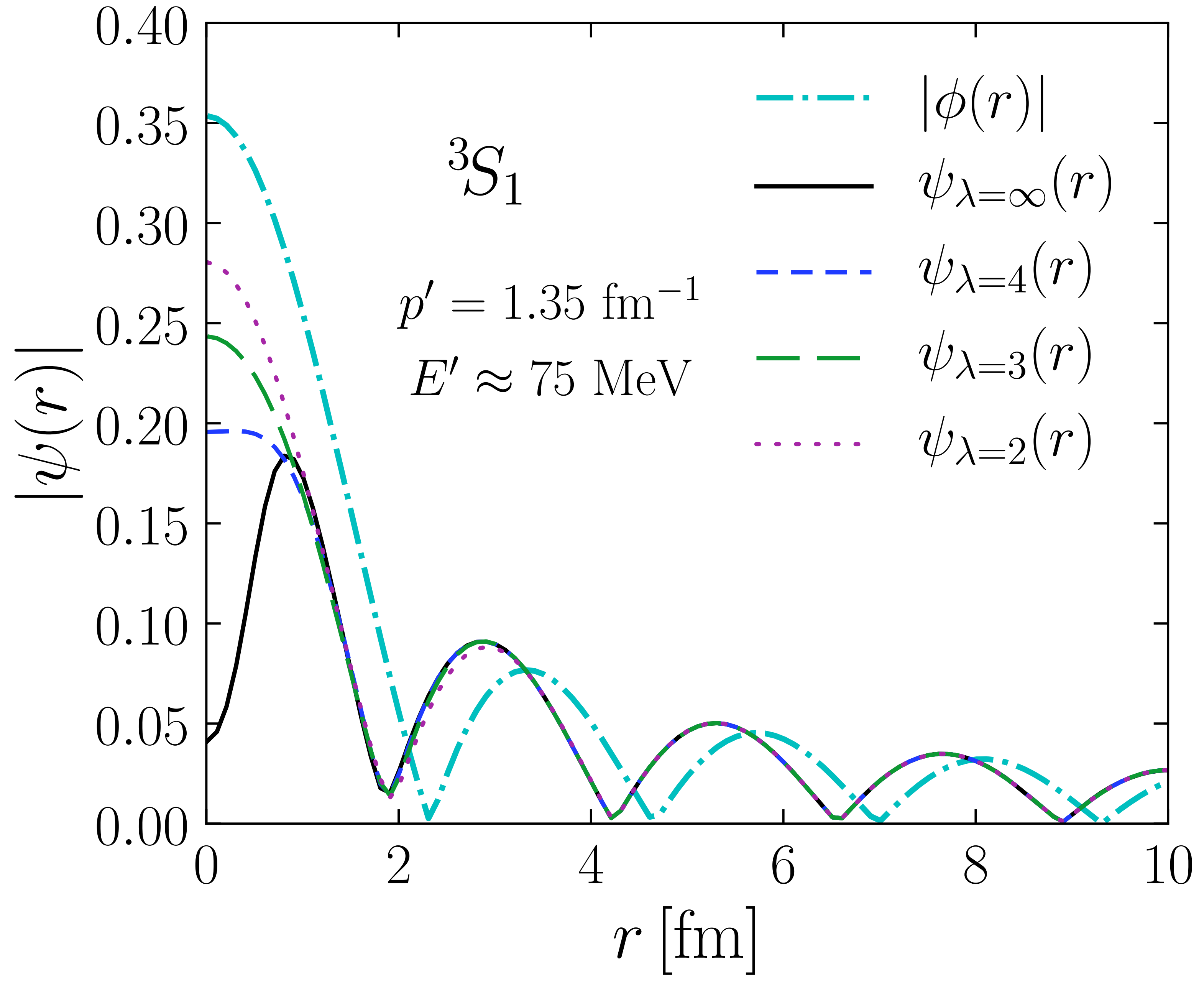
Can we use OPE + SRC/high-q measurements to extend reach of low-resolution theories ?

Can we use simpler low-resolution wf's + OPE for to do high-q electron scattering in medium mass nuclei?

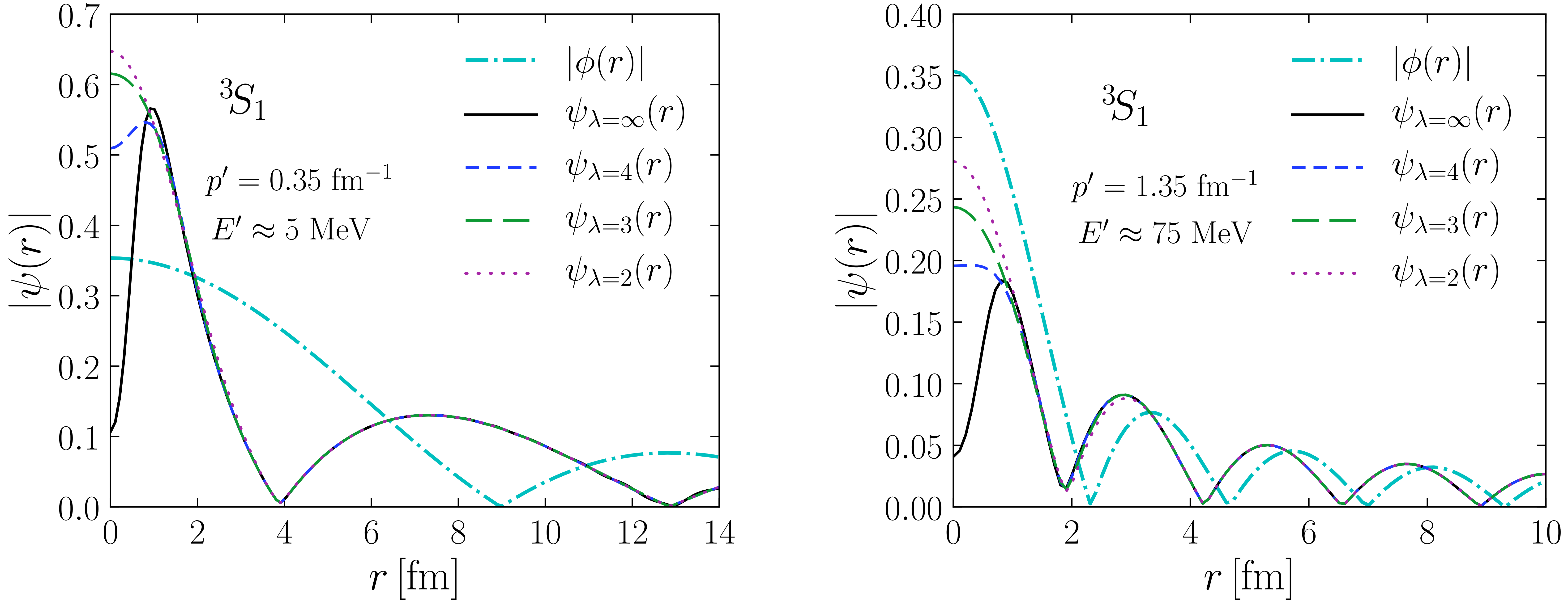
Extras

Final-state wave function evolution



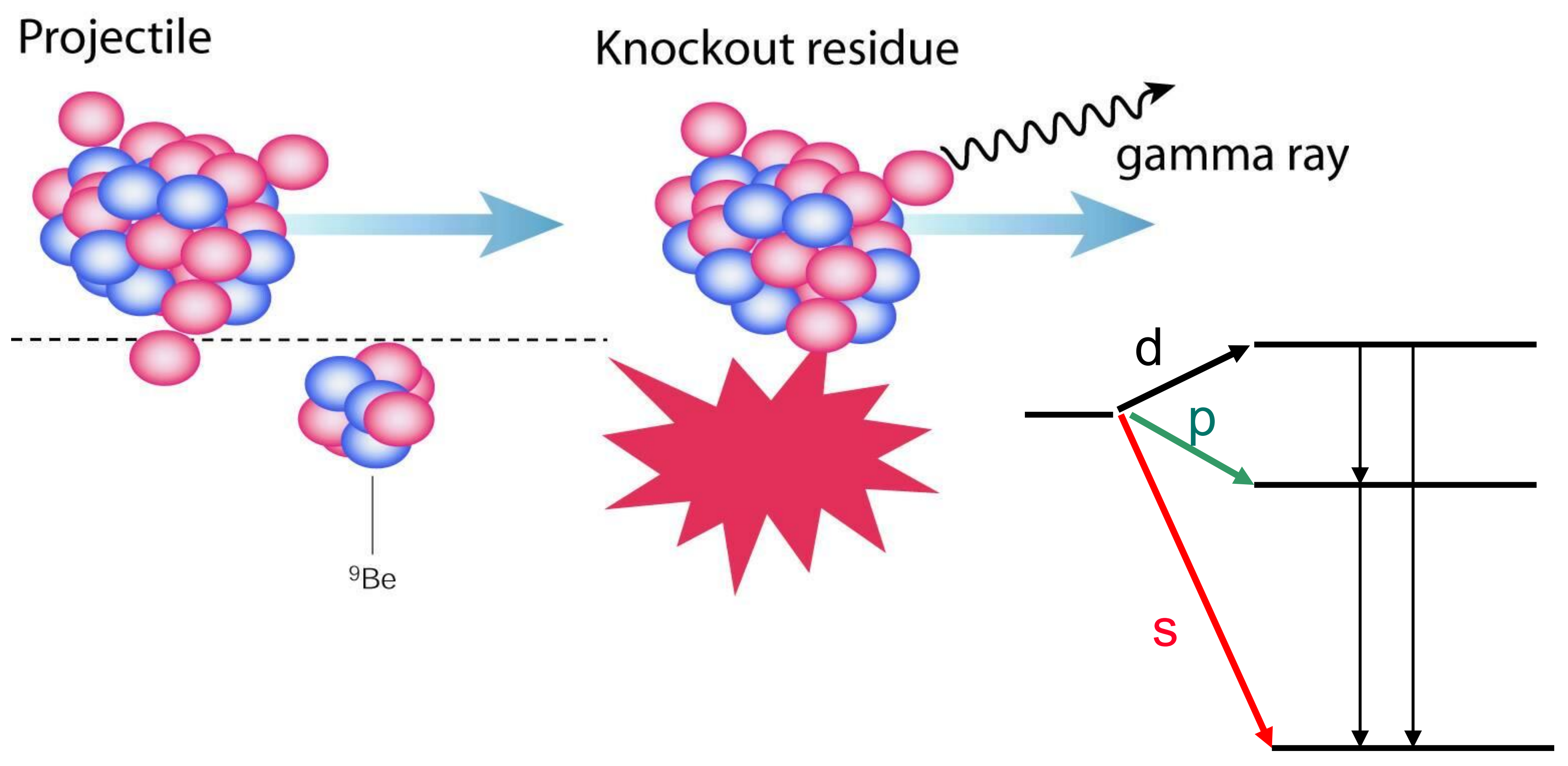


Final-state wave function evolution



- Correlation “wound” at small r smoothed out under evolution
- Long-distance tail invariant (phase shifts preserved)

Other exclusive knock-out reactions [pictures from A. Gade]



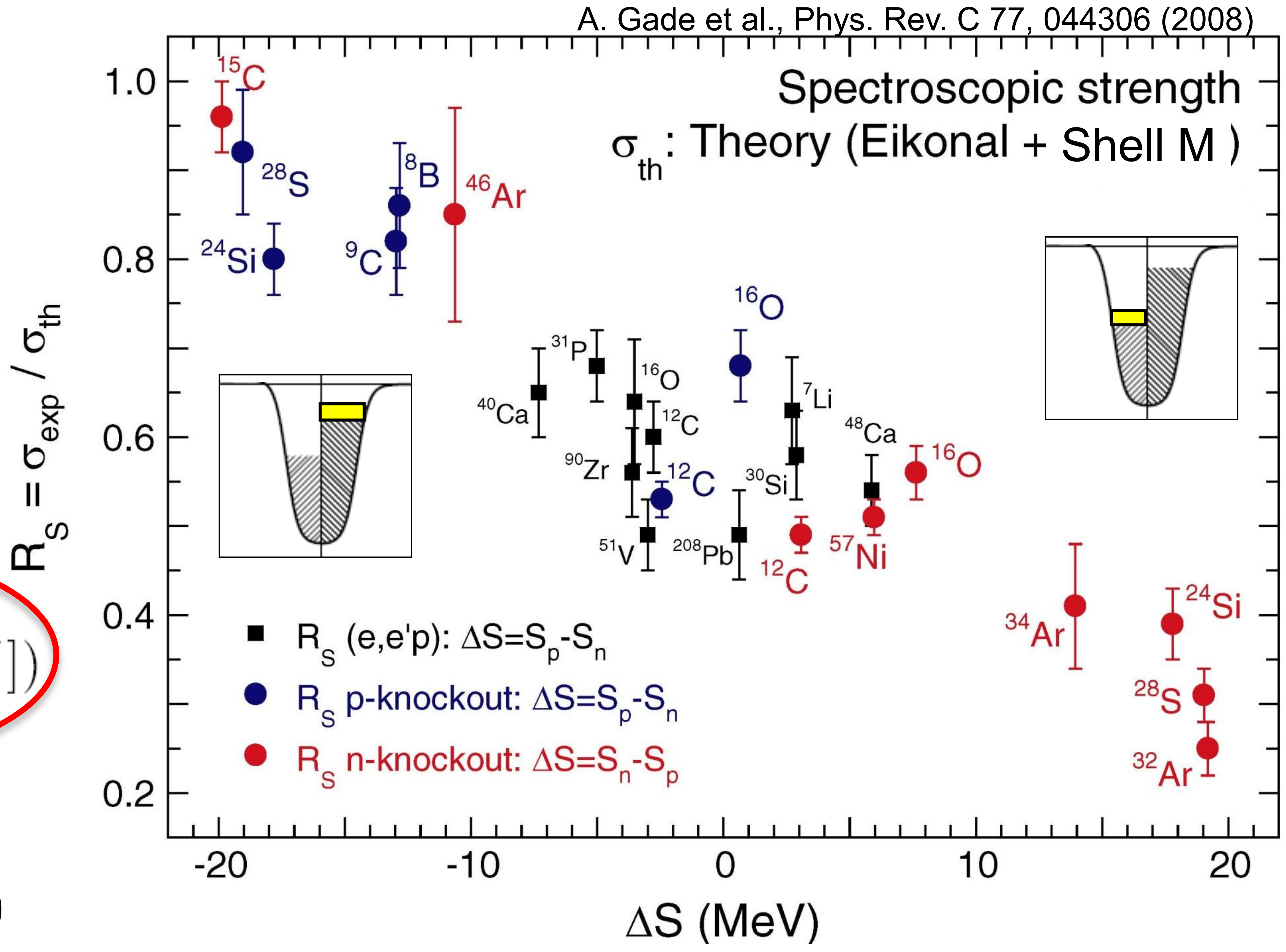
Exclusive reactions, theory vs. experiment

$$\sigma(j^\pi) = \left(\frac{A}{A-1}\right)^N C^2S(j^\pi)\,\sigma_{sp}(j, S_N + E_x[j^\pi])$$

Structure theory

Reaction theory

Origin and systematics of R = σ_{exp} / σ_{th} < 1 are not understood (includes e,e'p results)



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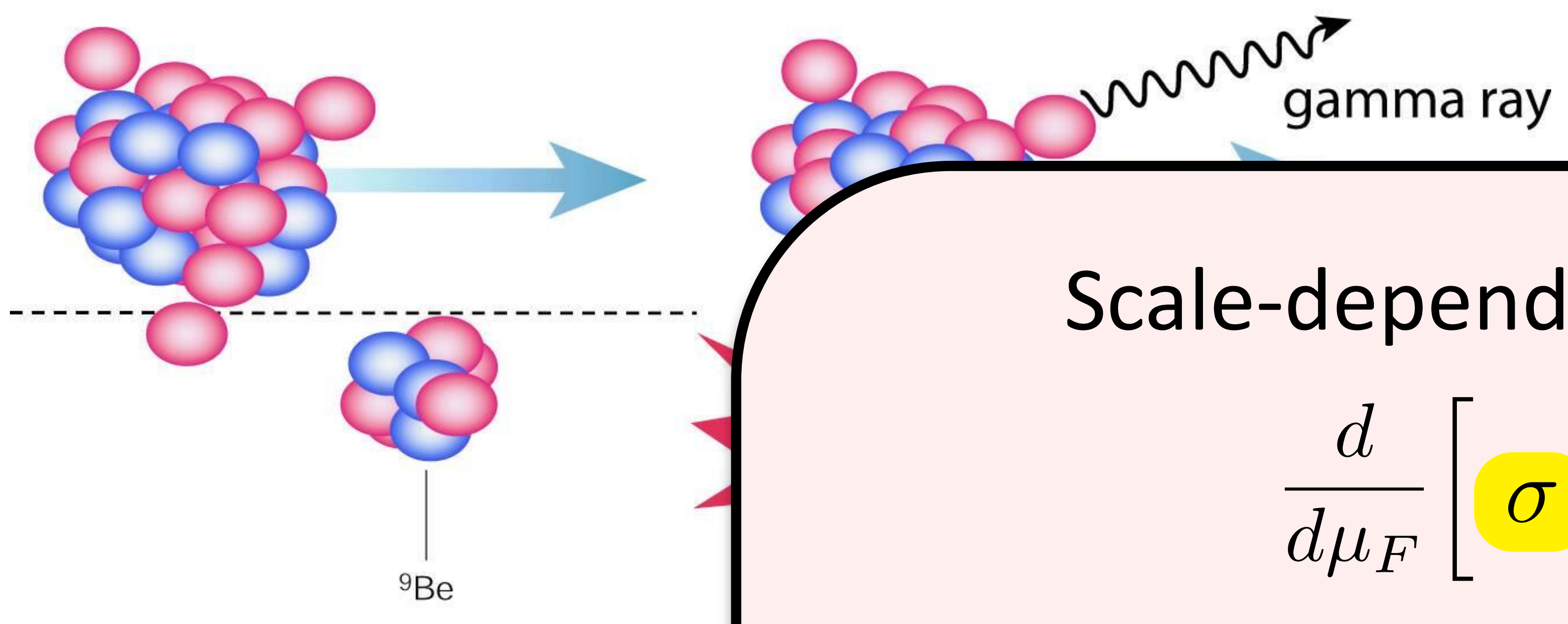

Exclusive reaction

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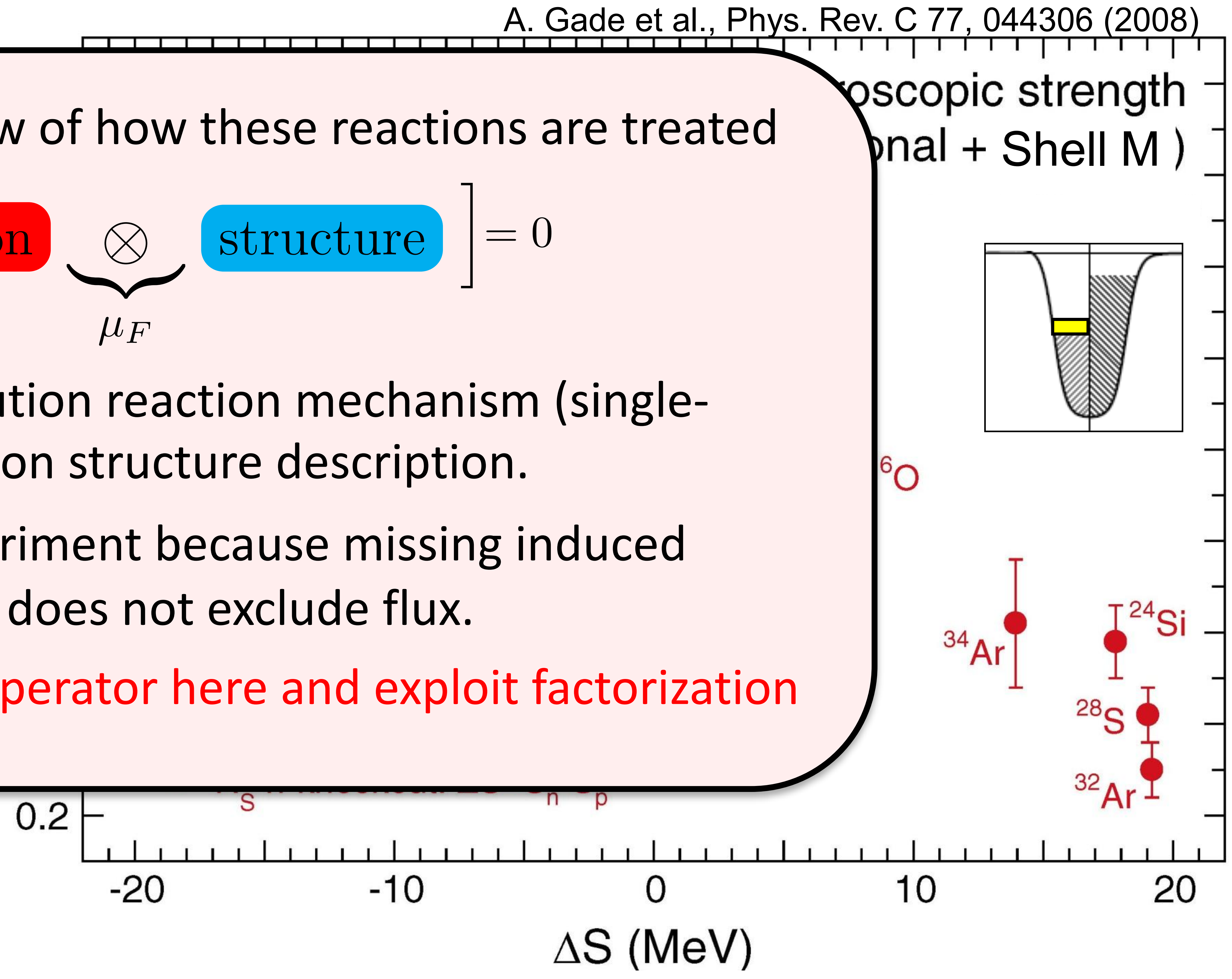

Scale-dependent (RG) view of how these reactions are treated

$$\frac{d}{d\mu_F}\left[\sigma = \text{reaction} \underbrace{\otimes}_{\mu_F} \text{structure}\right] = 0$$

- Analysis mixes a high-resolution reaction mechanism (single-particle) with a low-resolution structure description.
- Theory is greater than experiment because missing induced current (e.g., 2-body for e^-) does not exclude flux.
- Plan: use SRG on reaction operator here and exploit factorization

Deuteron electrodisintegration kinematics