Qualifying/Placement Exam
August 22, 2000

NAME: ______________________ STUDENT # ______________________

PUT YOUR NAME ON EVERY SHEET OF THIS 12 PROBLEM EXAM -- **NOW** --.

You have 3 hours to complete the 12 problems on this exam. Show your work! Full credit will not be given for answers without justification. Some partial credit may be earned for the correct procedure, even if the correct answer is not achieved. While waiting to begin, please enter your name and student number on the lines above.

Useful constants and conversion factors

Speed of light:	$c = 2.99792 \times 10^{8}$ m/s
Rydberg energy:	$E = 13.6$ eV.
Bohr radius:	$a_B = 5.29 \times 10^{-11}$ m.
Planck's constant:	$h = 6.63 \times 10^{-34}$ J·s $= 4.14 \times 10^{-15}$ eV·s.
English to SI units:	1 lb. = 4.45 N 1 ft. = .305 m

1 MeV = 1.6×10^{-13} J

Important masses and rest energies:

Electrons: $m_e = 9.1094 \times 10^{-31}$ kg. $m_e c^2 = 0.511$ MeV.

Pions: $m_\pi = 273\, m_e$.

Useful combinations:

$\hbar = 1.05 \times 10^{-34}$ J·s $= 6.58 \times 10^{-16}$ eV·s.

hc = 1240 eV·nm = 1240 MeV·fm.

$\hbar c = 197$ eV·nm $= 197$ MeV·fm.

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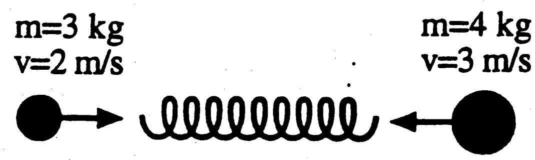
1. Two particles with masses 3 kg and 4 kg are traveling toward each other with speeds of 2 m/s and 3 m/s, respectively. Between them is a relaxed massless spring with spring constant 50 N/m. When the two masses hit the spring they compress it so that their relative velocity is zero when the spring has its maximum compression, which is equal to x.

(a) What is the velocity of the masses at the maximum spring compression?

(b) What is the maximum distance x by which the spring is compressed?



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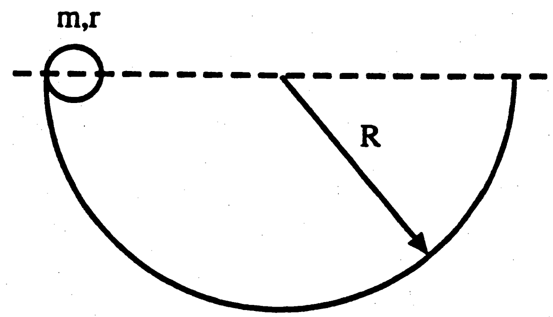
2. A sphere of radius r, mass m, and moment of inertia $I = 2mr^2/5$ is released from rest from the top of a semicircular track of radius R. It rolls down the track without slipping. Obtain an expression for the NORMAL force which the track exerts on the sphere when it is passing the bottom of the track.



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3. (a) *(4 points)* What test will determine if an external force acts on a system of two particles?

(b) *(3 points)* Test the following one-dimensional system for the existence of an external force: particles of mass m_1 and m_2 are observed to follow the paths

$$
\begin{aligned}
x_1(t) &= A_1 \sin \omega t + L + v_1 t\ , \\
x_2(t) &= A_2 \sin \omega t + v_2 t
\end{aligned}
$$

respectively, where constants A_1, A_2, v_1, v_2, L, are arbitrary except that $m_1 A_1 = -m_2 A_2$.

(c) *(3 points)* Test this system again if the paths of the two particles are

$$
\begin{aligned}
x_1(t) &= A_1 \sin \omega t \\
x_2(t) &= A_2 \sin(\omega t + \phi),
\end{aligned}
$$

where A_1 and A_2 are related as before and $\phi \neq 0$.

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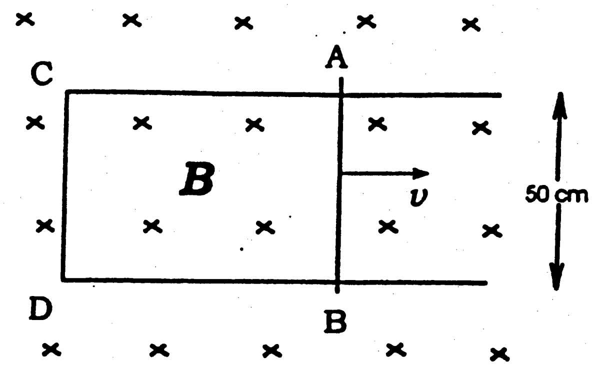
4. In the figure, a conducting rod AB is in contact with metal tracks CA and DB, which are immersed in a uniform magnetic field of $B = 0.5\,\mathrm{T}$ in the direction perpendicular ~~to~~ into the paper. (ACDB is a continuous conductor.)

(a) *(4 points)* If the rod moves toward the right with a speed of 4.0 m/s, find the magnitude and direction of the induced emf.

(b) *(3 points)* If the circuit has a resistance of 0.20 Ω when the rod is at a certain position, find the force exerted on the rod by the magnetic field.

(c) *(3 points)* Compare the power of the external force and the power lost in Joule heating. Ignore friction.



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5. The electric field in a linearly polarized electromagnetic plane wave is $\vec{E}(\vec{x},t) = E_0\hat{\jmath}\cos(kx - \omega t)$ where $\hat{\imath}$, $\hat{\jmath}$, $\hat{k}$ denote unit vectors along the x, y, z axes.

(a) (5 points) Determine the magnetic field.

(b) (5 points) Determine the Poynting vector.

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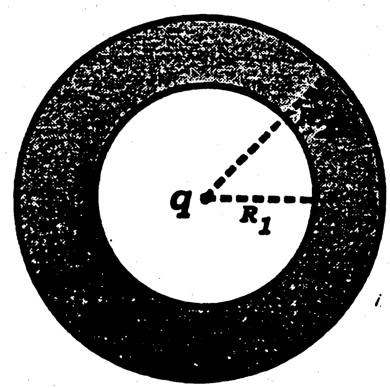
6. In the figure, a point charge $q = 4 \times 10^{-10}\,\text{C}$ is placed at the center of a conducting spherical shell which has inner and outer radii of $R_1 = 2\,\text{cm}$ and $R_2 = 3\,\text{cm}$ respectively. (charge ∅)

(a) *(5 points)* Find the electric field $\vec{E}$ as a function of r, the radial distance from the center.

(b) *(4 points)* Find the electric potential of the conducting sphere.

(c) *(1 point)* If the point charge is moved 1 cm from the center, then what is the electric potential of the conducting sphere?



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9. Write the two forms of the Heisenberg uncertainty principle involving momentum, energy, time and position, of a particle moving in the x – direction. Show that for a photon, the two forms are equivalent.

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10. 20 g of water at a temperature of 15° C are mixed with 25 g of water at a temperature of 95° C (Useful information: the specific heat capacity of water is 4.19 J/(g·K).).

a) (4 pts.) What is the final temperature of the system consisting of 45 g of water, after thermal equilibrium is established?

b) (6 pts.) What is the change in the entropy of the combined system, consisting of 45 g of water, caused by mixing the two volumes together?

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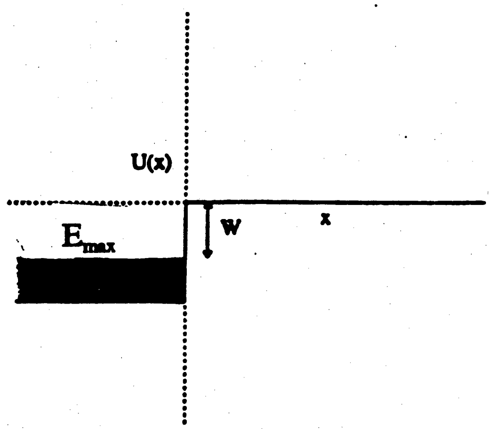
11. Assume that the surface of a metal can be modeled as a potential step as seen by the electrons in the metal. The height W of the potential barrier above the energy of the most energetic electron in the metal is known as the "work function". Let it have a value of W = 5 eV.



a) (6 pts.) What is the form of the wave function outside the surface of the metal for the highest energy electron?

b) (4 pts.) At what distance outside the surface is the probability distribution 0.1% of what it is at the surface?

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12. In the figure, a thermally conducting container with volume V_0 contains ideal gas of pressure p_0. If the gas is slowly compressed in thermal equilibrium with the environment to a volume $V_0/2$ by a piston, and the temperature of the surroundings remains constant in the process, then calculate: (a) the work done by the external force; (b) the heat exchange with the surroundings; (c) the change in entropy of the gas.