# Exercises to RPA

## 1 The Bolsterli Model

Consider two sets of single particle levels, $|n+\rangle$ and $|n-\rangle$ $(n = 1, \ldots, N)$ filled with $N$ particles with the Hamiltonian

$$\hat{H} = \frac{1}{2}\epsilon \sum_{n=1}^{N} (a^{\dagger}_{n+} a_{n+} - a^{\dagger}_{n-} a_{n-}) - \lambda \hat{D}^{\dagger} \hat{D}, \tag{1}$$

with

$$\hat{D} = \sum_{n}^{N} (a^{\dagger}_{n+} a_{n-} + a^{\dagger}_{n-} a_{n+}). \tag{2}$$

1) Determine the free response function $R^0_{DD}(\omega)$ for the operator $\hat{D}$

2) Derive the response equation for the full respone $R_{DD}(\omega)$ and determine the spectrum graphically.

3) Determine the energy of the collective state analytically

4) Calculate the transition density of the collective state as the residuum of the response function at the collective pole.

5) Determine the critical strength $\lambda_C$ where the RPA breaks down.

6) Calculate the RPA-matrix and solve it by diagonalization

## 2 Goldstone modes in the RPA spectrum

Consider a the generator $\hat{P}$ of a symmetry operator $\exp(i\alpha\hat{P})$, which commutes the Hamiltonian and assume that the HF-ground state breaks this symmetry show that the corresponding RPA-matrix

$$\hat{P}|\mathrm{HF}\rangle \neq 0. \tag{3}$$

1) Show that the RPA equation has a solution with zero energy, the Goldstone mode.

2) What is the energy of this mode?

# 3 Bosons and collective Fermion pairs

Consider a linear combination of $ph$-pairs $B^\dagger_{mi} = a^\dagger_m a_i$ and collective $ph$-pairs

$$O^+ = \sum_{mi} C_{mi} B^\dagger_{mi}$$

and determine the commutaion relations for the operators $B_{mi}, B^\dagger_{mi}$ and $O, O^\dagger$ in normal order with respect to the HF state for the TDA-approximation of the Bolsterli model.