Name: _____ Student No.: _____

Qualifying/Placement Exam
9:00 am, January 11, 2005

Put your NAME on every sheet of this 12 problem Exam -- NOW

You have 3 hours to complete the 12 problems on this exam. Show your work! Full credit will not be given for answers without justification. Some partial credit may be earned for the correct procedure, even if the correct answer is not achieved. Answers must be in the spaces provided. The **BACK** of the problem page may be used for lengthy calculations. Do not use the back of the previous page for this purpose!

You may need the following constants:

Speed of light in vacuum: $c = 3.00\times10^{8}$ m/s

Boltzmann constant: $k = 1.38\times10^{-23}$ J/K

Planck's constant: $h = 6.63\times10^{-34}$ J s

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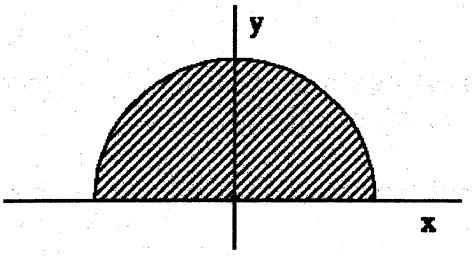
1. [10 pts] Find the position of the center of mass of a thin solid half-disk of radius R, which has a uniform mass distribution.



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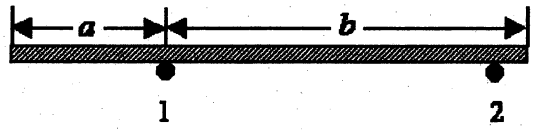
2. A uniform rod with mass-per-unit-length ρ is resting horizontally on two pencils as shown. Pencil 2 is suddenly removed.

   a. [4 pts] Find the moment of inertia of the rod for rotation about point 1. (Ignore the thickness of the rod in this calculation.)

   b. [6 pts] Find the angular acceleration of the rod immediately after pencil 2 is removed.



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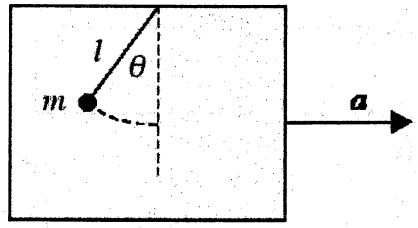
3. A simple pendulum of mass m and length l is suspended from the roof of a box. The box moves horizontally with a constant acceleration a.

   a. [5 pts] What is the angle that the pendulum makes with respect to the vertical direction, when it is in equilibrium?

   b. [5 pts] What is the period of small oscillations of the pendulum about this equilibrium angle?



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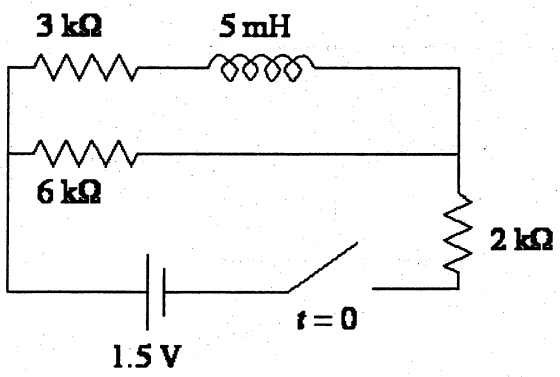
4. The switch in the circuit shown in the figure has been open a long time. It is closed at $t = 0$.

   a. [5 pts] What is the current in the battery just after the switch is closed?

   b. [5 pts] What is the current in the battery a long time after the switch is closed?



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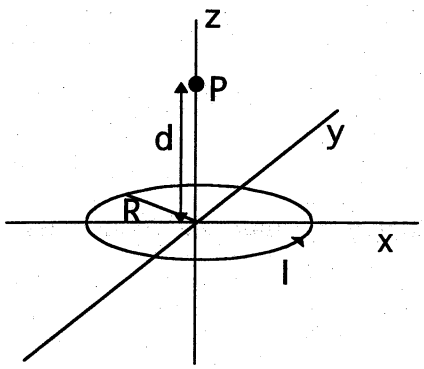
5. [10 pts] Calculate the magnetic field at point P which is a distance d along the z axis from a circular loop carrying current I. This loop has a radius R, lies in the x-y plane and is centered at the origin as shown below.



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6. An electromagnetic plane wave is passing through the vacuum. The components of the electric field are

$$E_x = p\cos(q\,y + r\,t)$$
$$E_y = 0$$
$$E_z = s\cos(q\,y + r\,t)$$

where p, q, r, s are constants.

   a. [3 pts] Find the condition that the constants p, q, r, s must satisfy to make the speed correct.

   b. [7 pts] Find the magnetic field.

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7. A beam of positively charged pions is created with a velocity of .99c at a production target at Fermilab. The mean lifetime of a pion is 26 ns. If the pions survive without decaying, they will be detected in a detector situated at a distance of 100 m from the production target.

   a. [3 pts] What is the momentum (in the units MeV/c) for these pions? ($m_\pi c^2$=140 MeV)

   b. [3 pts] In the rest frame of the pion, what is the distance between the production target and the detector?

   c. [4 pts] What fraction of these pions survive and reach the detector without decaying?

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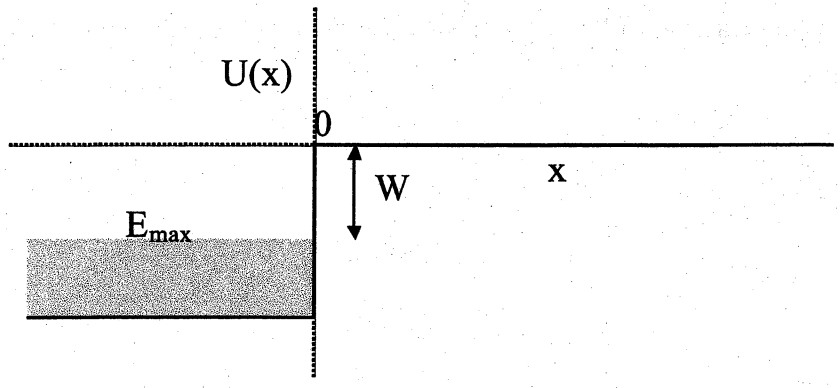
8. Assume that the surface of a metal can be modeled as a potential step as seen by the electrons in the metal. The height W of the potential barrier above the energy of the most energetic electron in the metal is known as the "work function".

   a. [6 pts] Provide an expression for the wave function outside the surface of the metal for the highest energy electron.

   b. [4 pts] Provide an expression for the spatial density (number of electrons per unit length) $n_e(x)$ of these highest energy electrons for $x>0$, in terms of x and the value for this density $n_e(0)$ just outside the metal at $x=0$.



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9. The energy eigenstates of the hydrogen atom are described by Schrodinger wave functions $\Psi_{nlm}(\mathbf{r})$, where the energy of the ground state $\Psi_{100}(\mathbf{r})$ is $E_1 = -13.6$ eV. (You do not need to remember the form of these functions to work this problem.)

   Suppose a hydrogen atom is in the state $0.8\,\Psi_{100} + 0.6i\,\Psi_{211}$.

   a. [5 pts] Find the expectation value of the energy.

   b. [5 pts] Find the expectation value of $\mathbf{L}^2$ (the square of the angular momentum vector).

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10. Assume that a 100 Watt light bulb emits light equally in all directions, and that the emitted photons have an average wavelength of 550 nm. How many photons/sec are being emitted by the light bulb?

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11. 10 g of water at a temperature of 10° C are mixed with 30 g of water at a temperature of 90° C. (Useful information: The specific heat capacity for water is 4.19 J/(g·K).)

    a. [4 pts] What is the final temperature of the system, consisting of 40 g of water, after thermal equilibrium is established?

    b. [6 pts] What is the change in total entropy of the combined system, consisting of 40 g of water, caused by mixing the two volumes of water together?

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12. The magnetic moment of a nucleus is of the order of 10^{-26} A m^2. Estimate the magnetic field required at 0.01 K to produce appreciable (90%) alignment of such nuclei. Assume that the nucleus is spin-1/2, so that each nucleus has its magnetic moment either aligned or anti-aligned with the magnetic field.