# Excercises to Pairing

## 1   Seniority model for two particles

Consider the seniority model with a single $j$-shell

$$\hat{H} = -GS_+S_- \quad \text{with} \quad S_+ = \sum_{m>0} c_m^\dagger c_{\tilde{m}}^\dagger, \quad \text{and} \quad S_- = S_+^\dagger \qquad (1)$$

for two particles and diagonalize the Hamiltonian in the subspace of pairs

$$\Psi = \sum_{m>0} X_m c_m^\dagger c_{\tilde{m}}^\dagger \qquad (2)$$

and determine the spectrum and the wave functions.

## 2   Decomposition of BCS-state

We have seen that the BCS-state

$$|\text{BCS}\rangle = \prod_{k>0}(u_k + v_k a_k^+ a_{\tilde{k}}^+)|-\rangle = \sum_N c_N|N\rangle, \qquad (3)$$

is a linear combination of normalized states with good particle number $|N\rangle$. The coefficients are given by

$$|c_N|^2 = \langle\text{BCS}|\hat{P}^N|\text{BCS}\rangle \qquad (4)$$

where

$$\hat{P}^N = \delta(\hat{N} - N) = \int \frac{d\varphi}{2\pi} e^{i(\hat{N}-N)} \qquad (5)$$

is the projector on good particle number.

    a) Give an analytical expression for $|c_N|^2$ for a single $j$-shell with a monopole pairing force.

    b) calculate $|c_N|^2$ numerically by using the expression

# 3 The Richardson model

Consider $M$ pairs of levels $(k, \tilde{k})$ $(k = 1, \ldots, M)$ with the Hamiltonian

$$\hat{H} = \hat{H}_0 - GS_+S_- \tag{6}$$

where

$$\hat{H}_0 = \sum_{k>0} \epsilon_k (c_k^\dagger c_k + c_{\tilde{k}}^\dagger c_{\tilde{k}}) \quad \text{and} \quad S_+ = \sum_{k>0} c_k^\dagger c_{\tilde{k}}^\dagger \tag{7}$$

where $\epsilon_k = k\epsilon$ Solve this model in the BCS-Approximation for $N$ particles and study the solution for various values of the parameters $\epsilon$ (level spacing) and $G$ (strength of the pairing force)