Qualifying Exam
Department of Physics and Astronomy
Michigan State University
January 8, 2008

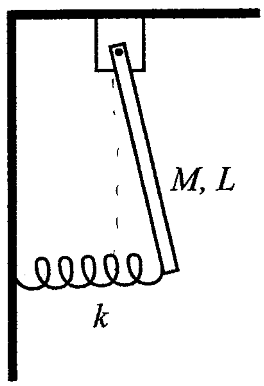
Put your NAME on every sheet of this 12 problem Exam – NOW

You have 3 hours to complete the 12 problems on this exam. Show all your work! Unsupported answers are not likely to earn the full credit and may even earn no credit at all. A partial credit may be earned for correct procedures, even if the correct answer is not reached. Answers must be in the spaces provided. The BACK of the problem sheet may be used for lengthy calculations. Do not use the back of the preceding sheet for this purpose!

You may need the following constants:

Speed of light in vacuum:	$c = 3.00 \times 10^8\,\mathrm{m/s}$
Boltzmann constant:	$k = 1.38 \times 10^{-23}\,\mathrm{J/K}$
Planck's constant:	$h = 6.63 \times 10^{-34}\,\mathrm{J\,s} = 4.14 \times 10^{-15}\,\mathrm{eV\,s}$
	$h\,c = 1240\,\mathrm{eV\,nm}$ $\quad \hbar\,c = 197\,\mathrm{MeV\,fm} = 197\,\mathrm{eV\,nm}$
Gas constant:	$R = 8.31\,\mathrm{J/(K\,mol)}$
Permittivity of free space:	$\epsilon_0 = 8.99 \times 10^9\,\mathrm{C^2/(N\,m^2)}$
Permeability of free space:	$\mu_0 = 4\pi 10^{-7}\mathrm{N\,s^2/C^2}$
Atomic mass unit:	$1\,\mathrm{u} = 1.661 \times 10^{-27}\,\mathrm{kg} = 931.5\,\mathrm{MeV/c^2}$
Electron mass:	$m_e = 9.11 \times 10^{-31}\,\mathrm{kg} = 511.0\,\mathrm{keV/c^2}$
Elementary charge:	$e = 1.602 \times 10^{-19}\,\mathrm{C}$

1. [10 pts] A uniform rod of mass M and length L is pivoted at one end, as shown in the figure. The pivot allows the rod to swing in a vertical plane, coinciding with the plane of the figure. The other end of the rod is attached to a horizontal spring of spring constant k. When the rod is hanging vertically, the spring is at equilibrium (neither stretched nor compressed). What is the oscillation period of the rod for small oscillations? *Note:* The moment of inertia of the rod about its *center of mass* is $I_{\text{cm}} = \frac{1}{12}ML^2$.



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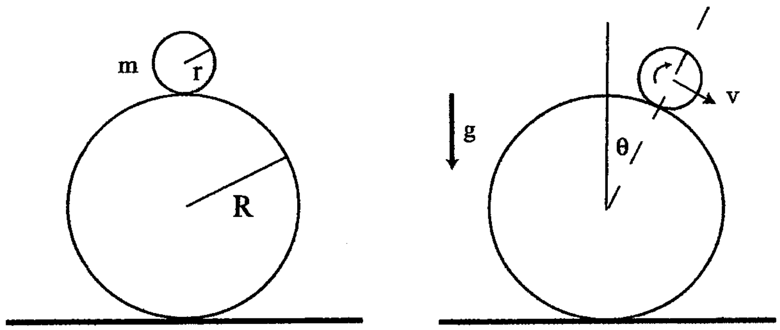
2. A small ball of radius r and mass m starts rolling down from rest at the top of a large fixed cylinder of radius R, see the figure. The ball rolls without slipping and its moment of inertia about its center of mass is $I = 2m\,r^2/5$.

   (a) [3 pts] Obtain a condition in terms of the angle θ and ball speed v for the ball detaching from the cylinder.

   (b) [7 pts] Exploit a conservation law to find the angle θ at which the ball detaches from the cylinder.



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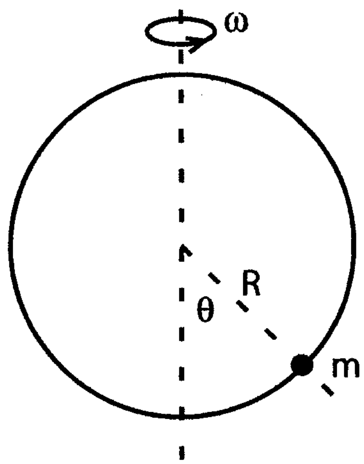
3. A bead of mass m can slide without friction on a vertical hoop of radius R. The hoop is rotating at constant angular speed ω about a vertical axis passing through the hoop's center.

   (a) [5 pts] Determine the Lagrangian for the bead, using the indicated angle θ as the generalized coordinate.

   (b) [5 pts] Obtain an equation of motion for the bead in terms of the angle θ.



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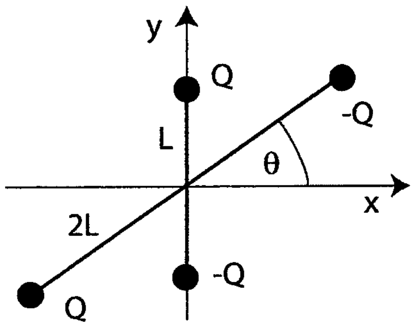
4. [10 pts] Four charges are arranged in the xy plane as follows: $+Q$ at $(0, L)$, $-Q$ at $(0, -L)$, $+Q$ at $(-2L\cos\theta, -2L\sin\theta)$, and $-Q$ at $(2L\cos\theta, 2L\sin\theta)$, where the angle θ is shown in the figure. Calculate the components of dipole moment vector $\vec{d}$. Obtain the magnitude of the moment d and plot it as a function of the angle θ.



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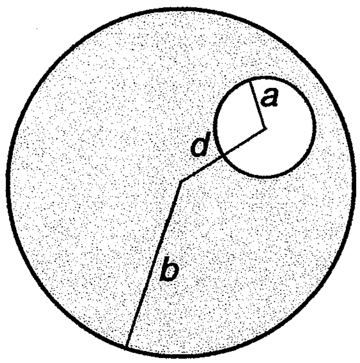
5. [10 pts] A hole of radius a is bored parallel to the axis of a solid conducting cylinder of radius b ($b > a$). The two axes are at a distance d apart. A current of I amperes flows *along* the cylinder. What is the magnitude of magnetic field at the center of the bored hole? Hint: Use the principle of superposition.



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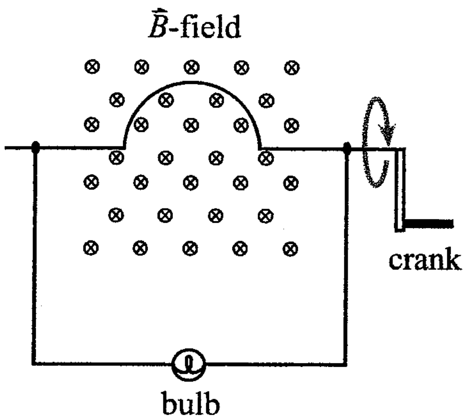
6. You are trying to build a hand-cranked generator, using a powerful horseshoe magnet and a circuit that contains a semicircular portion that can be rotated through the magnetic field, as shown in the figure. The semicircle has a radius of 5.0 cm, and the magnet produces a field of 0.20 T, which is constant throughout the full range of the rotating portion of the circuit. You want to use the generator to light a small 0.5-Ω lightbulb.

   (a) [5 pts] Find an expression for the induced current as a function of time, if you turn the crank at frequency f. Assume that the semicircle is at its highest point when $t = 0$.

   (b) [5 pts] At what frequency must you turn the crank to get a peak power of 3.0 W?



7. The energy eigenstates of a hydrogen atom are described by the wavefunctions $\Psi_{n\ell m_\ell}(\vec{r})$. Here, n, ℓ and m_ℓ are, respectively, the main quantum number of hydrogen atom, the quantum number of the square of orbital angular momentum L^2 and of the projection of orbital momentum onto the z-axis, L_z. The energy of the ground state, described by the wavefunction $\Psi_{100}(\vec{r})$, is $E_0 = -13.6\,\text{eV}$. Suppose a hydrogen atom is, at a given instant, described by the wavefunction:

$$\Psi = 0.6\,\Psi_{100} + 0.8\,i\,\Psi_{211}\,.$$

(a) [5 pts] Find the expectation value of energy for the above state.

(b) [5 pts] Determine the probability that a measurement of the z-component of orbital angular momentum would produce the value of 0.

(Over)

8. [10 pts] An ω^0 meson can decay into a neutral pion and a photon: $\omega^0 \to \pi^0 + \gamma$. The rest energy of ω^0 is $m_{\omega^0} c^2 = 780\,\text{MeV}$ and of π^0 is $m_{\pi^0} c^2 = 135\,\text{MeV}$. The photon is massless. For an ω^0 decaying while at rest, calculate the energy of the photon. Note: It is necessary to use relativistic kinematics.

(Over)

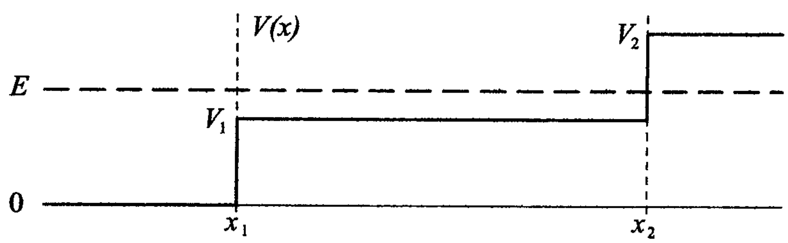
9. A free electron, moving in one dimension, comes in from the left and scatters off a series of step-function potentials, as shown in the figure below. The potential is

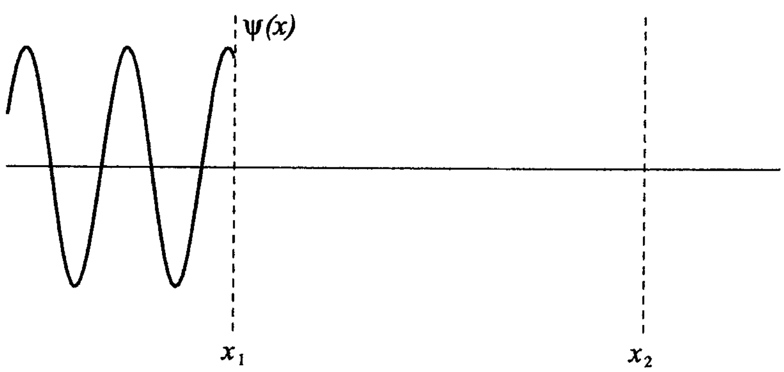
$$V(x) = \begin{cases} 0 & \text{for } x < x_1, \\ V_1 & \text{for } x_1 < x < x_2, \\ V_2 & \text{for } x_2 < x, \end{cases}$$

and the electron energy E satisfies $V_1 < E < V_2$.

(a) [5 pts] A sketch of the real wavefunction $\psi(x)$, describing the electron, is drawn for $x < x_1$. Continue this sketch to the region of $x > x_1$, being sure to account for all special features of the wavefunction as it crosses the potential steps. Augment your sketch with a few words identifying the special wavefunction features.

(b) [5 pts] Let $V_2 - E = 3.0\,\text{eV}$. By what factor will the probability distribution fall, over the distance from x_2 to x, if $x - x_2 = 0.10\,\text{nm}$?





10. Neutron mass is 939.57 MeV/c^2 and proton mass is 938.27 MeV/c^2. In free space, neutron is unstable and decays with a lifetime of 920 s.

(a) [3 pts] What is the dominant mode of decay of neutrons in free space? What interaction is responsible for that decay?

(b) [7 pts] The following decay modes of neutrons are not observed:

$n \to p + \gamma$

$n \to p + \pi^-$

$n \to p + e^-$

$n \to \pi^+ + e^-$

$n \to \gamma + \gamma$

For each of the unobserved decays list the conservation laws that forbid that decay. *Clearly* indicate what laws pertain to what decay.

(Over)

11. According to Planck's original theory, the energy of an quantum oscillator can be represented by

$$E = nh\nu\,,$$

where n is the number of quanta of energy $h\nu$ each, for the frequency ν.

(a) [5 pts] Assuming that the probability distribution for occupation of the oscillators is governed by the Boltzmann's law, compute the mean energy of an oscillator for a given absolute temperature T.

(b) [5 pts] Demonstrate that for sufficiently high temperature T the obtained mean energy reduces to one expected classically. Estimate how high the temperature needs to be for the reduction to the classical result.

Potentially useful formula: $\displaystyle\sum_{n=1}^{\infty} n\, x^n = \frac{x}{(1-x)^2}\,.$

(Over)

12. Two identical bodies, characterized each by a constant heat capacity C_p and at initial temperatures, respectively, of T_1 and T_2 ($T_1 < T_2$), are used as reservoirs for a heat engine. During the exchange of heat with the engine, the reservoir bodies are maintained at constant pressure and, except for the contact with engine, may be considered thermally isolated from the environment.

(a) [5 pts] Find the total amount of work W provided by the engine when the reservoir bodies achieve the same final temperature T_f (the bodies undergo no change of phase).

(b) [5 pts] Show that, to make the engine's operation the most efficient, the temperature T_f must be set to a value of $T_f = \sqrt{T_1 T_2}$.