PHY-820: CLASSICAL MECHANICS
FINAL EXAM
December 11, 2008

PROBLEM 1. /20/ a. Find the capture cross section for particles of mass m_1 moving with velocity v to the surface of the spherical body of radius R and mass M that attracts the particles by the gravitational force. Particles are considered captured if they fall on the surface of the big body.

b. Under what conditions is this cross section equal to πR^2?

PROBLEM 2. /20/ Determine the motion $x(t)$ of a particle of mass m with energy $E = 0$ in the potential

$$U(x) = -Ax^4, \quad A > 0. \tag{1}$$

Consider the initial condition $x(0) > 0$ and two cases, $\dot{x}(0) > 0$ and $\dot{x}(0) < 0$.

PROBLEM 3. /20/ An atom of mass m moving with velocity $\mathbf{v}$ undergoes an elastic head-on collision with one of the atoms of a diatomic molecule. The molecule consists of two atoms of mass $m/2$ each and can be modeled by a dumbbell with distance a between the atoms. Before the collision, the molecule is at rest being oriented perpendicular to $\mathbf{v}$ and not rotating. Consider the atoms as massive point-like objects. For the situation after the collision determine:

a. the velocity $\mathbf{v}'$ of the first atom,

b. translational velocity $\mathbf{u}$ of the molecule,

c. angular momentum L of the molecule, and

d. angular velocity Ω of rotation of the molecule.

PROBLEM 4. /15/ Two equal charges Q are fixed at the ends of a vertical line of length b. A charged bead of mass m and unknown charge q of the same sign as Q can slide along the line.

a. Write down the Lagrange function for the bead and derive the equation of motion (including the gravitational potential).

b. The position of equilibrium of the bead was measured to be at the height x_0 from the lower end of the line. Determine the charge q.

c. Find the frequency of small oscillations of the bead around the equilibrium.

d. Find the solution $x(t)$ for small oscillations started at rest at $t = 0$ with the initial velocity v_0 directed up the line.

PROBLEM 5. FAST QUESTIONS. /25/

1. Find the Poisson bracket $\{\ell_i; (\mathbf{r} \cdot \mathbf{p})\}$, where $\mathbf{r}$ and $\mathbf{p}$ are the radius-vector and the momentum of a particle, respectively, and ℓ_i is the ith component of the angular momentum.

2. A charged particle is moving in the static field of two fixed point-like charges. List the constants of motion.

3. A particle is freely moving inside a sphere of radius R being elastically reflected from the surface of the sphere. How does the energy of the particle change if the sphere is slowly expanding?

4. Two particles with masses m and m' are moving along identical trajectories in the same potential field. What is the relation between the times of motion of the particles between given points?

5. Enumerate normal modes of small oscillations of a linear molecule CO_2; in the equilibrium, distances OC and CO are equal.