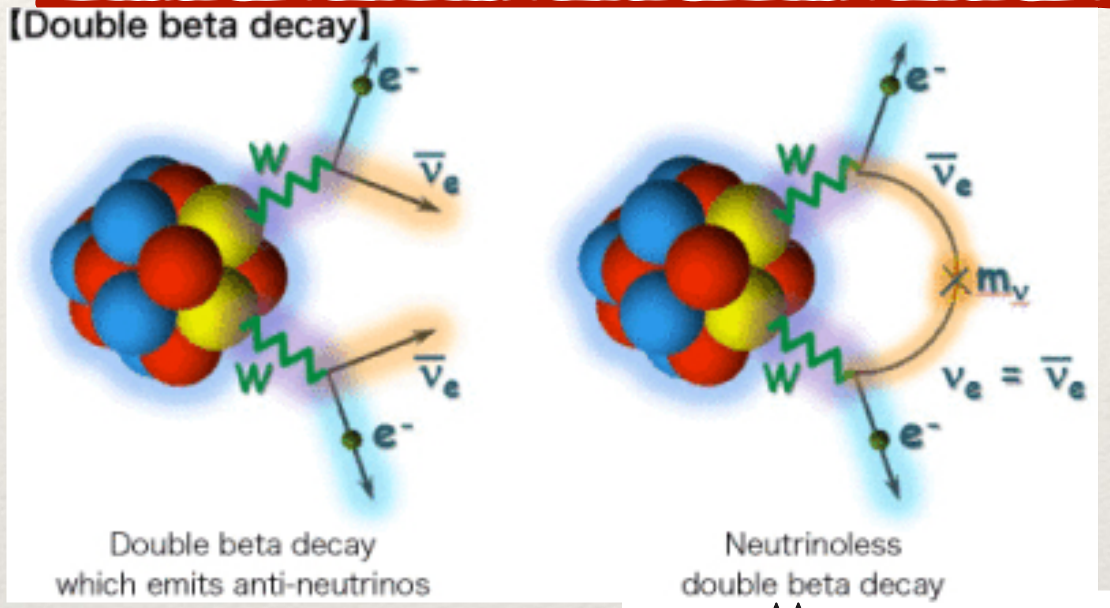
Understanding lattice QCD applications to the two-nucleon system

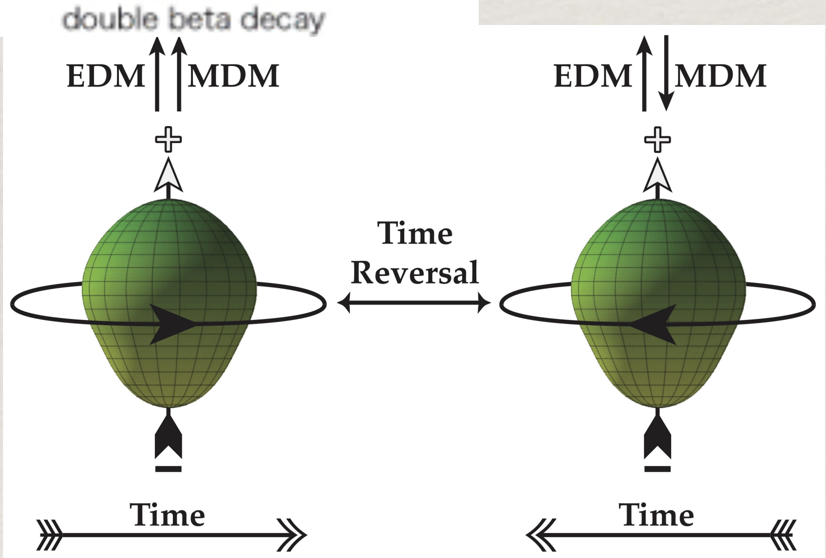
NSCL / FRIB Nuclear Theory Seminar
16th February, 2021

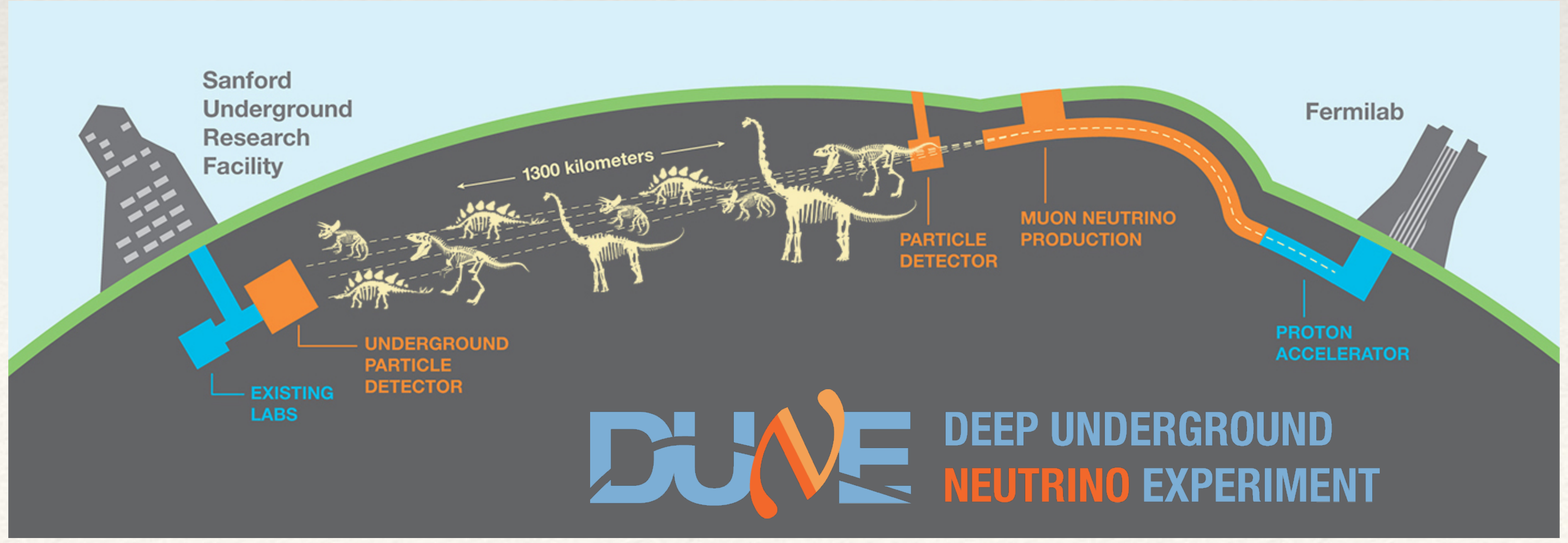
André Walker-Loud

Science Questions

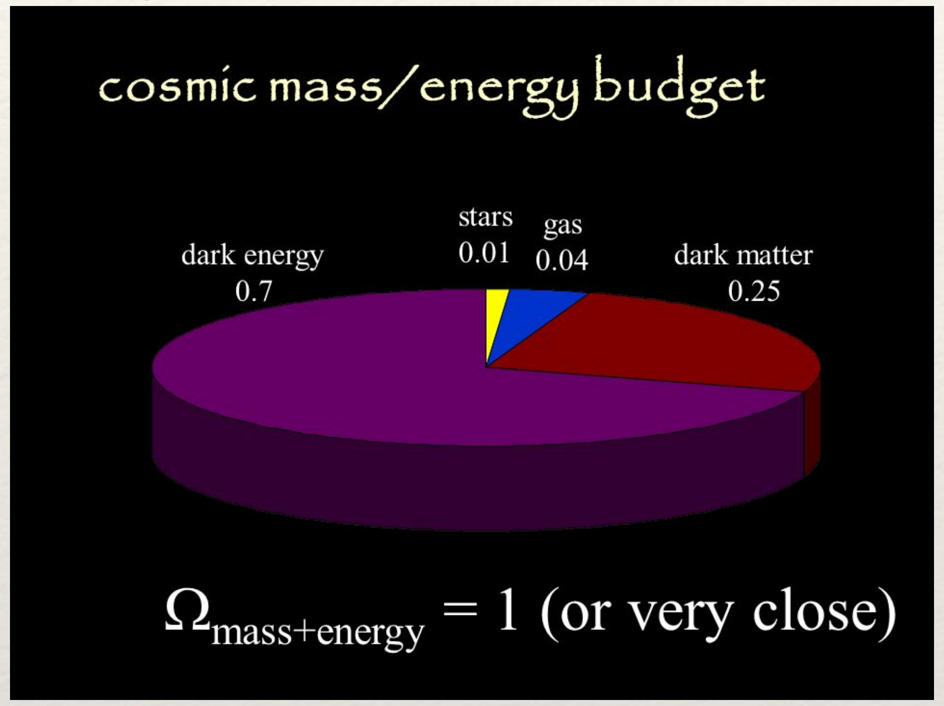
Why is the universe composed of matter? (and not anti-matter)

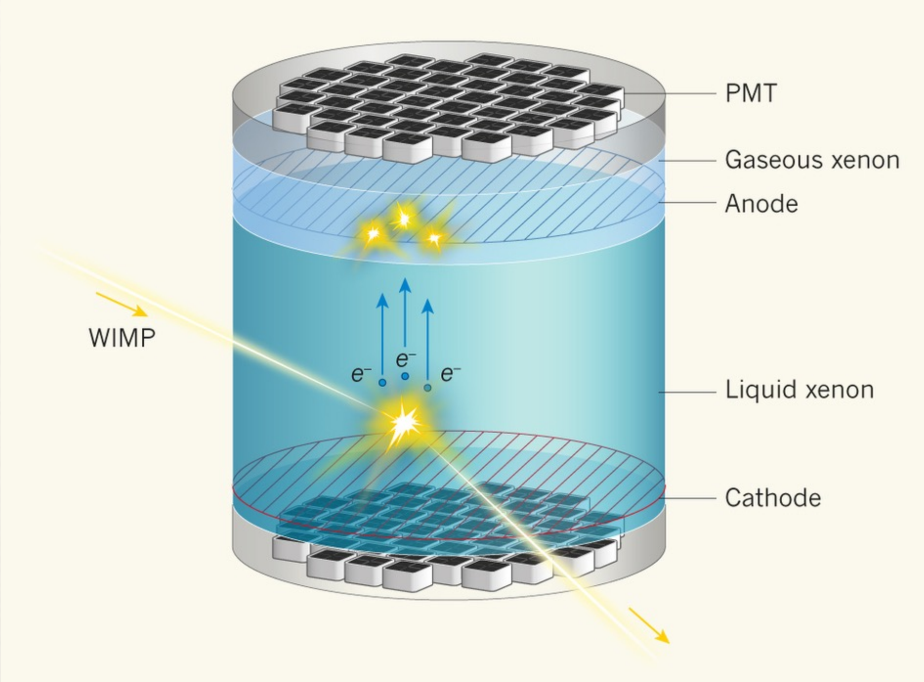




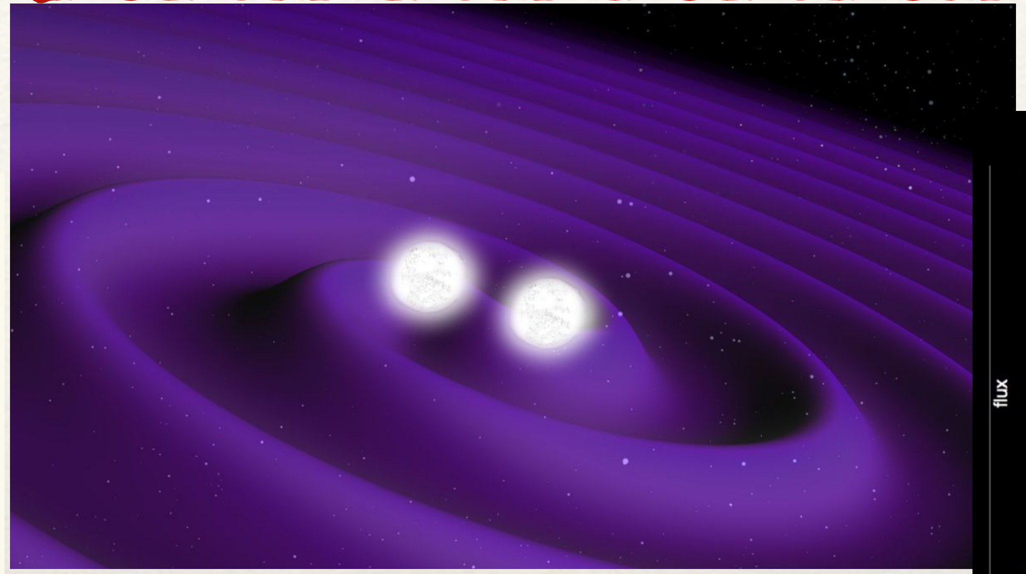


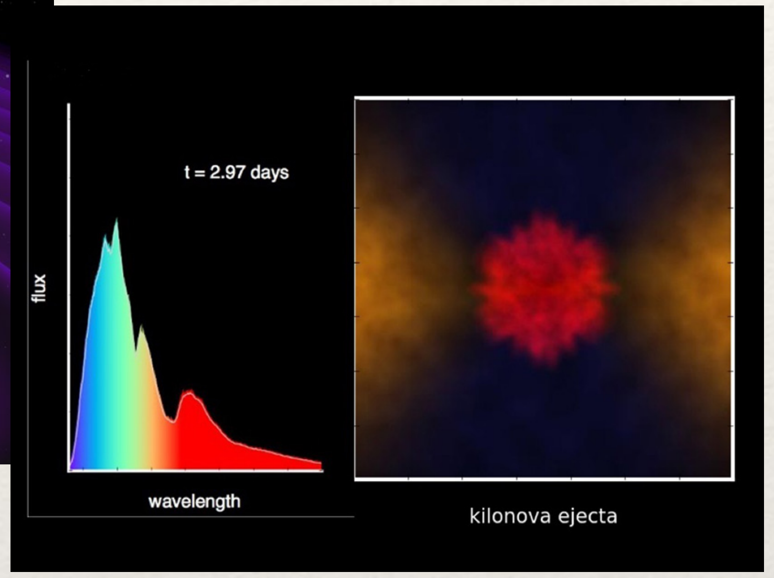
Does dark matter interact with matter? (beyond gravitationally)





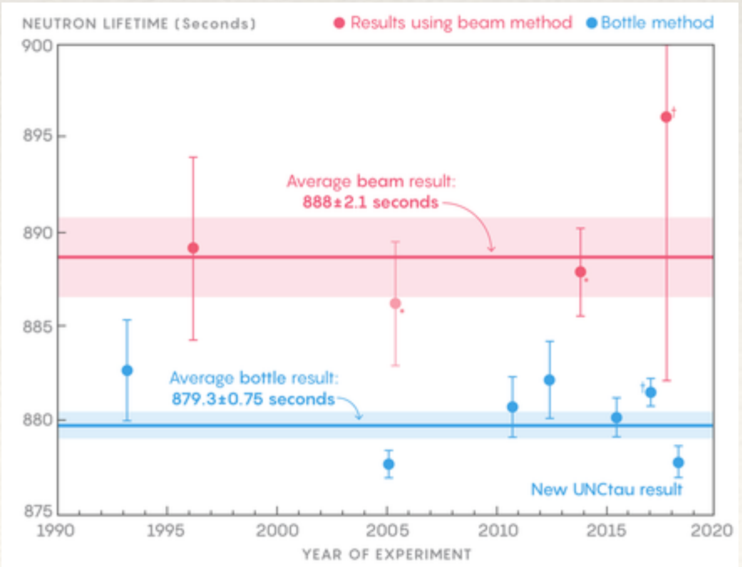
What are the properties of dense nuclear matter?

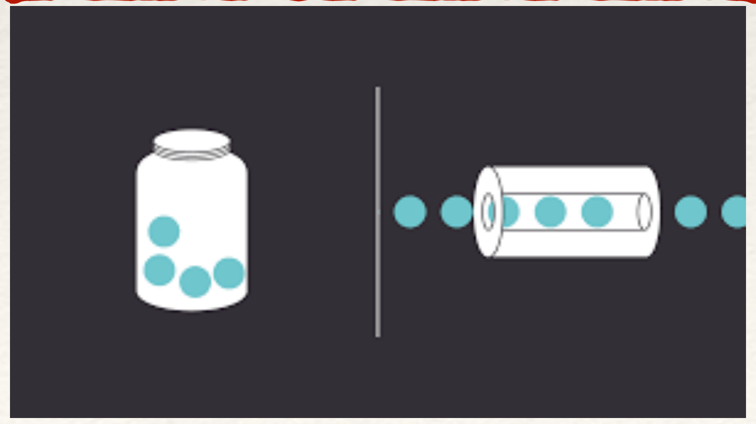




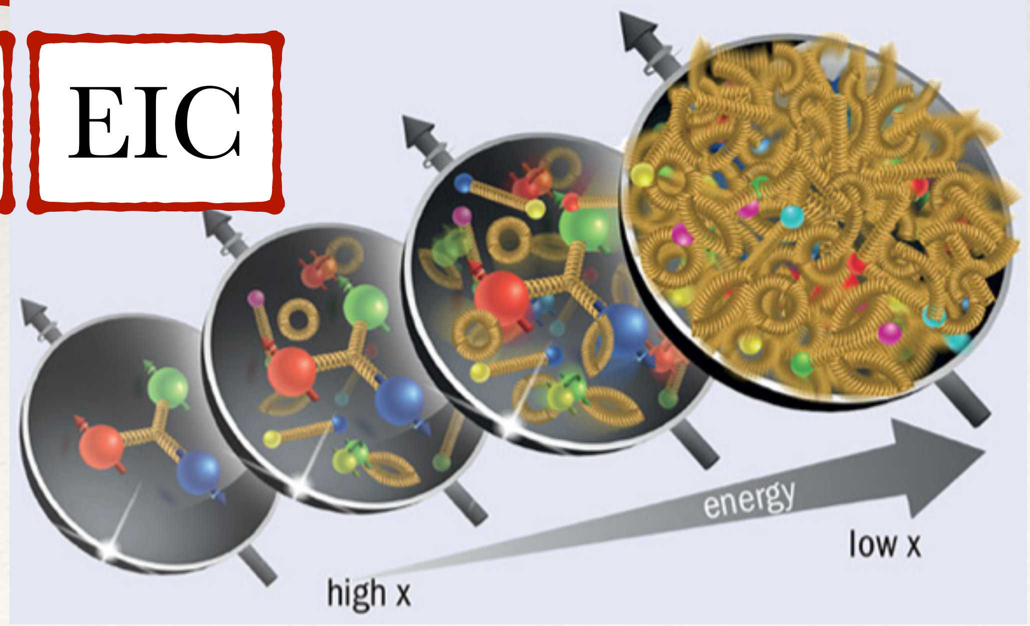
What are the properties of the proton?

neutron lifetime





EIC

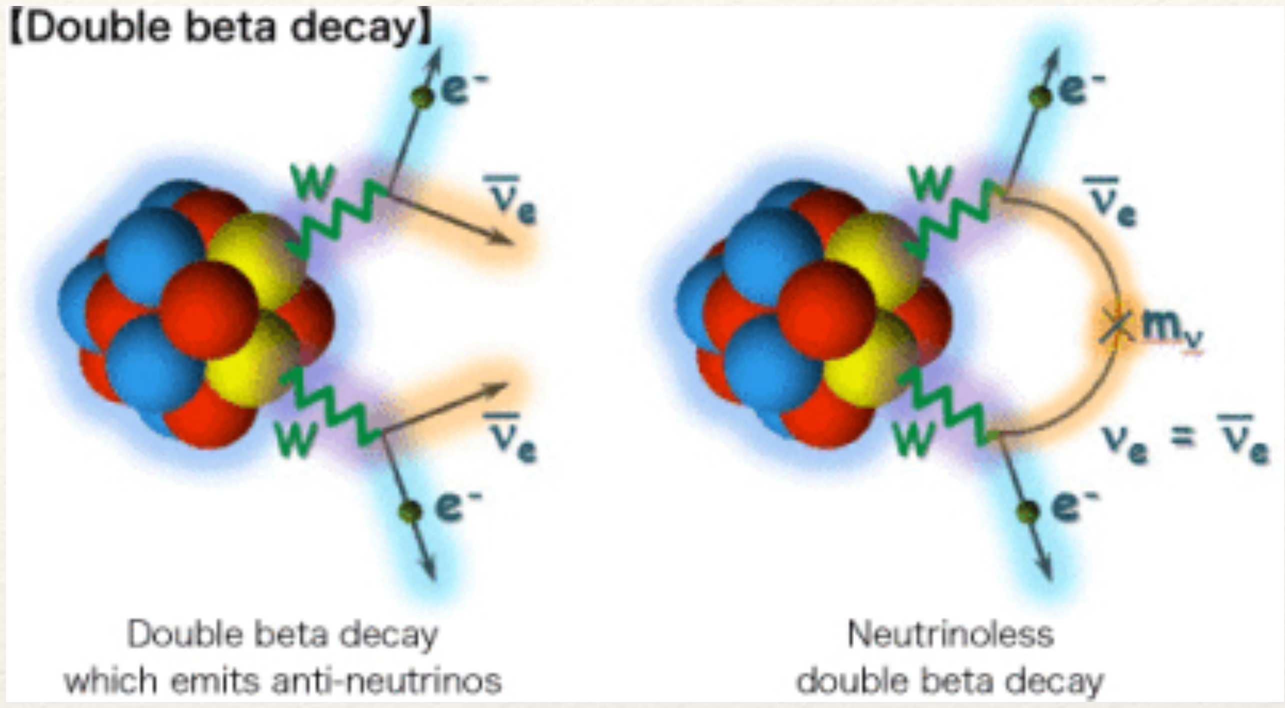


Science Questions

- Of course, we will not use LQCD to directly compute most of these processes
- In each case, there are key pieces of information that are challenging or impossible to determine from experimental information alone - and which we can address with LQCD
- Success requires a coordinated effort between
  - Lattice QCD
  - Effective Field Theory (EFT)
  - Theories of many body nuclear physics
- Allowing for the propagation of a quantitative theoretical uncertainty, rooted in the Standard Model, into theories of nuclear physics

Science Questions

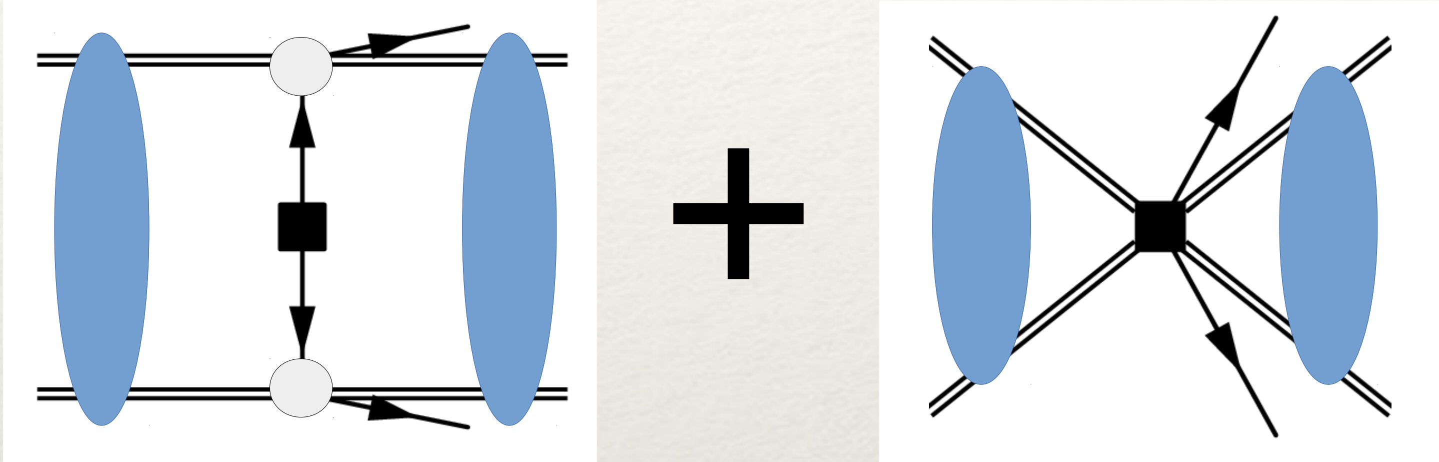
One example (of a few) related to 0νββ



- Recently, it was pointed out that in NN Effective Field Theory (EFT), there is a short-range operator, nominally higher order - that must be promoted to leading order (LO) to properly renormalize the nn⟶pp(ee) amplitude

  Cirigliano et al. PRL 120 (2018) [1802.10097]

  Cirigliano et al. PRC 100 (2019) [1907.11254]



known (predictive) unknown coupling

- The issue is this new term comes with an unknown coefficient - rendering the LO prediction to have an O(100%) uncertainty (on top of the challenging many-body nuclear uncertainties)

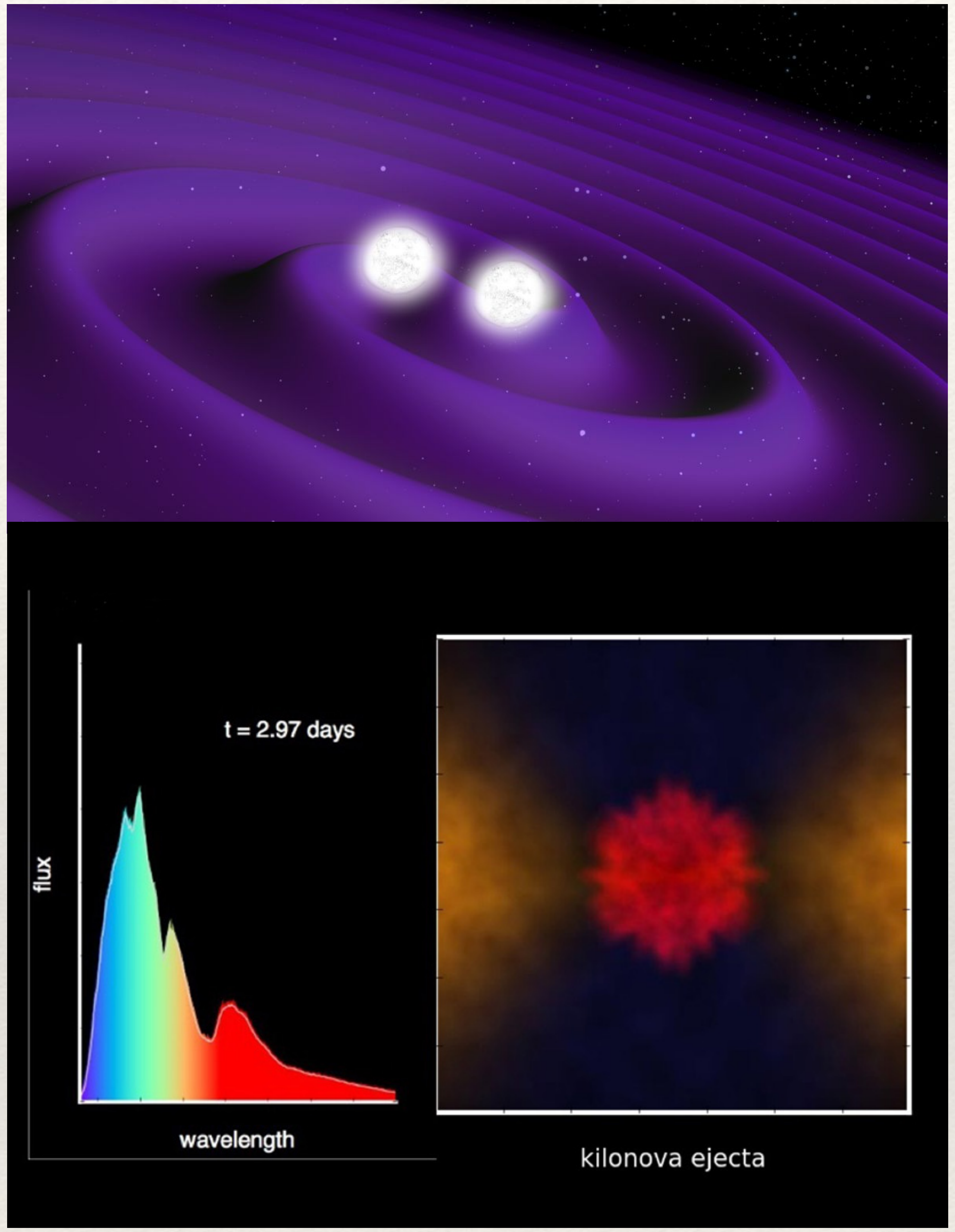
- It is not practically possible to measure nn⟶pp(ee) experimentally to constrain this term

- We can perform a lattice QCD calculation of the nn⟶pp(ee) amplitude which will allow for a determination of the unknown low-energy-constant (LEC) restoring the predictive capability

Science Questions

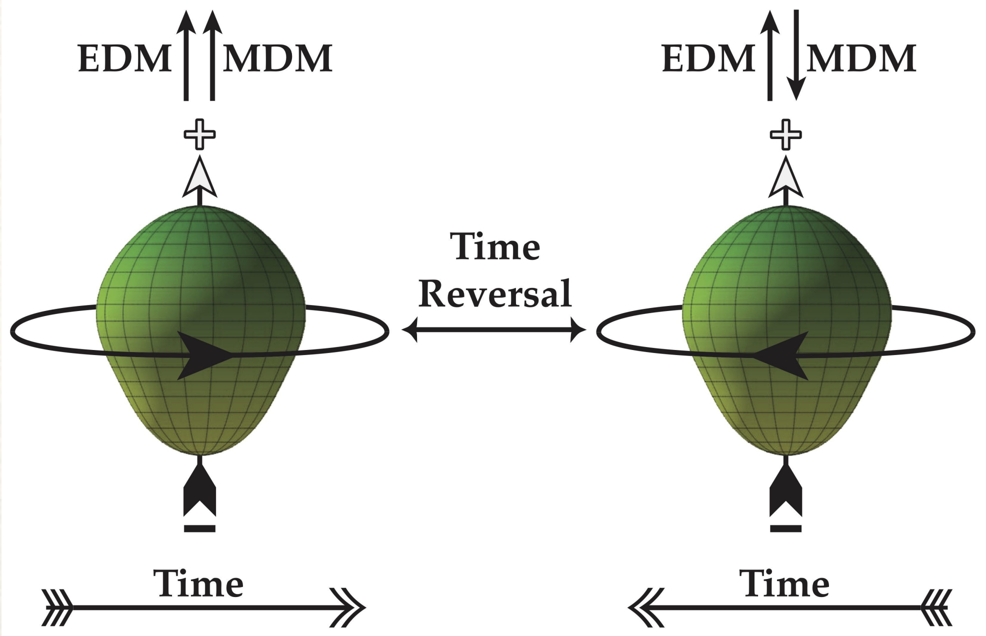
Multi-messenger era and neutron-star mergers

- ❑ The ability to measure neutron star mergers has brought to reality the possibilities of constraining the nuclear equation of state with much better precision than previously possible
- ❑ It is difficult to make models with hyperons that can support the heavy ~2M neutron stars
- ❑ It is difficult to imagine that hyperons are irrelevant in the core of neutron stars (based upon the anticipated energy/density)
- ❑ The three-neutron interaction plays an important role in stabilizing neutron stars - but it is challenging to constrain
- ❑ Hyperon-Nucleon (YN) interactions are challenging to measure (since hyperons decay rapidly)
- ❑ If hyperons exist in neutron stars - it is probable that YNN interactions are also important
- ❑ The NNN and YN and YNN are interactions in principle we can determine with Lattice QCD

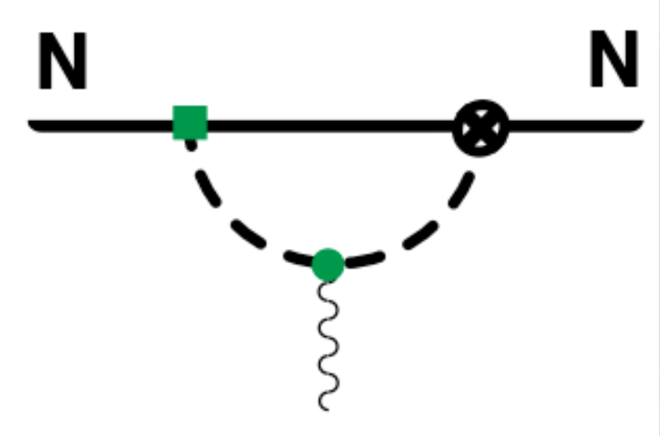


Science Questions

Why is the universe composed of matter (and not anti-matter)?



- Our current understanding of the universe requires orders of magnitude larger CP-violation than exists in the Standard Model (SM) in order to give rise to the observed abundance of matter over anti-matter $\frac{n_B}{n_\gamma} = \eta \approx 6 \times 10^{-10}$
- CP-violation $\rightarrow$ permanent electric dipole moments (EDMs) in (quarks) nucleons and nuclei
- Significant experimental effort to measure EDMs in nuclei - including those with enhanced sensitivity to EDMs (like ^{225}Ra) - eg. Jaideep Signh group at NSCL/FRIB measuring EDMs
- In large nuclei, we have an expectation that the EDM might be dominated by the long-range pion-exchange involving a CP-odd pion-nucleon coupling



- With Lattice QCD, we can compute these CP-odd couplings arising from various sources the Θ_{QCD}-term, quark-chromo-EDMs, etc.
and then use them in nuclear-models to predict the nuclear EDMs (constraint on Θ_{QCD} from ^{199}Hg)
eg. Andrea Shindler's group at NSCL/FRIB computing nucleon EDMs with Lattice QCD

Application of Lattice QCD to multi-nucleon systems

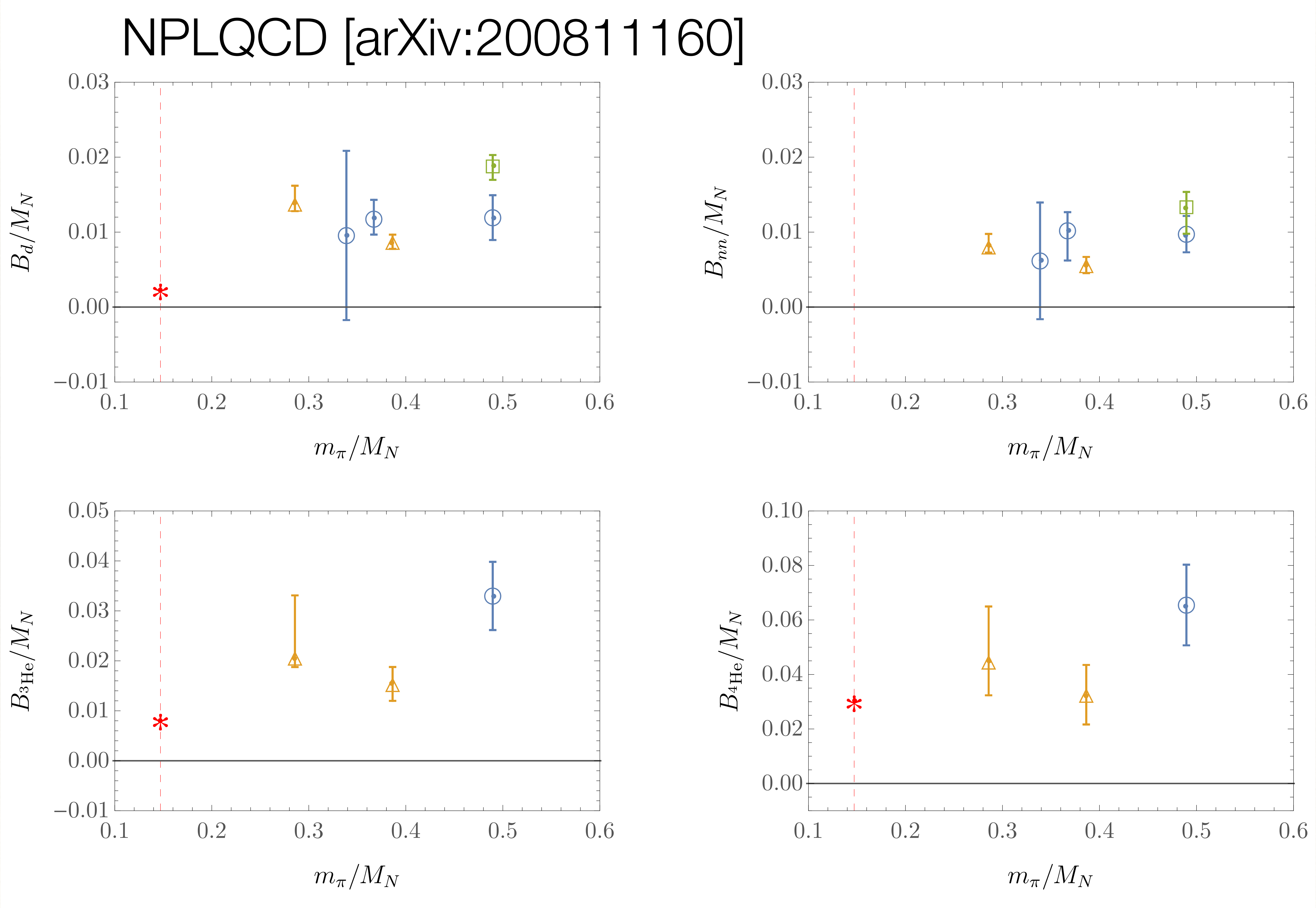
Towards grounding nuclear physics in QCD
C. Drischler, W. Haxton, K. McElvain,
E. Mereghetti, A. Nicholson, P. Vranas,
A. Walker-Loud
arXiv:1912.03580

- Why are the computations so difficult?
- What is the two-nucleon controversy from lattice QCD?
- What can we do to resolve the controversy?
  - A first step towards resolution
    B.Hörz et al. PRC 103 (2021) [arXiv:2009.11825]

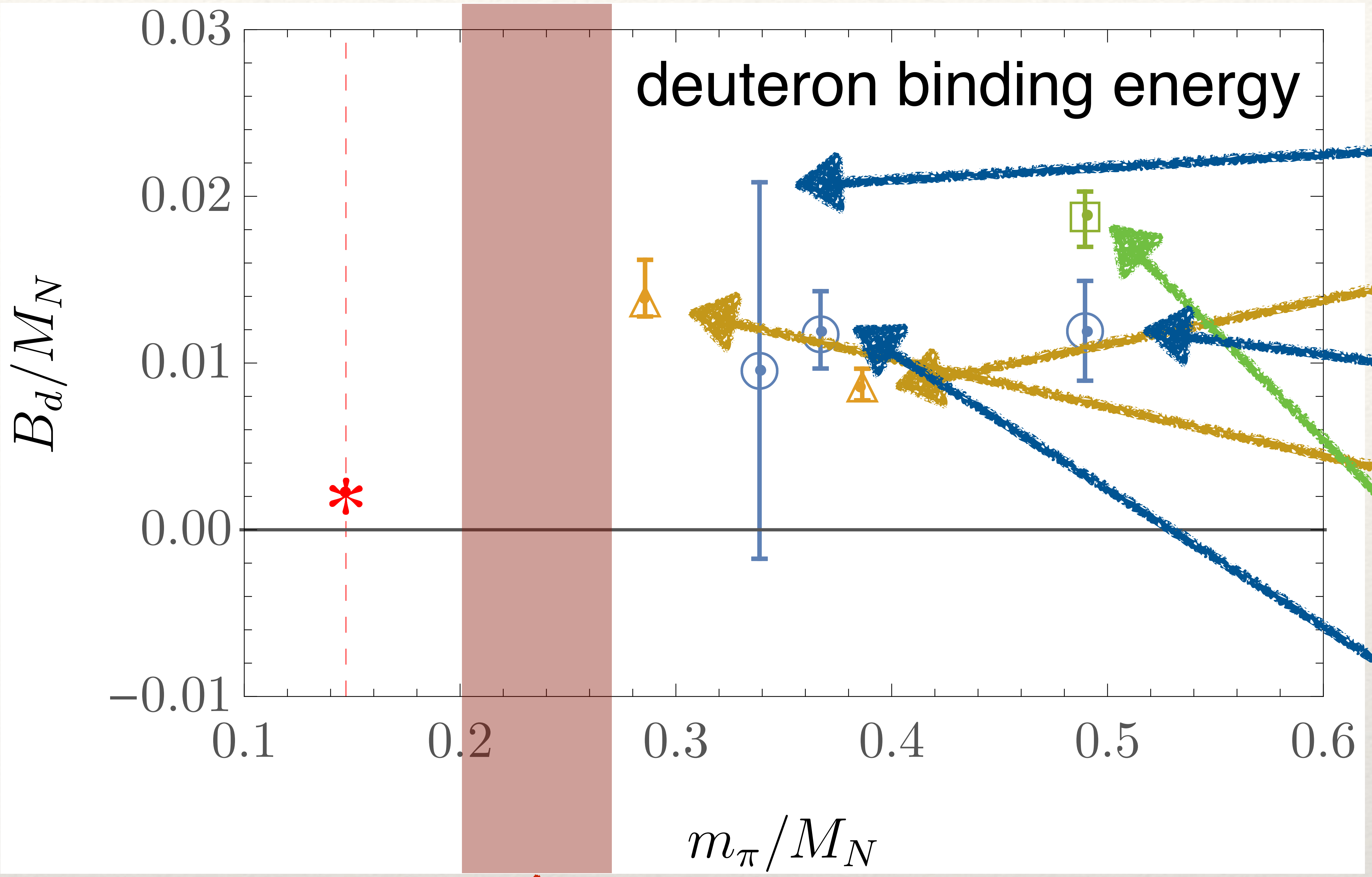
- Some of my goals today are to provide you with
  - A more self-critical look at the state of lattice QCD (LQCD) applications to two-nucleon (NN) systems
  - An understanding of why the computations are challenging
  - Questions to ponder when listening to this and other lattice talks on NN and NN matrix elements

Survey of lattice QCD results for two-baryons

Typical summary plot of LQCD calculations of binding energies



Survey of lattice QCD results for two-baryons



Approximate upper range of validity of NN EFT

Why have we (the community) failed to achieve the anticipated results?

2006 NPLQCD - first dynamical LQCD calculations of NN

2011 NPLQCD $M\pi \simeq 390$ MeV

2012 Yamazaki et al. $M\pi \simeq 510$ MeV

2012 NPLQCD $M\pi \simeq 800$ MeV

2015 Yamazaki et al. $M\pi \simeq 310$ MeV

2015 CalLat $M\pi \simeq 800$ MeV + P,D,F waves

2015 NPLQCD $M\pi \simeq 450$ MeV

2020 NPLQCD $M\pi \simeq 450$ MeV

2009 Scientific Grand Challenge Report
"Sustained 10-Petaflops will deliver NN at physical pion mass" (Titan 27 PFlop machine that ran 2012-2017)

2016 DOE Exascale Requirement for NP
"Complete calculations of NN and YN amplitudes at the physical pion mass by 2020"
(complete = continuum, infinite volume limits)

Why is the application of LQCD to NP so challenging?

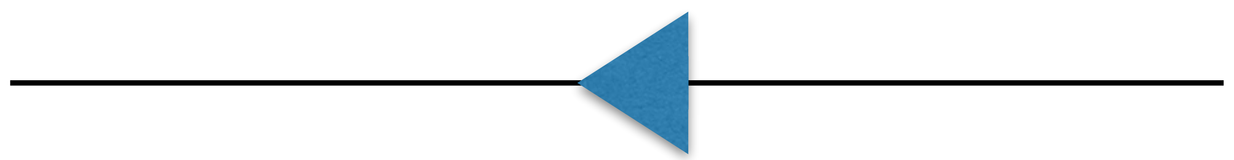
Why is the application of LQCD to NP so challenging?

LQCD challenges for NP

Most difficult challenge: an exponentially bad signal-to-noise problem

Parisi, Phys. Rep. 103 (1984) 203

Lepage, TASI 1989

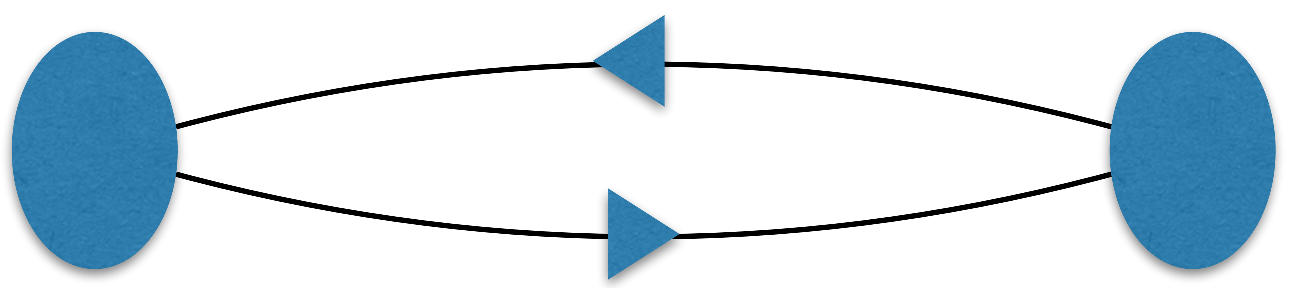


$$\sim e^{-\frac{1}{2}m_\pi t} + e^{-\frac{1}{3}m_N t} + \cdots$$

Each **quark propagator** carries information about pions and nucleons (conversations with David Kaplan)

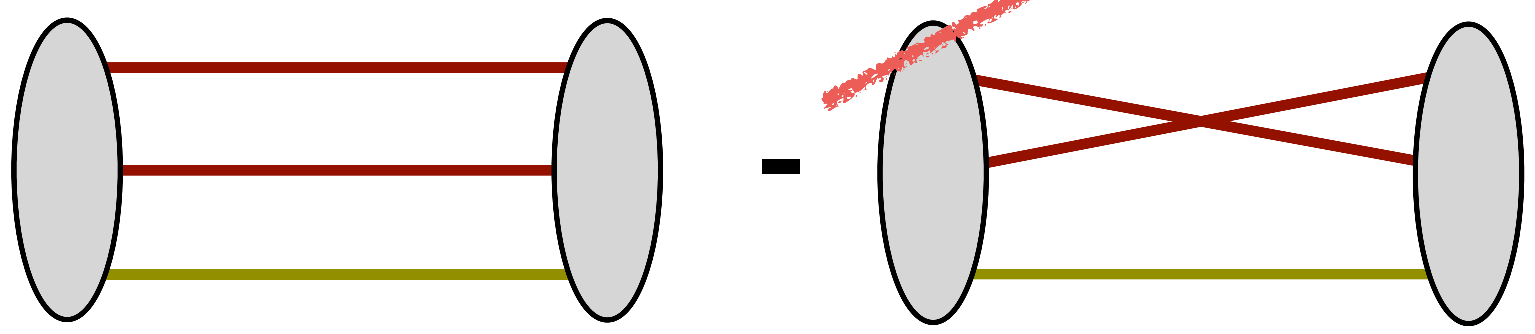
$$\lambda_\pi(t) \gg \lambda_N(t)$$

$$\lambda_i(t) \sim e^{-E_i t}$$



$$\bar{d}\gamma_5 u : C(t) = A_\pi e^{-m_\pi t} + \cdots$$

Large pion eigenvalues must cancel to expose small nucleon eigenvalues



$$(u^T C\gamma_5 d)u : C(t) = A_N e^{-m_N t} + \cdots$$

$$\frac{\text{Signal}}{\text{Noise}} \sim \sqrt{N} \exp\left[-A\left(m_N - \frac{3}{2}m_\pi\right)t\right]$$

exponential noise
power-law statistics

LQCD challenges for NP

2-point correlation function

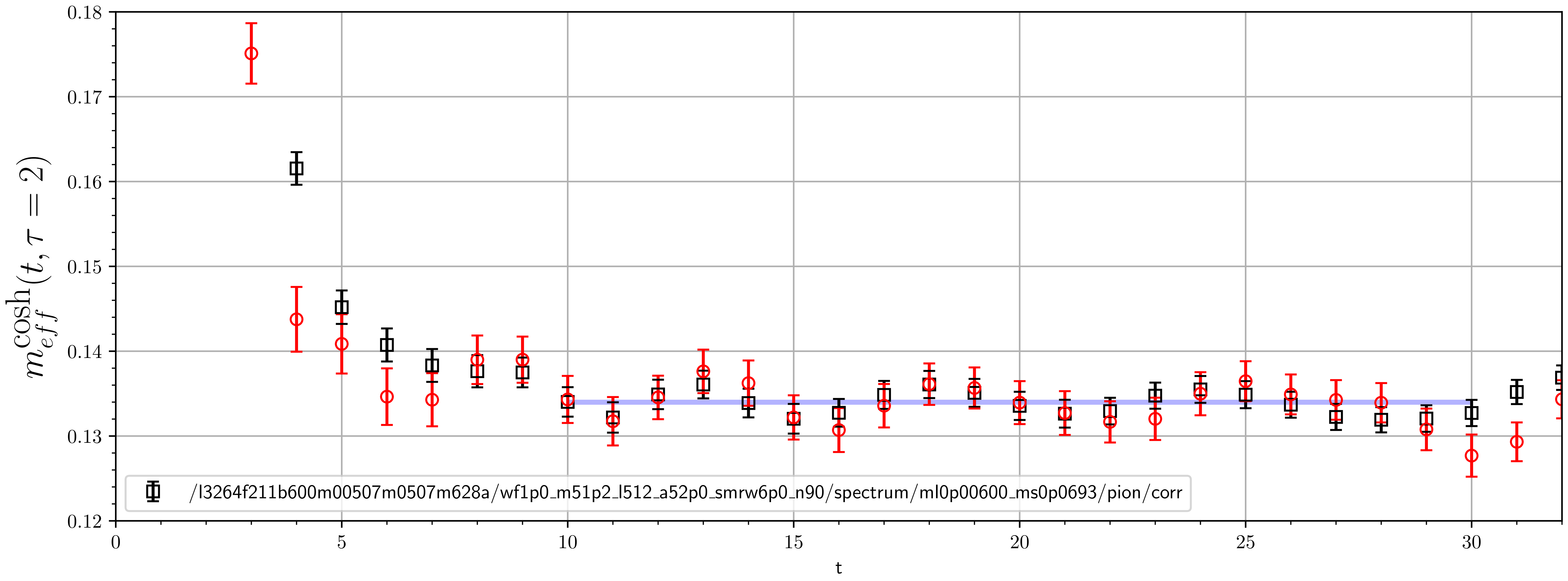
$$C(t) = \sum_n z_n z_n^\dagger e^{-E_n t}$$

$$m_{eff}(t) = \frac{1}{\tau} \ln\left(\frac{C(t)}{C(t+\tau)}\right)$$

For pions, need to consider leading finite temperature effects

$$C(t) = \sum_n z_n z_n^\dagger \left(e^{-E_n t} + e^{-E_n (T-t)}\right)$$

$$m_{eff}^{\cosh}(t,\tau) = \frac{1}{\tau}\cosh^{-1}\left(\frac{C(t+\tau)+C(t-\tau)}{2C(t)}\right)$$

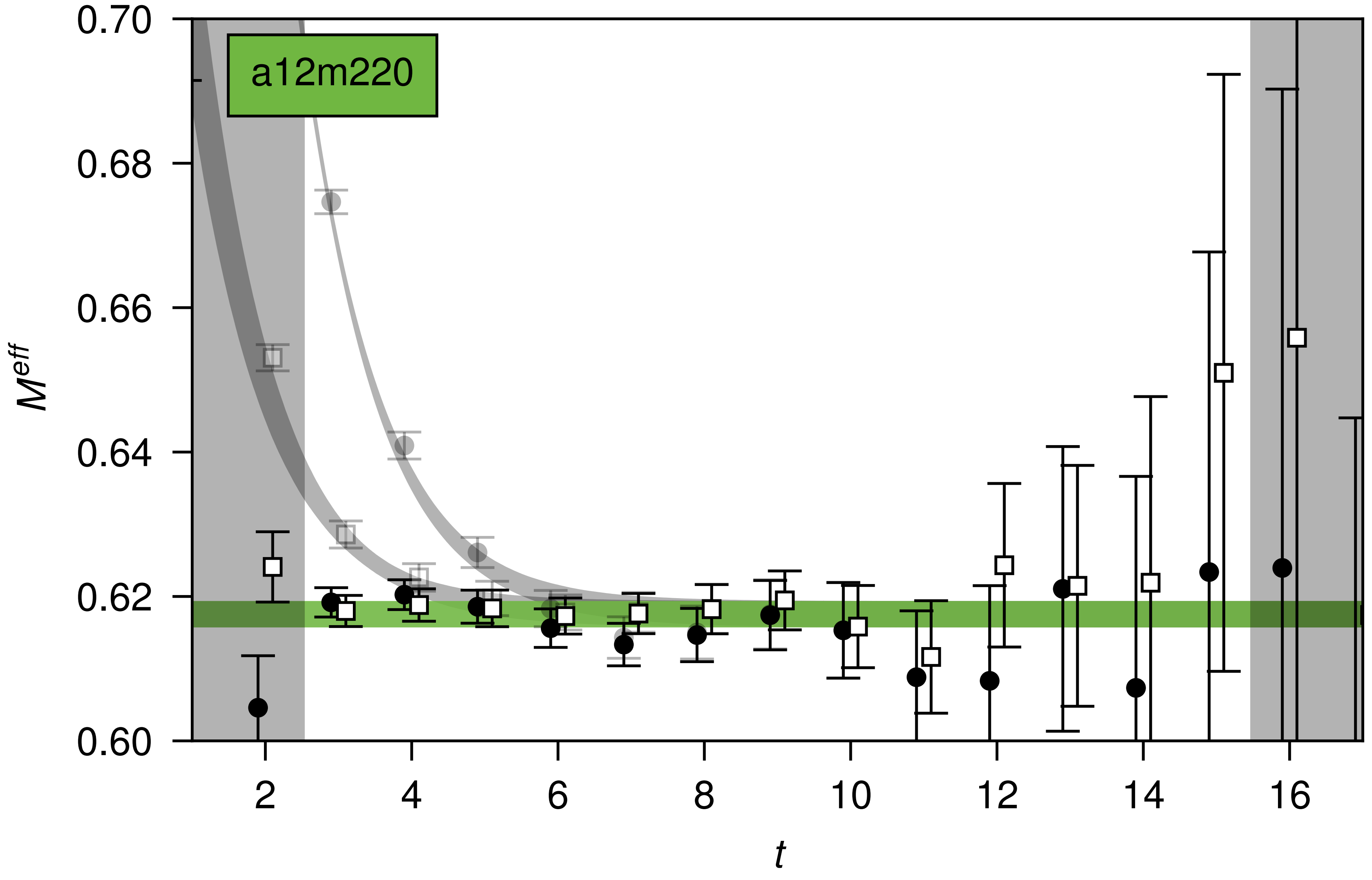


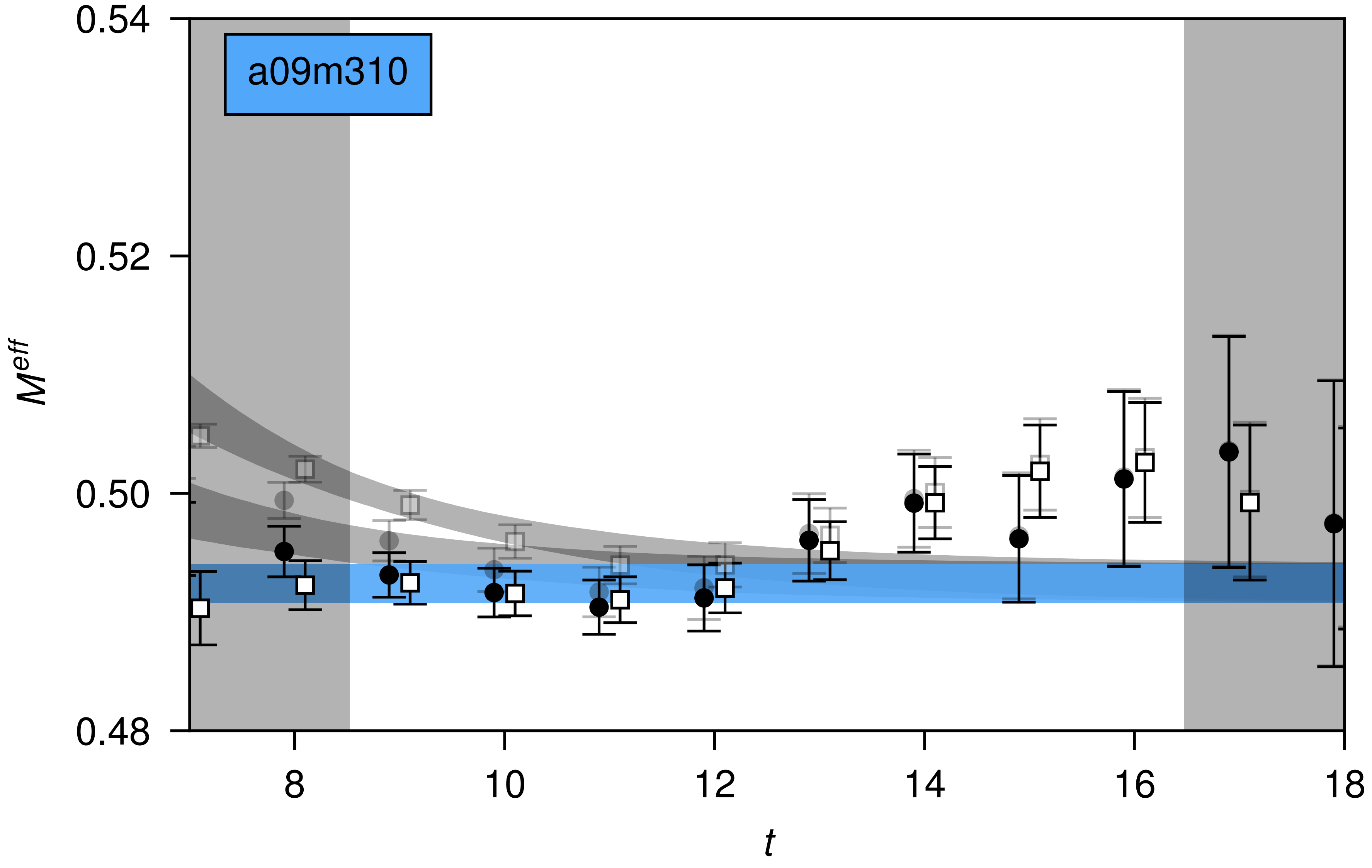
Effective mass of Pion 2-point correlation function
red and black “data” are from different choices of *interpolating* operators

Noise is constant in time - can determine very clean ground state (blue band)

LQCD challenges for NP

2-point correlation function





Two examples of nucleon effective mass

Noise is growing in time - can not simply go to the long-time limit without exponentially increasing the amount of statistics needed

$$\frac{Signal}{Noise} \rightarrow \sqrt{N_{state}} e^{-(m_N - \frac{3}{2}m_\pi)t}$$

Correlated late-time fluctuations… what is the ground state?

Need sophisticated analysis to ensure you are not susceptible to correlated fluctuations

This problem is exacerbated with form-factor calculations (g_A) and 2+ nucleons

- quark contraction cost becomes dominant
- density of excited states grows significantly and gap becomes small (nuclear interaction energies instead of pion mass gap)

Why is the application of LQCD to NP so challenging?

For two (and more) nucleons, the physics of interest lies in the interaction energy

$\Delta E = E_{AN} - AM_N \ll AM_N$ (for the deuteron, ~2 MeV out of ~2 GeV - a per-mille effect)

for a weakly interacting system: $\Delta E \approx -\frac{4\pi a}{ML^3}$ scattering length

The excited states of the NN system arise from

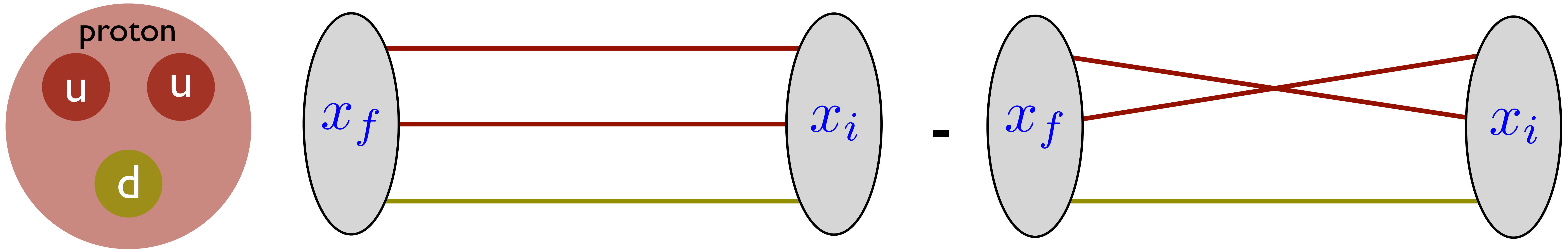
- inelastic nucleon excitations, ΔE ~ 2 mπ
- two-nucleon elastic scattering states, ΔE ~ q²/2M ~ 20 MeV

$$C_{NN}(t) \approx a_0 e^{-E_0 t}\left[1 + a_1 e^{-\Delta E t} + \cdots\right]$$

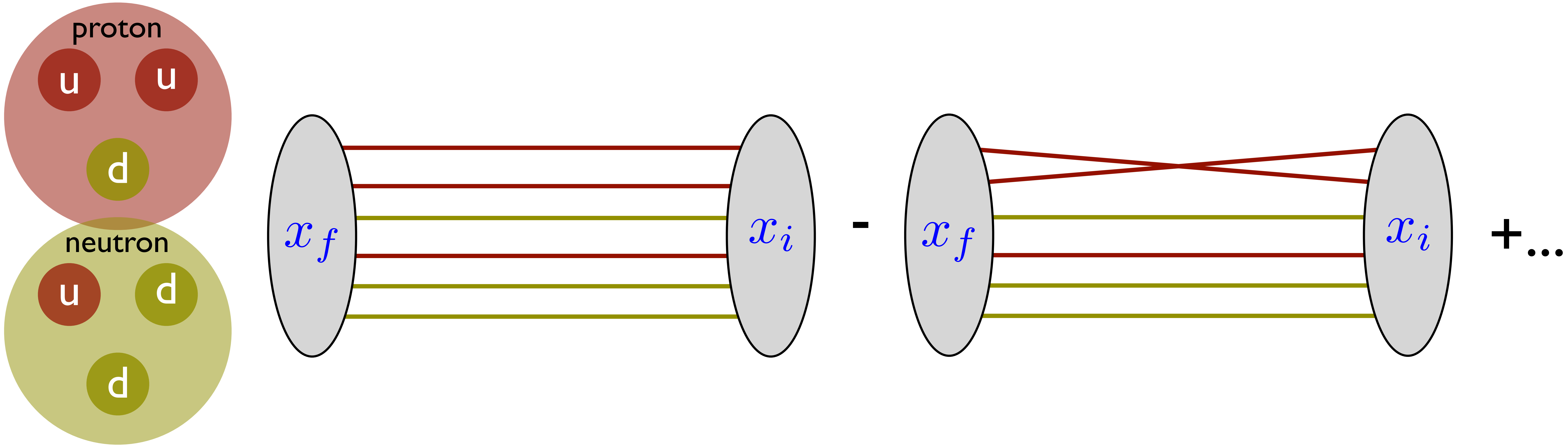
$\Delta E t \approx 1 \longrightarrow t \approx 5 - 10$ fm
signal is lost to noise before 2 fm

we have to precisely tease out a per-mille-level energy from a noisy system…

LQCD challenges for NP

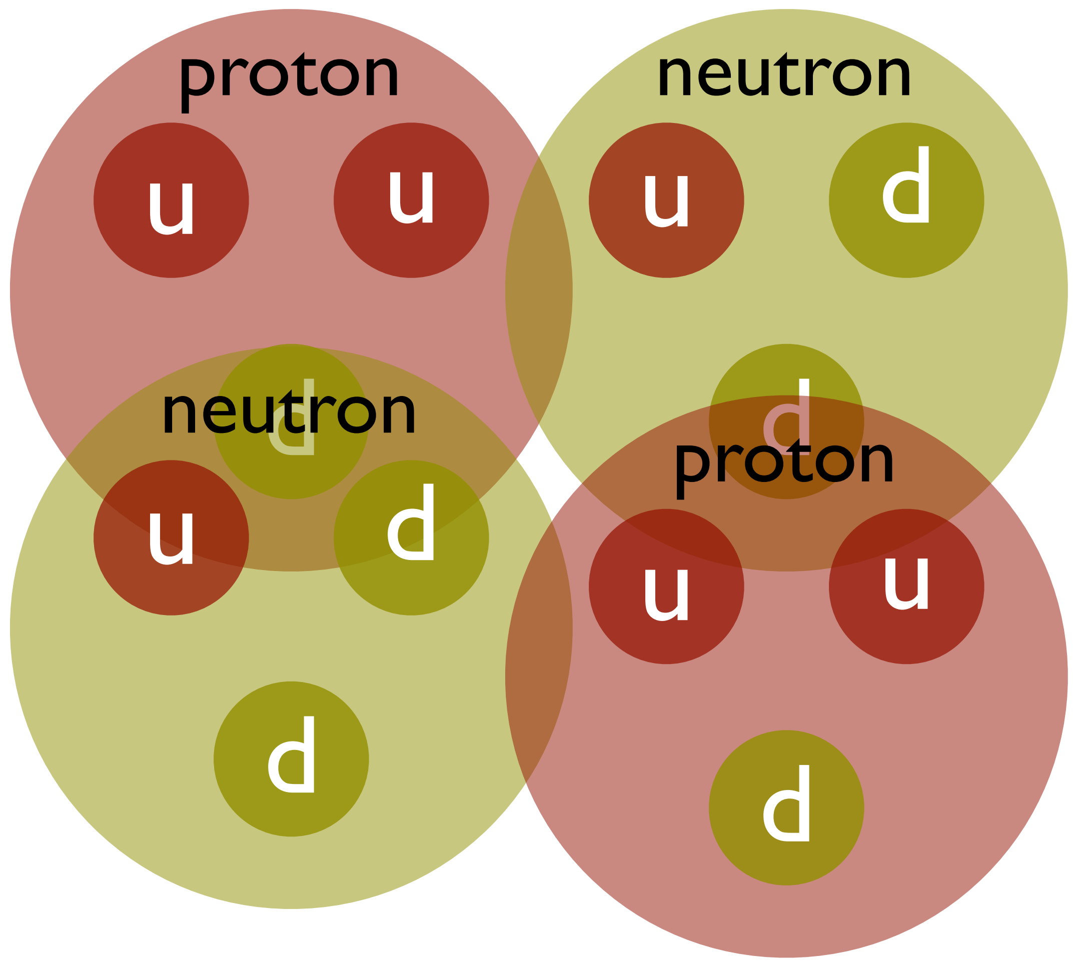


$2! \times 1! = 2$
contractions



$3! \times 3! = 36$
contractions

LQCD challenges for NP



+...

$6! \times 6! = 518400$

contractions

numerical cost exceeds HMC + props

quark-exchange diagrams are source of fermion sign problem

expensive AND noisy :(

There are clever solutions that reduce this cost significantly

Yamazaki, Kuramashi, Ukawa arXiv:0912.1383,

Doi, Endres arXiv:1205.0585, Detmold, Orginos arXiv:1207.1452

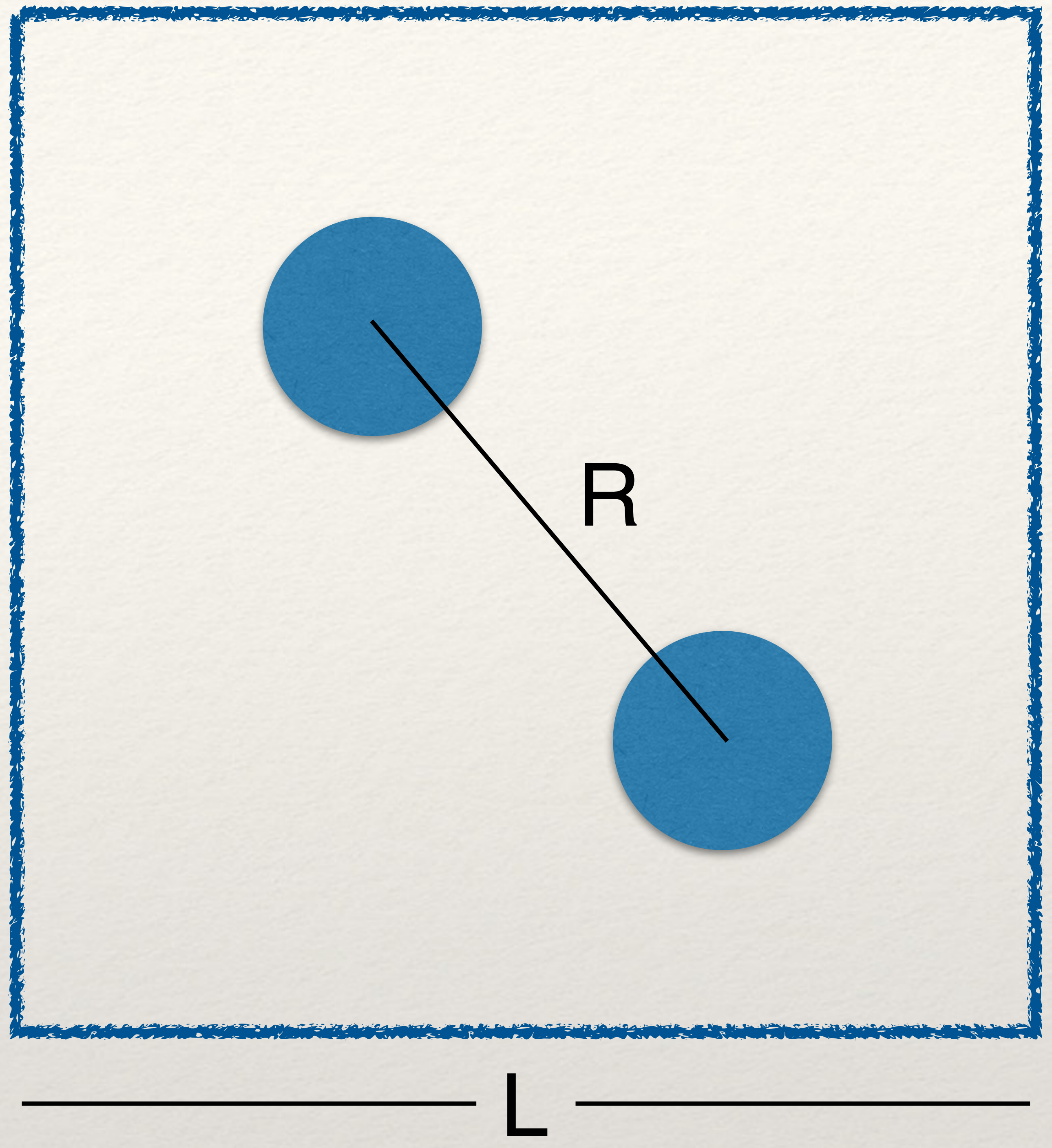
Günther, Toth, Varnhorst arXiv:1301.4895

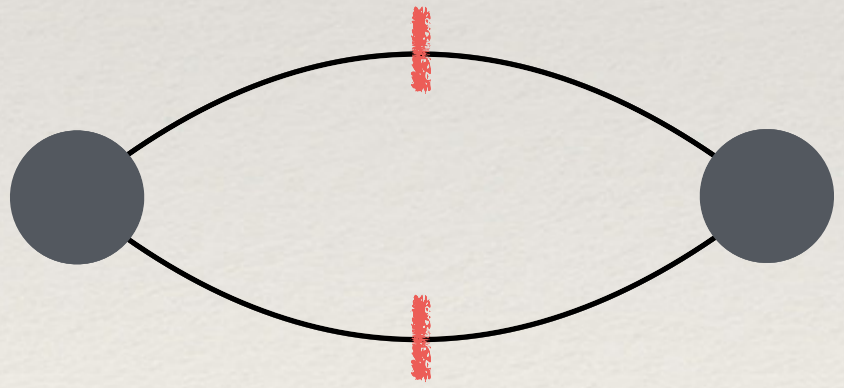
But unfortunately, they only work with unrealistically simplistic interpolating operators in which all quarks originate from the same spacetime point, or are otherwise, identical

How are the calculations performed?

- The calculations are performed in a finite (typically) periodic volume
  - We can not do scattering as we typically think of it (take initial and final states asymptotically far apart as single-particle states)
  - If R ≲ L/2, we can use the "Lüscher" method which provides a map from E → q cot δ(q)
    R = range of interaction, L = size of volume

    Consider the s-channel interaction
    - the intermediate particles have enough energy to go "on-shell" and propagate a long-distance and "feel" that they are in a finite volume





$$E = 2\sqrt{M^2 + q^2}$$

non-interacting particles: $\vec{q}_n = \vec{n}\dfrac{2\pi}{L}$

distortion of q from non-interacting modes is proportional to the interaction

Weakly interacting particles:

$$\Delta E = E - 2M \approx -\frac{4\pi a}{ML^3}\left[1 + \mathcal{O}\left(\frac{a}{L}\right)\right]$$

"Signal" scales inversely with the volume

How are the calculations performed?

- HAL QCD has been developing an alternative method - the HAL QCD Potential Method

nucl-th/0611096
arXiv:0909.5585
arXiv:1203.3642
…

$$\left[\frac{\partial_t^2}{4M} - \partial_t - H_0\right] R(\mathbf{r},t) = \int d^3s\, U(\mathbf{r},\mathbf{s}) R(\mathbf{s},t)$$

$$R(\mathbf{r},t) = \frac{C_{NN}(\mathbf{r},t)}{[C_N(t)]^2} \qquad C_{NN}(\mathbf{r},t) = \sum_{\mathbf{x}} \langle N(\mathbf{x}+\mathbf{r},t) N(\mathbf{x},t) N^\dagger(0) N^\dagger(0)\rangle$$

$$U(\mathbf{r},\mathbf{s}) = \delta^3(\mathbf{r}-\mathbf{s}) \left[V_C(r) + V_\sigma(r)\sigma_1\cdot\sigma_2 + V_{LS}(r)\mathbf{L}\cdot\mathbf{S} + \cdots + V_{N^2LO}(r)\frac{\nabla^2}{\Lambda^2} + \cdots\right]$$

- One assumes that R($\mathbf{r}$,t) is free from "inelastic" excited states (both from the single nucleon and from NN)
- Then, this time-dependent equation is valid for all elastic NN scattering states
- Use numerical results to determine potential - then - solve the Schrödinger Equation

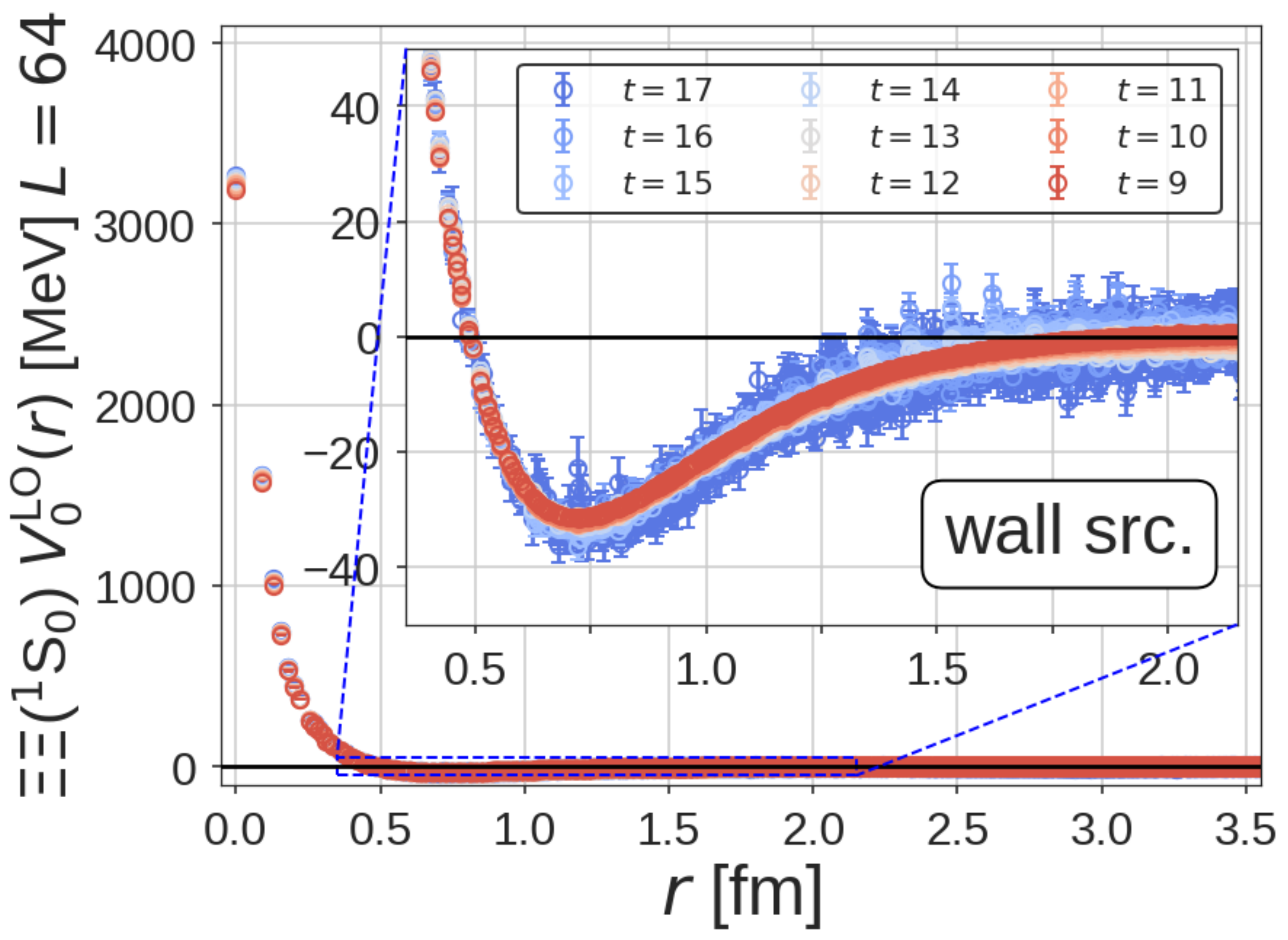
How are the calculations performed?

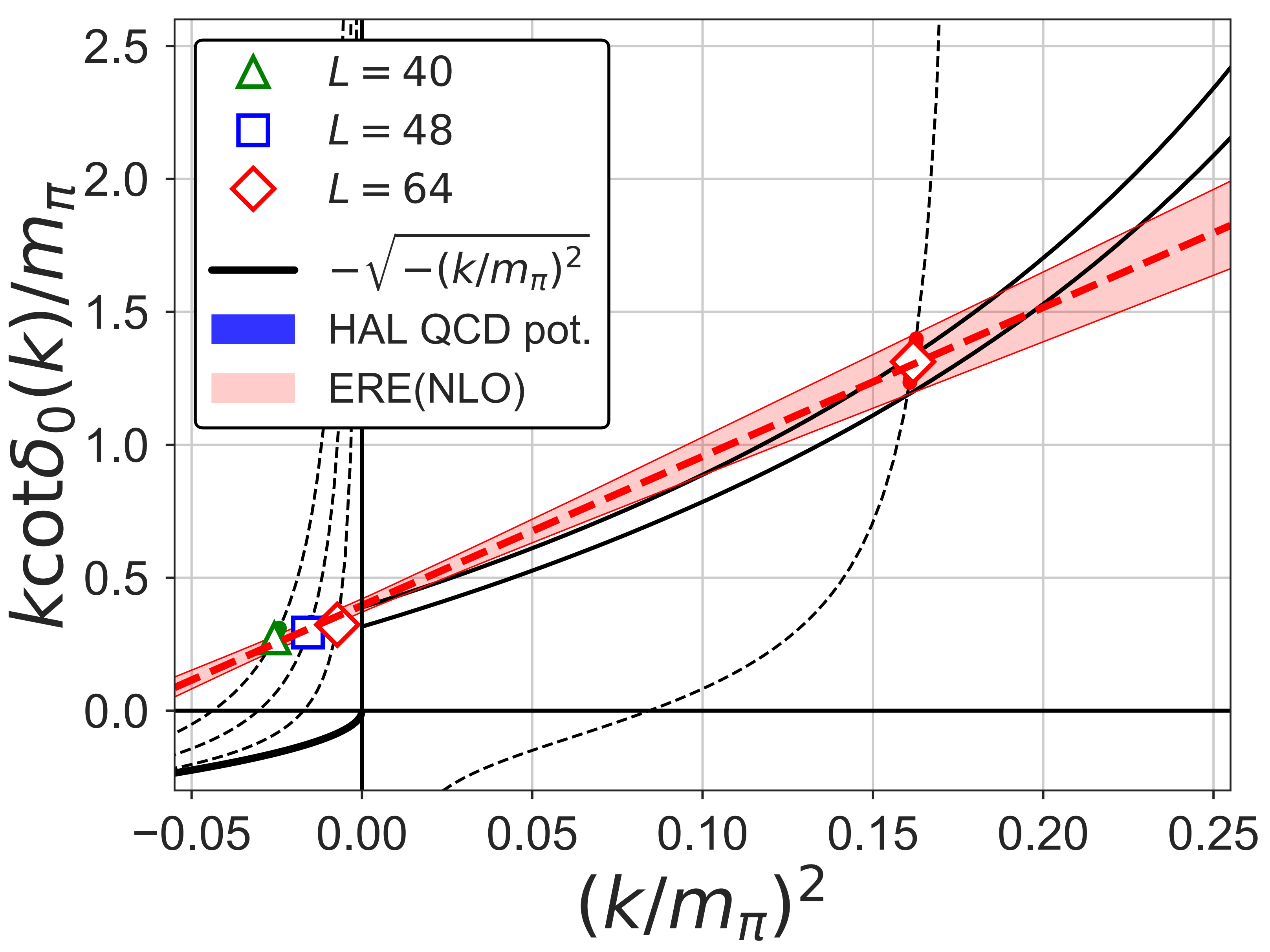
- HAL QCD Potential Method
  nucl-th/0611096
  arXiv:0909.5585
  arXiv:1203.3642
  …

- The potential is not an observable - and is arbitrary up to unitary transformations - but - the combination of the potential with the "wave function" R(**r**,t) produces observable results - the phase shifts

  - At the values of **q** (**k**) allowed by the Lüscher quantization condition - the HAL QCD Potential method agrees with the Lüscher method

  - In between these values, the HAL QCD Potential method provides a smooth interpolating function between the allowed values of **q** (**k**) similar to an effective range expansion (ERE)





Summary of Challenges

- In addition to the standard extrapolations
  - continuum, infinite volume, physical pion mass limits
- We have to tease out a per-mille-level interaction energy from a system with
  - exponentially growing noise
  - dense spectrum
  - exponentially growing quark-line contraction cost

$$\frac{\mathrm{S}}{\mathrm{N}}(t) \approx \sqrt{N_{\mathrm{samp.}}}\, e^{-(2M_N - 3m_\pi)t}$$

- Trying to get the physics "just" by increasing the statistical samples is an exponentially challenging approach
- Until recently, this was the strategy of a significant fraction of the community
- HAL QCD has been developing an alternative method which they believe is more cost-effective - but difficult to do three-nucleons or incorporate electroweak matrix elements

Two-nucleon LQCD controversy

- LQCD calculations that utilize compact (hexaquark) creation operators observe deeply bound di-nucleon systems at heavier than physical pion masses ($300 \lesssim M\pi \lesssim 800$) MeV (NPLQCD, Yamazaki et al, CalLat)
- Calculations that utilize diffuse creation operators
  - HAL QCD potential
  - displaced nucleons (CalLat),
  - momentum space creation operators (Mainz, sLapHnn)
- observe there are no bound di-nucleons

- The spectrum of the theory does not depend upon the creation operator
  - at least one method has to be wrong (the creation operators have introduced an unrecognized systematic uncertainty)
  - or there have to be significant discretization effects (this is the worst-case scenario in terms of computing cost)

Two-nucleon LQCD controversy

- When the discrepancy first arose (~2012), it was assumed by most to be an issue with the HAL QCD Potential method - it requires a few more assumptions than the "standard"
- HAL QCD has extensively studied their method in comparison with the standard method (as it is applied in most calculations)
  - When wall source creation operators are used, a consistent spectrum between Lüscher and potential are observed
  - When a local (hexaquark) creation operator is used, the spectrum is different than potential
- HAL QCD have uncovered possible problems with the "standard method"
  - They claim the standard application is susceptible to a false plateaux of the ground state (misidentification of the true spectrum) [arXiv:1607.06371]
  - They catalogued a set of "consistency checks" that results must satisfy provided all systematic uncertainties are under control (and they showed that most results failed these consistency checks) [arXiv:1703.07210]

Two-nucleon LQCD controversy

- How can one get a "false plateaux"?

$$R(t) = \frac{C_{NN}(t)}{[C_N(t)]^2} \qquad C_{NN}(t) = \sum_m A_m e^{-E_m t} \qquad C_N(t) = \sum_n A_n e^{-E_n t}$$

- The excited states of NN will have both elastic NN excitations as well as interactions between inelastic single nucleon states

$$C_{NN}(t) = \underbrace{\sum_q B_{00,q} e^{-(2E_0+\Delta E_{00}(q))t}}_{\text{tower of elastic states}} + \underbrace{\sum_{\tilde{q}} B_{01,\tilde{q}} e^{-(E_0+E_1+\Delta E_{10}(\tilde{q}))t}}_{\text{inelastic states}} + \cdots$$

$$R(t) \approx b_{00,0} e^{-\Delta E_{00}(q_0)t} + \underbrace{b_{00,1} e^{-\Delta E_{00}(q_1)t}}_{} + \underbrace{b_{10,0} e^{-(E_1-E_0+\Delta E_{10}(\tilde{q}_0))t} - 2a_1 e^{-(E_1-E_0)t}}_{} + \cdots$$

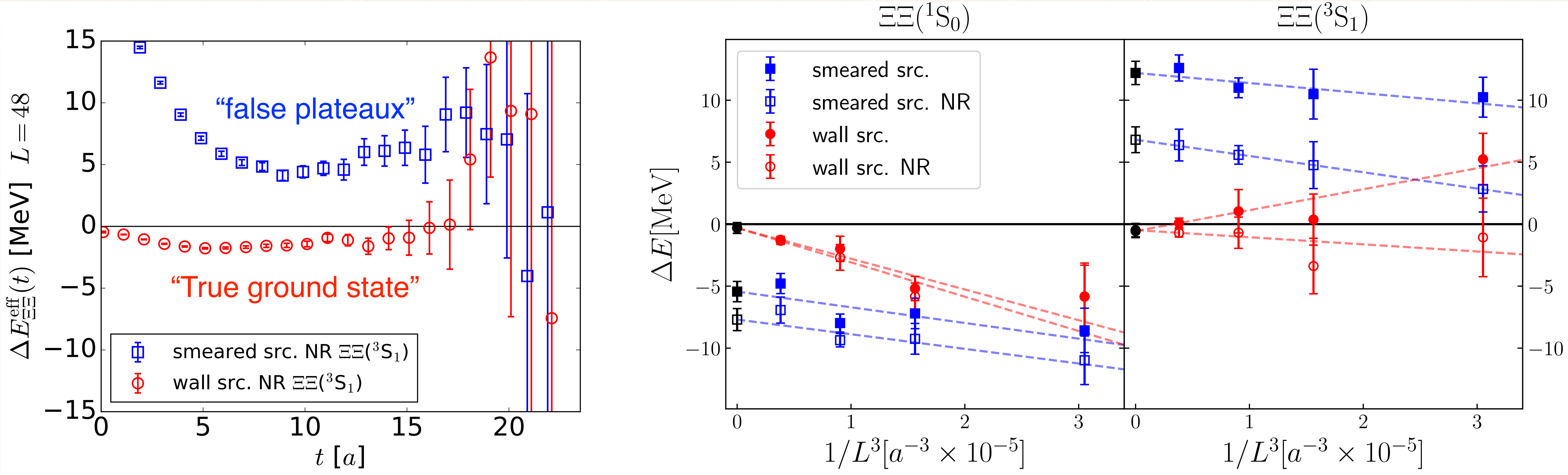
excited states that persist for 4-10 fm
possible opposite sign coefficients

competing opposite sign terms

$$\Delta E_{00}(q) \approx 10 - 40 \text{ MeV} \qquad E_1 - E_0 \approx \mathcal{O}(2m_\pi)$$

Two-nucleon LQCD controversy

- HAL QCD [1607.06371] - issue with local, gaussian smeared quark source creation operators



Whether bound or scattering, attractive interactions → ΔE < 0

Bound states have non-vanishing energy as L → ∞

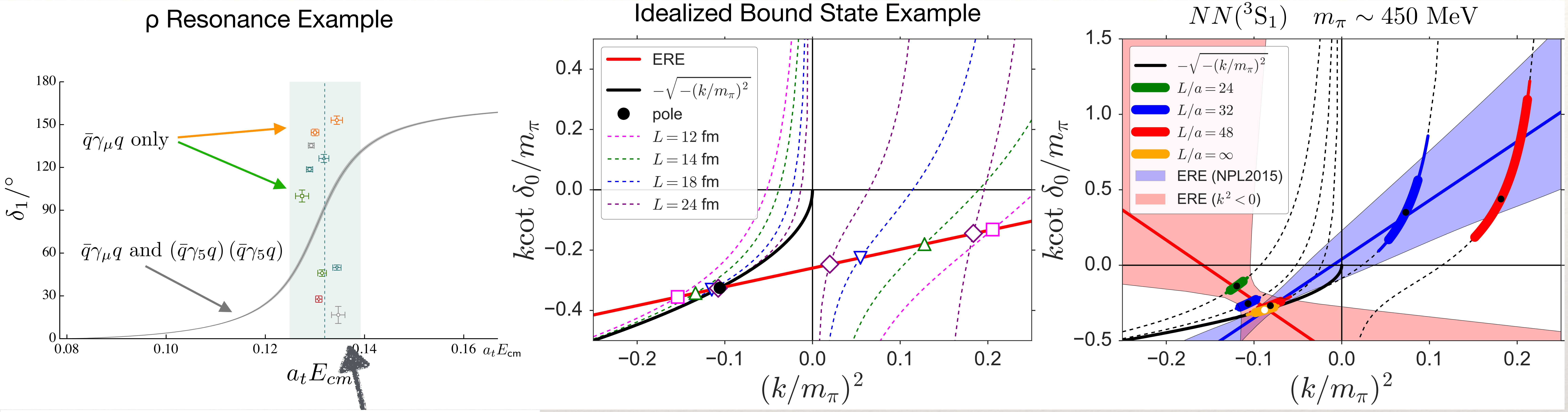
Scattering states have energies that must vanish as L → ∞

$$\Delta E = E - 2M \approx -\frac{4\pi a}{ML^3}\left[1 + \mathcal{O}\left(\frac{a}{L}\right)\right]$$

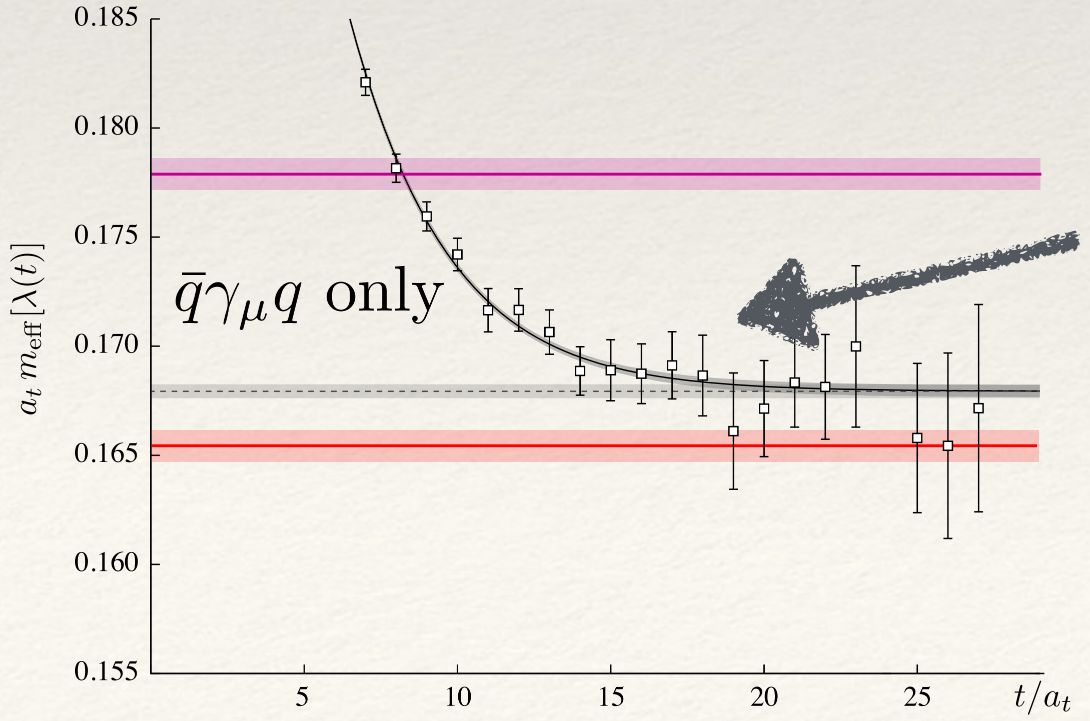
Two-nucleon LQCD controversy

- HAL QCD Consistency Checks [1703.07210]

NPLQCD PRD 92 (2015) [1508.07583]



False energy levels can be detected from not obeying expectations from Lüscher



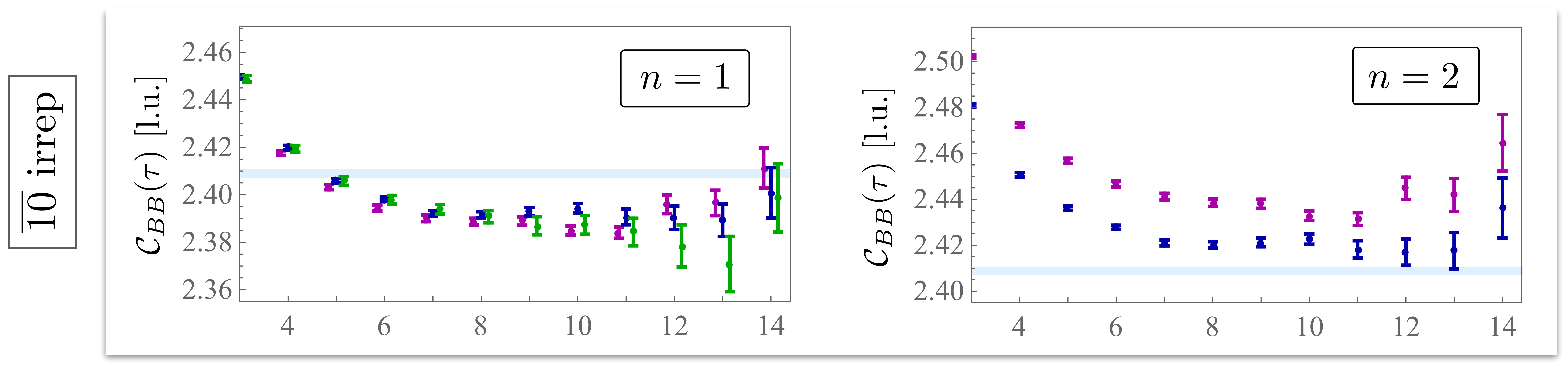
seemingly healthy effective mass (gray) but does not follow Breit-Wigner

True energy levels must
- Follow the "Lüscher lines" (dashed curves)
- Have correct sign for pole of residue slope of ERE less than slope of bound-state curve, $-\sqrt{-(k/m_\pi)^2}$

- InConsistency of the below and above threshold ERE
- Above-threshold ERE crosses bound-state curve with too large of a slope
- These results have a clear systematic uncertainty issue
- See updated result: 2009.12357

Two-nucleon LQCD controversy

- NPLQCD Mπ ≃ 800 MeV [arXiv:1706.06550]



- On the other hand - the NPLQCD results at Mπ ≃ 800 MeV are behaving as one would expect
  - Bound states are ~volume independent
  - Scattering states show clear volume dependence

Two-nucleon LQCD controversy

- We're left with a murky situation
  - At heavy pion masses ($M\pi \simeq 800$ MeV)
    - There is nothing obviously wrong with the HAL QCD Potential Method
    - There is nothing obviously wrong with the "standard" results (NPLQCD, CalLat)
    - Even at such heavy pion masses where the signal-to-noise problem is least challenging, there is this fundamental disagreement in the qualitative spectrum between the methods
  - Lighter pion mass results do not pass the consistency checks
  - The community can not seem to produce NN results below $M\pi \simeq 300$ MeV,
  - Results at $M\pi \lesssim 200$ MeV are necessary to provide input to NN EFTs that can be used to describe light-nuclei

The Path Forward

- How do we make progress and resolve this discrepancy?
  - We must resolve the discrepancy: suppose the NPLQCD method has an un-recognized systematic uncertainty - then all of the matrix-elements they are determining are arbitrarily wrong

- One key shortcoming of the standard applications with Lüscher's method (NPLQCD, Yamazaki et al, CalLat) is that they utilize local (hexaquark) creation operators and diffuse momentum-space sink operators
  - The correlation functions are not positive definite
  - The computations require significantly more stochastic samples as they do not take advantage of the volume-averaging provided by momentum-space creation operators

- A community consensus has emerged: momentum-space creation operators should be used to allow for a variational calculation with positive-definite operator basis (and full use of all irreps of the lattice symmetry)

- We should also perform a computation with all methods on the same configurations to eliminate all sources of systematic uncertainty besides the method
  - HAL QCD has analyzed the potential method and NPLQCD method on the same configurations
  - There is no use of the CalLat (displaced source method) or variational method on the same configurations

The Path Forward

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Two-nucleon S-wave interactions at the SU(3) flavor-symmetric point with $m_{ud} \approx m_s^{\text{phys}}$: A first lattice QCD calculation with the stochastic Laplacian Heaviside method

Ben Hörz, Dean Howarth, Enrico Rinaldi, Andrew Hanlon, Chia Cheng Chang (張家丞), Christopher Körber, Evan Berkowitz, John Bulava, M. A. Clark, Wayne Tai Lee, Colin Morningstar, Amy Nicholson, Pavlos Vranas, and André Walker-Loud

Phys. Rev. C **103**, 014003 – Published 19 January 2021

arXiv:2009.11825

for lack of a better name - sLapHnn Collaboration
(stochastic Laplacian Heaviside NN)

The Path Forward

- Standard application with Lüscher method:

$$C_{NN}(t,\mathbf{p}+\mathbf{q}) = \sum_{\mathbf{x}}\sum_{\mathbf{y}} e^{i\mathbf{p}\cdot\mathbf{x}} e^{i\mathbf{q}\cdot\mathbf{y}} \langle 0|\underline{N(t,\mathbf{x})N(t,\mathbf{y})}\,\underline{N^\dagger(0,\mathbf{0})N^\dagger(0,\mathbf{0})}|0\rangle$$

each nucleon is separately projected to a state of definite momentum (but only **P=p+q** is conserved)

all 6 quarks originate from the same spacetime point - hexaquark

- With such operators - the overlap of the creation operator onto the excited states is as large, or larger than the overlap onto the ground state $A_1 \gtrsim A_0$

$$C_{NN}(t,\mathbf{p}+\mathbf{q}) \approx A_0 e^{-E_0 t} + A_1 e^{-E_1 t} + \cdots$$

  - The two-nucleons "do not like" being at the same spacetime point

- CalLat observed that if the source operators were displaced PLB 765 (2017) [1508.00886]

$$C_{NN}(t,\mathbf{p}+\mathbf{q},\mathbf{\Delta}) = \sum_{\mathbf{x}}\sum_{\mathbf{y}} e^{i\mathbf{p}\cdot\mathbf{x}} e^{i\mathbf{q}\cdot\mathbf{y}} \langle 0|N(t,\mathbf{x})N(t,\mathbf{y})N^\dagger(0,\mathbf{0})N^\dagger(0,\mathbf{0}+\mathbf{\Delta})|0\rangle$$

the overlap onto the ground state became much larger than onto the excited state $A_1 < A_0$
  - The deep bound state is not observed
  - The correlation functions are still not positive definite

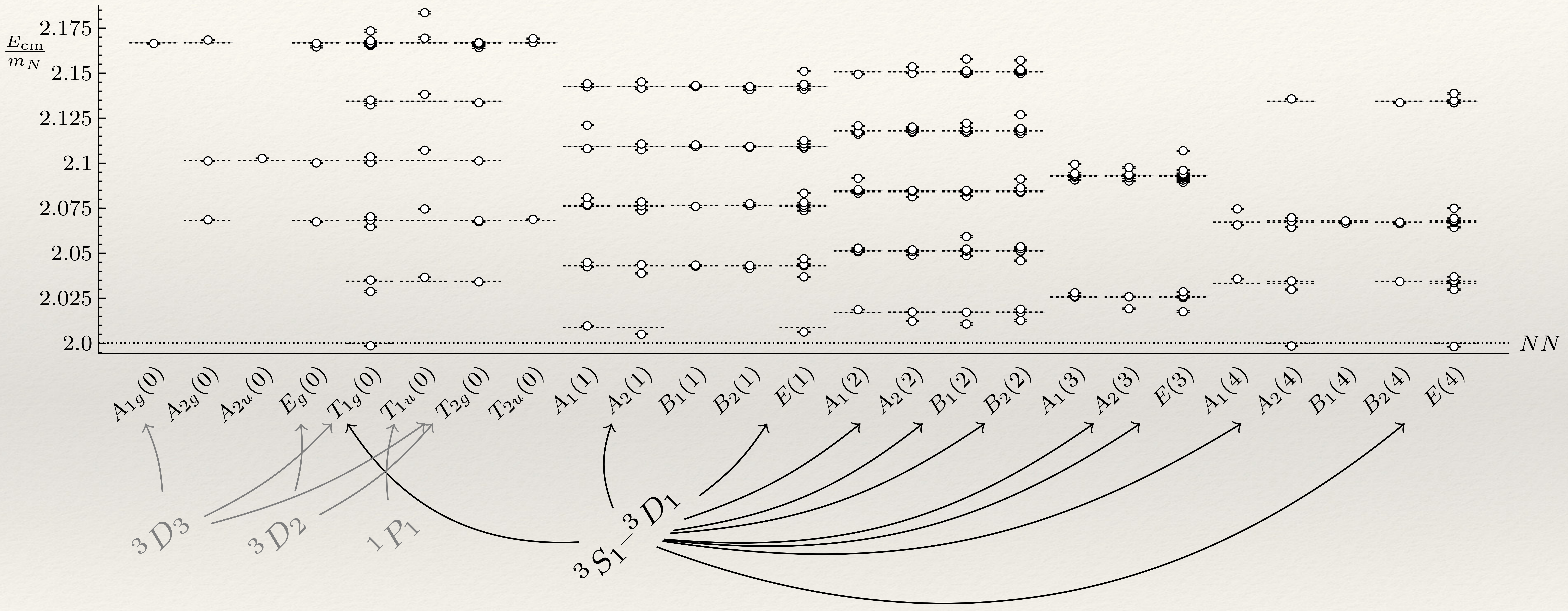
The Path Forward

- Alternatively - one can solve quark propagators from the eigenvectors of the 3D smearing kernel that is typically used (instead of one propagator per source for many sources) [arXiv:0905.2160]
  - This allows one to construct momentum-based creation operators
  - The quark-level contraction cost significantly increases (instead of 6-quarks from one source, we have N_{eig} sources for each quark → $N_{eig}{}^4$ contractions)
    - This also provides a volume averaging at the source (as well as the sink)
  - We used a stochastic variant which holds N_{eig} fixed as the volume varies [arXiv:1104.3870]
  - The correlation functions are now positive-definite: $A_{ii} = \langle 0|NN_i|n\rangle\langle n|NN_i^\dagger|0\rangle \geq 0$

$$C_{ij}^{NN}(t, \mathbf{p_f}, \mathbf{q_f}, \mathbf{p_i}, \mathbf{q_i}) = \sum_{\mathbf{x_f},\mathbf{y_f}} \sum_{\mathbf{x_i},\mathbf{y_y}} e^{i(\mathbf{p_f}\cdot\mathbf{x_f}+\mathbf{q_f}\cdot\mathbf{y_f})} e^{-i(\mathbf{p_i}\cdot\mathbf{x_i}+\mathbf{q_i}\cdot\mathbf{y_i})} \langle 0|NN_i(t, \mathbf{x_f}, \mathbf{y_f}) NN_j^\dagger(0, \mathbf{x_i}, \mathbf{y_i})|0\rangle$$

The Path Forward

- arXiv:2009.11825



(only shown for total zero momentum)

(in the following: assume negligible $S - D$ mixing)

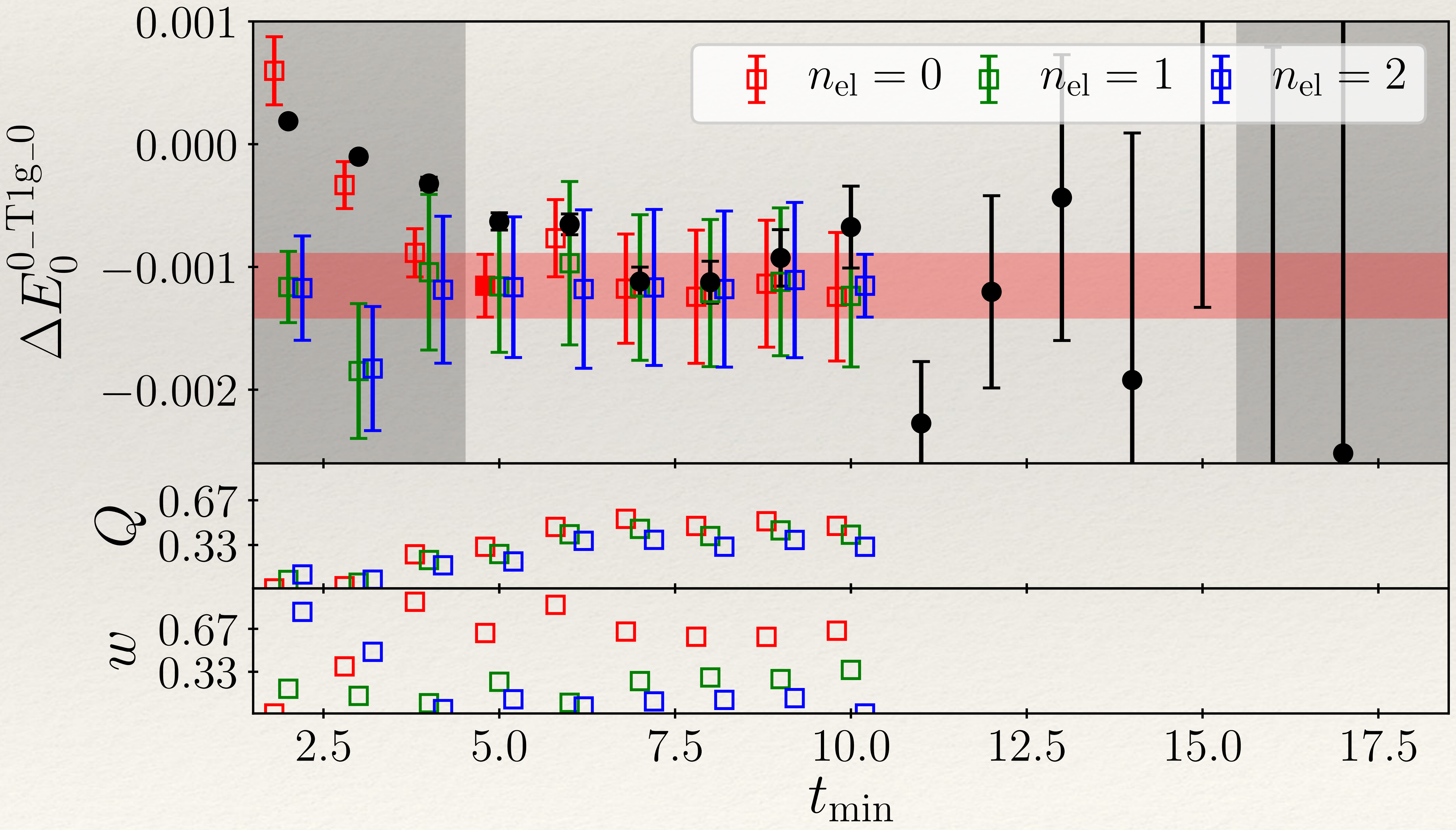
The Path Forward

- In order to take advantage of the positive-definite nature of the NN correlation function, we use the following fit function (implicit sum over l, q, p)

$$R(t) = \frac{r_0^2 e^{-\Delta E_0^{NN} t}\left(1 + r_l^2 e^{-\Delta E_{l,0}^{NN} t}\right)}{\left(1 + z_{q,n}^2 e^{-\Delta E_{n,0}^{q} t}\right)\left(1 + z_{p,m}^2 e^{-\Delta E_{m,0}^{p} t}\right)}$$

  - $r_l \sim O(1) \geq 0$
  - $z_{q,n} \sim O(1) \geq 0$

  - We include the same number of inelastic excited states in NN as in N, and then study the ground state vs the number of additional elastic excited states included in the analysis



The Path Forward

- In this first work - we focus on states below the t-channel cut which also have only predominant S-wave interactions
- Given the spectrum and resulting phase-shift values, we perform an effective range expansion analysis

$$q\cot\delta(q) = -\frac{1}{a} + \frac{1}{2}r_0 q^2 + \frac{1}{6}r_1 q^4 + \cdots$$

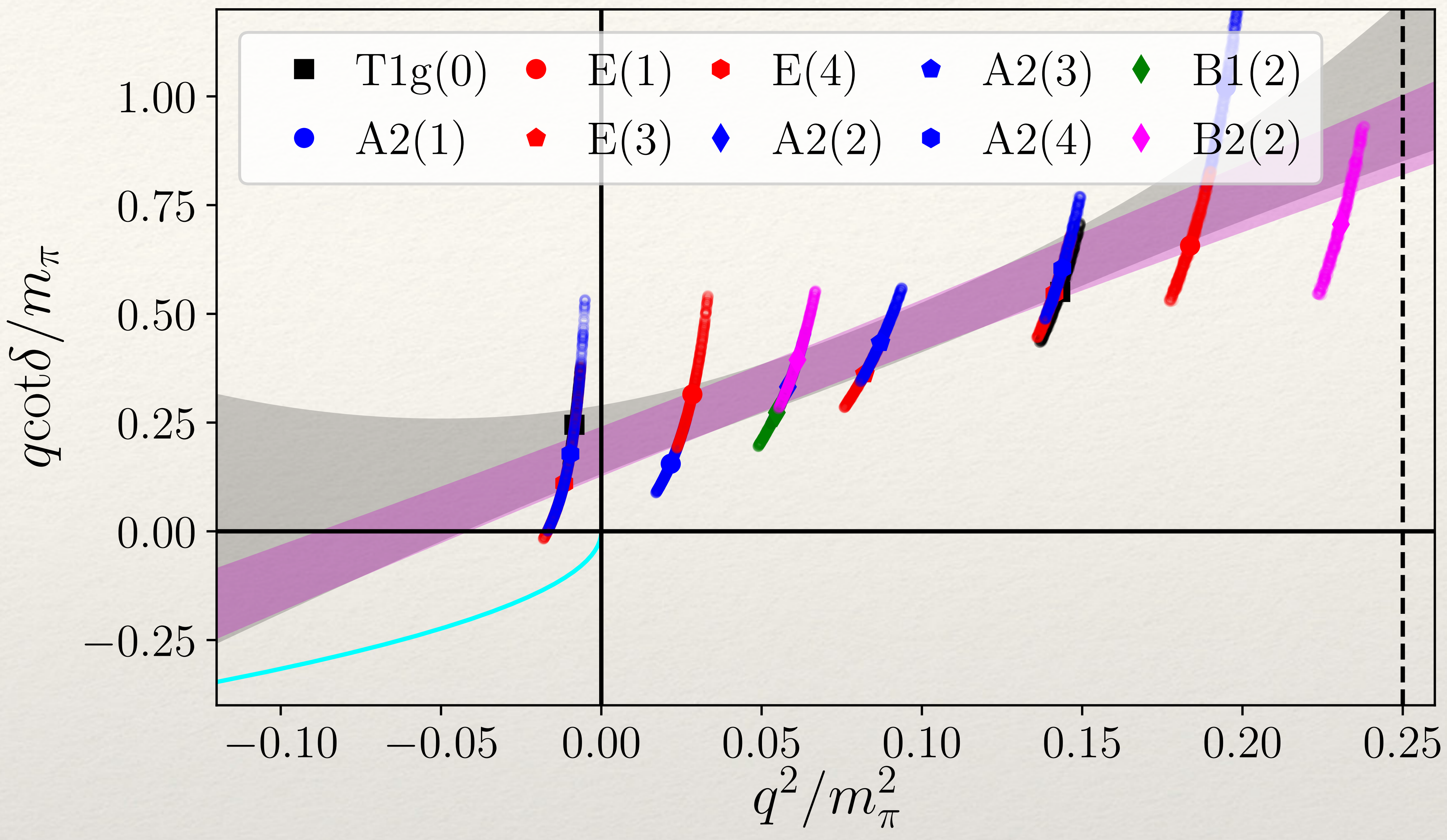
$$m_\pi a = -5.5(1.6), \quad m_\pi r_0 = 5.82(.71)$$

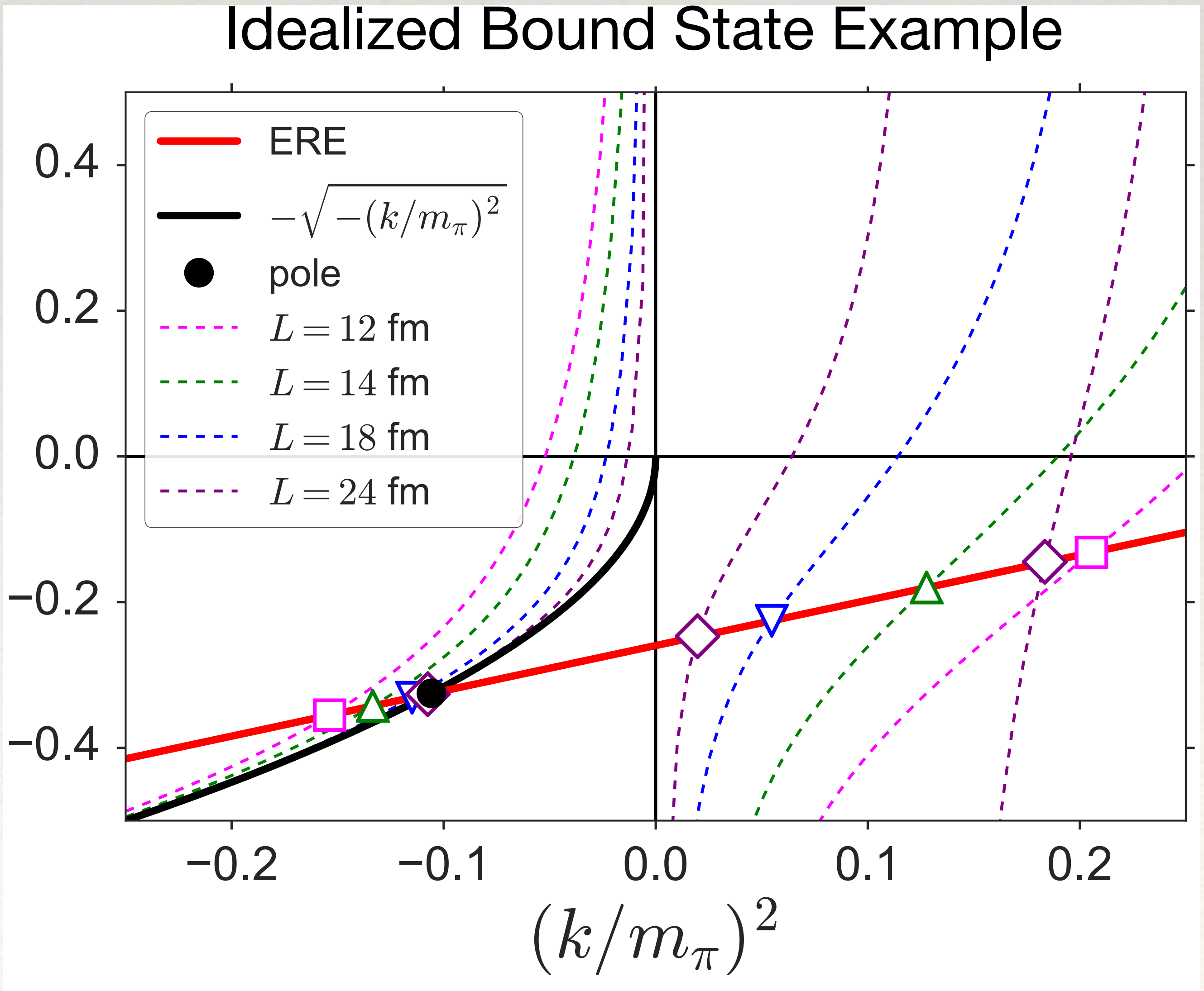
- A bound state solution requires $q\cot\delta(q)\Big|_{q=0} < 0$ and a slope < tangent to $-\sqrt{-q^2}$
- We find a virtual bound state (like dineutron) - a purely imaginary solution with negative sign

$$\frac{q_-^{\text{deuteron}}}{m_\pi} = -i\,0.132(32)$$

- We can infer the size of the potential from causality and unitarity: Wigner PRD 98 (1955), Phillips and Cohen PLB 390 (1997)

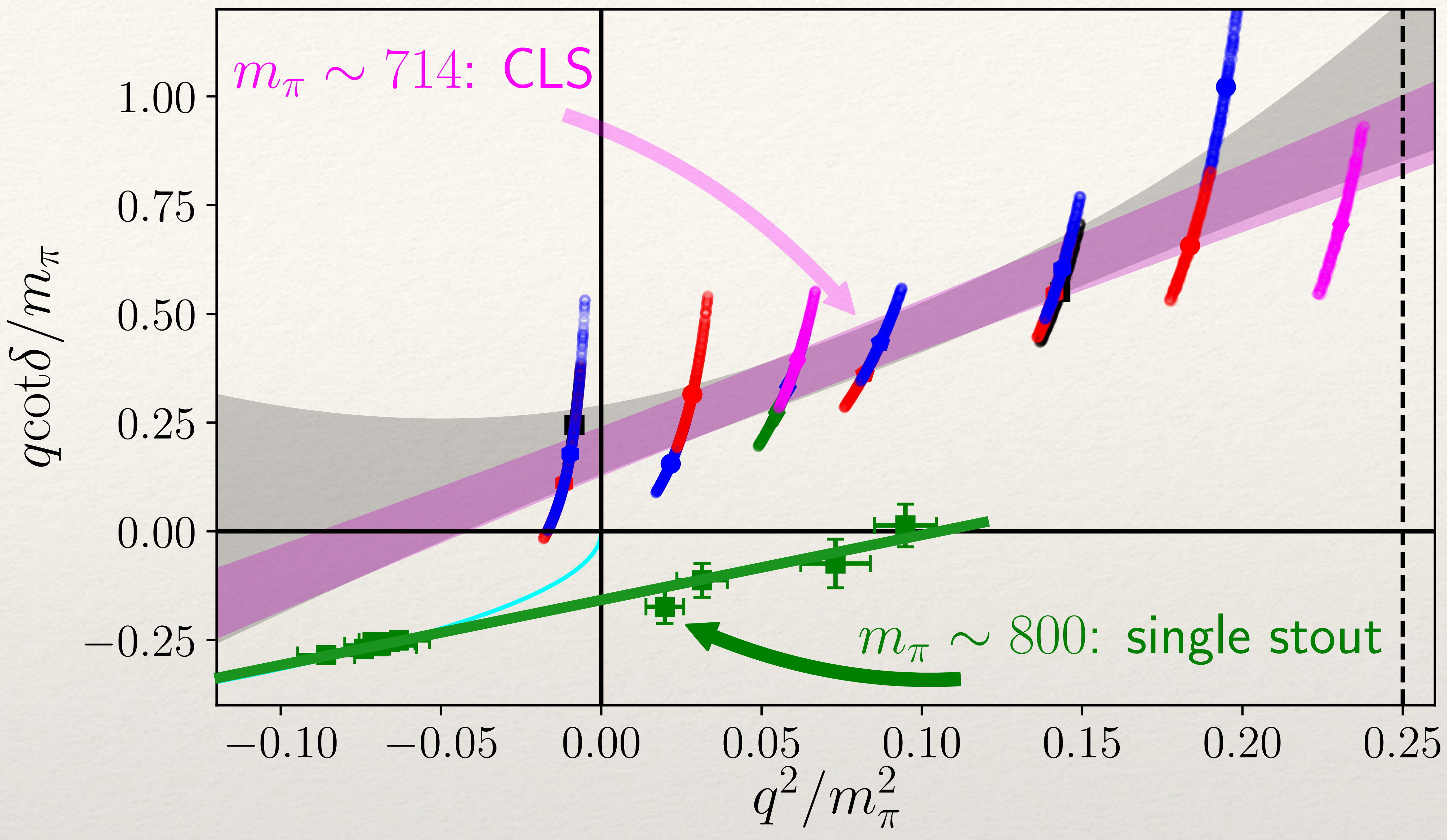
$$r_0 \leq 2\left[R - \frac{R^2}{a} + \frac{R^3}{3a^2}\right] \qquad m_\pi R \gtrsim 2.0, \quad R \gtrsim 0.55 \text{ fm}$$





Comparing with NPLQCD

- Our results are in clear contradiction to NPLQCD at a similar pion mass
  - NPLQCD results were generated with local hexaquark creation operators
    - could this local operator couple more strongly to a deep bound state?
    - If so - our entire spectrum would have to shift down (similarly, NPLQCD would have to shift up)
  - Could it be a discretization effect?
    - Community expectation that discretization effects will not qualitatively alter the nature of bound or not bound



- We are in the process of
  - Analyzing higher lying states with partial-wave mixing
  - Adding a hexaquark operator to our basis
  - Performing the NPLQCD, CalLat and HAL QCD calculations on the same gauge configurations - this will make all systematic uncertainties the same except for the method

Outlook

- Lattice QCD + Effective Field Theory will enable us to ground our understanding of nuclear physics in the Standard Model - provided we can bring the lattice QCD calculations under-control

- Such a quantitative rooting is important for
  - A number of high-impact, high-profile experiments aimed at testing the limits of the Standard Model in low-energy nuclear environments
  - Enabling us to constrain the contribution of n-n-n, YN, YNN contributions to the nuclear equation of state, at least in the low-density regime

- I have tried to present a more critical summary of lattice QCD results for NN interactions than typical
  - This self-introspection is important:
    - To overcome the present difficulties (develop and push new methods)
    - Resolve the bound-state discrepancy
    - Obtain results with Mpi $\lesssim$ 200 MeV (the existing strategies have failed to materialize anticipated results)
  - These lattice QCD calculations are extremely expensive (numerically) compared to other NP applications
    - We must ensure we are making a wise investment
    - So far, no meaningful (relevant to experiment) contact with NN EFT - we keep selling the promise but can we be quantitatively useful?

- I have presented an inconclusive new result related to the discrepancy
  - Suggestive the old results generated with hexaquark creation operators are problematic
  - We hope to have a complete set of results with all methods in the literature by the end of the year

- I don't want to leave you with a negative impression - I am optimistic about this endeavor - connecting our understanding of nuclear physics to the Standard Model - and I believe we are on the right track
  - with sLapHnn, I am involved in a calculation with Mpi~200 MeV that looks promising, for example
- In our review - we highlight some interesting connections that can be made through effective theories of nuclear physics
  - Connecting to E(F)Ts - Chiral EFT and HOBET,
  - neutrinoless double beta decay
  - Christian Drischler added a very nice discussion on the nuclear-matter equation of state

Towards grounding nuclear physics in QCD*

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Amy Nicholson,[4,**] Pavlos Vranas,[5,2,††] and André Walker-Loud[2,1,5,‡‡]

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