Qualifying/Placement Exam
6:00 pm, January 10, 2006

Put your NAME on every sheet of this
12 problem Exam -- NOW

You have 3 hours to complete the 12 problems on this exam. Show your work! Full credit will not be given for answers without justification. Some partial credit may be earned for the correct procedure, even if the correct answer is not achieved. Answers must be in the spaces provided. The **BACK** of the problem page may be used for lengthy calculations. Do not use the back of the previous page for this purpose!

You may need the following constants:

Speed of light in vacuum: $c = 3.00 \times 10^8$ m/s

Boltzmann constant: $k = 1.38 \times 10^{-23}$ J/K

Planck's constant: $h = 6.63 \times 10^{-34}$ J s $= 4.14 \times 10^{-15}$ eV s

$hc = 1240$ eV nm

Gas constant: $R = 8.31$ J/(mol K)

Permittivity of free space $\varepsilon_0 = 8.99 \times 10^9$ $C^2 / (N\ m^2)$

Permeability of free space $\mu_0 = 4\pi \times 10^{-7}$ N s^2 / C^2

Mass of electron $m_e = 511$ keV/c^2

Useful formulae:

$$\cos\frac{\alpha}{2} = \sqrt{\frac{1+\cos\alpha}{2}}$$

$$\sin\alpha\sin\beta = \frac{1}{2}\left[\cos(\alpha-\beta) - \cos(\alpha+\beta)\right]$$

$$\cos\alpha\cos\beta = \frac{1}{2}\left[\cos(\alpha-\beta) + \cos(\alpha+\beta)\right]$$

1. A container, partially filled with water, slides down a ramp inclined at an angle α relative to the horizontal. The coefficient of friction between the container and the ramp is μ. After some initial transient motion, the surface of the water in the container eventually reaches equilibrium, and is found to maintain a constant angle θ relative to the horizontal.
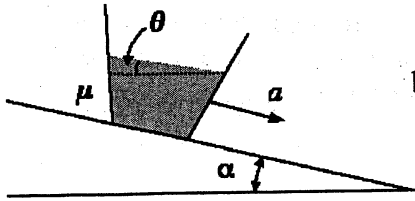
a. [5 pts] What is the acceleration a of the container, in terms of μ, α, and g?

b. [5 pts] Now assume $\mu = 0$. What is the inclination angle θ of the water surface relative to the horizontal? Give a concise but complete explanation of how you arrived at this answer.

2. A bowling ball (mass $M = 20$ kg and radius $R = 0.1$ m) is released on the surface of the lane with initial velocity $v_0 = 10$ m/s and no initial angular momentum. The ball has moment of inertia of $I = 2MR^2/5$ and the coefficient of friction between the ball and the lane is $\mu = 0.2$.

    a. [6 pts] What is the final velocity of the ball after it begins to roll without slipping?

   b. [4 pts] How far does the ball travel down the lane before it starts to roll without slipping?

3. [10 pts] A rocket starts from rest in free space by emitting mass at a constant rate α and with an ejection velocity u, relative to the rocket. At what fraction of the original rocket mass is the kinetic energy of the rocket a maximum? (Assume the mass of the empty rocket to be negligible.)

4. A long straight conducting wire has been bent around an insulated cylinder of radius
    R, within the plane perpendicular to the cylinder axis, as shown below. The ends of
    the wire make an angle ϕ relative to each other.
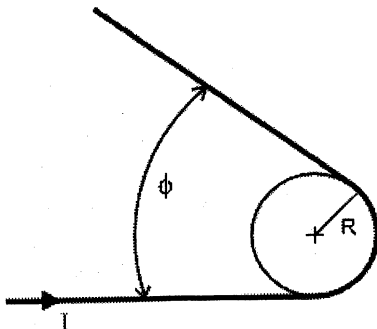
a. [2 pts] What is the direction of the magnetic field in
    the wire plane, at the center of the cylinder, if a
    current I is passed through the wire?

b. [8 pts] Find the value of the magnetic field at the
    point discussed above.

5. [10 pts] If the electric field vector is inversely proportional to the radial distance to the center of a spherical charge distribution (i.e. $\bar{E}(\bar{r}) = -\dfrac{\alpha}{r}\hat{e}_r$), find the function describing the density $\rho = \rho(r)$ within the sphere.

6. [10 pts] The circuit shown below consists of two capacitors of capacitances C_1 and C_2, a resistor R, and a switch S, connected in series. With the switch open, batteries are used to charge the two capacitors to the voltages V_1^0 and V_2^0 respectively. At time $t=0$, the switch S is closed. Find the current $I(t)$ across the resistor as a function of time.
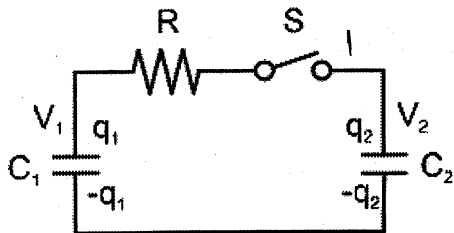
7. A parallel beam of mono-energetic electrons with De Broglie wavelength λ strikes a thin sheet of metal with two parallel slits in it. The distance between the centers of the slits is d. You can assume the two slits to be of the same size and extremely narrow.

a. [5 pts] Show that the intensity of electrons observed at a small angle θ is given approximately by
$$I = 4I_0\cos^2(\pi\theta d / \lambda)$$
where I_0 is the intensity observed at $\theta = 0$ with one slit blocked off.

b. [5 pts] If the electrons have kinetic energy of 5 keV, what value of d is necessary for the first diffraction minimum to occur at $\theta = 1\times10^{-5}$ radians?

8. Consider the atom $_{13}$Al in its ground state. (Note: you may find the periodic table below useful.)

a. [4 pts] What is the electron configuration for this atomic state?

b. [6 pts] What is the square of the total orbital angular momentum $\vec{L}^2$, the orbital angular momentum quantum number ℓ, the square of the total spin angular momentum $\vec{S}^2$ and total spin quantum number s for this atomic state?

9. An electron confined to a simple-harmonic oscillator potential is initially in the state $\psi(0) = 0.6\psi_0 + 0.8\psi_1$, where ψ_0 and ψ_1 are the two lowest energy eigenstate solutions of the stationary Schroedinger equation, normalized to 1, with energies $\hbar\omega/2$ and $3\hbar\omega/2$, respectively.

    a. [3 pts] What is the form of the wave function $\psi(t)$ at a later time t ?

    b. [4 pts] What is the expectation value of the energy as a function of t ?

    c. [3 pts] What is the expectation value of the position as a function of t, given that $\langle\psi_0|x|\psi_0\rangle = \langle\psi_1|x|\psi_1\rangle = 0$ and $\langle\psi_0|x|\psi_1\rangle = \sqrt{\hbar/(2m\omega)}$?

10. [10 pts] A charged pion decays into a muon and a neutrino, $\pi^+ \rightarrow \mu^+ + \nu_\mu$. If the final muon is at rest, what was the original momentum of the pion? Masses of the pion and muon are 140 MeV/c^2 and 105 MeV/c^2, respectively. Assume that the neutrino is massless, use relativistic kinematics, and give the result in MeV/c.

11. [10 pts] Consider a microscopic system that contains two non-degenerate energy levels. The system has ground state energy 0 and upper state energy ε. Find the heat capacity $C = d\langle E \rangle / dT$ of N such non-interacting identical systems in thermal equilibrium at a temperature T.

12. A cylinder contains n moles of monatomic Ar gas capped on the top by a piston, as shown in the drawing below.

a. [4 pts] The bottom of the cylinder is placed in contact with a reservoir at temperature T_h and comes to equilibrium with it. Derive an expression for **the work done by the system** and **the entropy change of the system** as the system undergoes an isothermal expansion from an initial volume V_1 to a final volume V_2. Express the answers in terms of given quantities such as n, the temperature T_h and the initial and final volumes.

Isothermal expansion

Energy reservoir at T_h
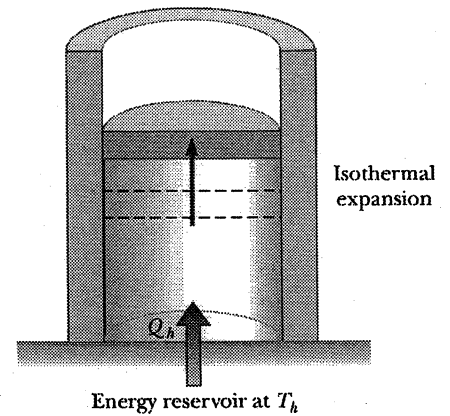
b. [6 pts] Suppose instead that the system is removed from contact with the reservoir or any other external heat source or sink. The system expands reversibly and adiabatically from V_1 to V_2. Derive expressions for **the pressure P_2**, the **entropy change $\Delta S = S_2 - S_1$** and the **change in internal energy of the system** due to the expansion. Express the answers in terms of given quantities such as n, the temperature T_h, and the initial and final volumes.