Qualifying/Placement Exam
9:00 am, August 20, 2002

Put your NAME on every sheet of this 12 problem Exam -- NOW

You have 3 hours to complete the 12 problems on this exam. Show your work! Full credit will not be given for answers without justification. Some partial credit may be earned for the correct procedure, even if the correct answer is not achieved. Answers must be in the spaces provided. The **BACK** of the problem page may be used for lengthy calculations. Do not use the back of the previous page for this purpose!

You may need the following constants:

$$k = 1.38 \times 10^{-23}\ \text{J/K}$$

$$h = 6.63 \times 10^{-34}\ \text{J}\cdot\text{s}$$

$$\hbar = 1.05 \times 10^{-34}\ \text{J}\cdot\text{s}$$

$$= 6.58 \times 10^{-22}\ \text{MeV}\cdot\text{s}$$

$$c = 3.0 \times 10^{8}\ \text{m/s}$$

$$\hbar c = 197\ \text{MeV}\cdot\text{fm}$$

$$e = 1.6 \times 10^{-19}\ \text{C}$$

$$\frac{1}{4\pi\varepsilon_0} = 9.0 \times 10^{9}\ \text{kg}\cdot\text{m}^3/\text{s}^2\text{C}^2$$

$$\mu_0 = 4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}$$

$\sigma = 5.7 \times 10^{-8}\ \text{W/m}^2\text{K}^4$ (Stephan-Boltzmann constant).

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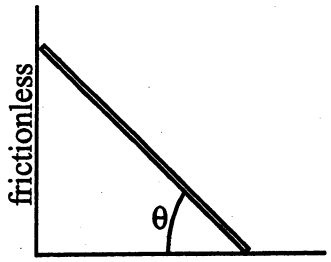
1. [10 pts] Leaning against a frictionless vertical wall is a uniform ladder making an angle θ, with the horizontal floor, as shown in the figure. The coefficient of static friction between the ladder and the floor is μ. In terms of μ, find the minimum angle, θ_m, for which the ladder will not slip.



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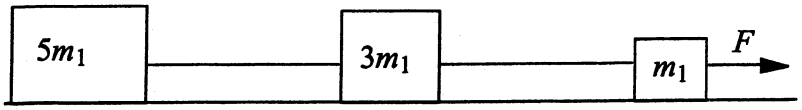

2. A force F accelerates three masses tied together with strings along a frictionless surface as shown in the figure above.

a) [6 pts] What is the net force acting on the middle mass?

b) [2 pts, each] What is the tension in each (specify) of the two strings?

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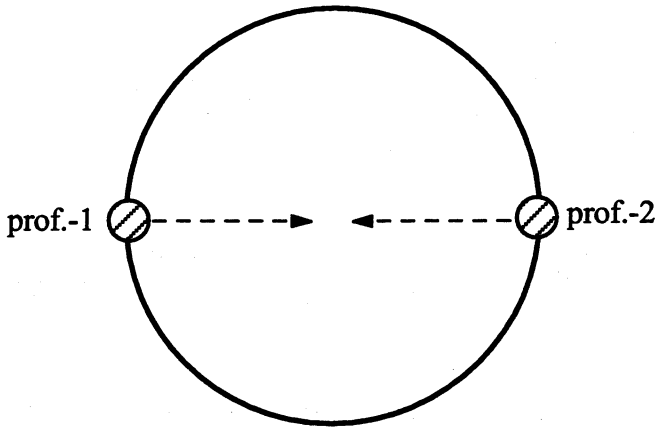
3. [10 pts] Two 100 kg (but thin) professors stand at opposite edges of a merry-go-round, which can be considered a solid cylinder of mass 300 kg, and radius 3 m. Initially, the merry-go-round makes 1 revolution every 2 seconds.



The two professors walk toward the center at the same rate and reach it at the same time. What is the angular velocity, ω, of the merry-go round when they reach the center. (Hint: the moment of inertia of a solid cylinder about its axis is $MR^2/2$.)

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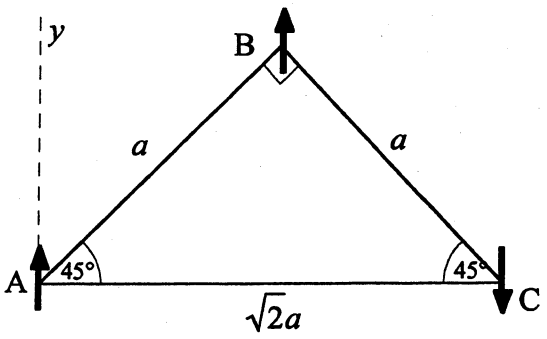
4. [10 pts] Consider the 45° - 90° - 45° triangle with sides $a, a, a\sqrt{2}$. At each vertex there is an electric dipole of magnitude p, with

$$\vec{p}_A = p\,\hat{j}, \quad \vec{p}_B = p\,\hat{j}, \quad \vec{p}_C = -p\,\hat{j},$$

as shown in the diagram.



a) What is the electric field $\vec{E}$ at point B, due to the dipoles at A and C?

b) How much work does it take to reverse the direction of $\vec{p}_B$, if $\vec{p}_A$ and $\vec{p}_C$ are held fixed?

c) How much work does it take to turn $\vec{p}_B$ clockwise through an angle of 90°, if $\vec{p}_A$ and $\vec{p}_C$ are held fixed?

The electric field of a point dipole is $\vec{E}(\vec{r}) = \dfrac{1}{4\pi\varepsilon_0 r^3}\left[3(\vec{p}\cdot\hat{r})\hat{r} - \vec{p}\right]$, in which $\hat{r}$ is the unit vector from the dipole to the field point.

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5. A singly charged positive ion with an initial kinetic energy, $K = 100$ keV, follows a circular path with a radius, $r = 0.41$ m, in a magnetic field, $B = 0.223$ T, normal to the plane of the path.

a) [6 pts] What is the mass of the ion?

b) [4 pts] What electric field (magnitude and direction) must be superimposed if the ions are to pass undeflected through the magnetic field?

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6. [10 pts] When time-dependent magnetic fields are present within a conducting material, they can induce dissipative currents in the material that are called "eddy currents". As a simple example of this, consider a spatially uniform time-dependent magnetic field,

$$\mathbf{B} = B_0 \sin(\omega t)\hat{\mathbf{z}}$$

which perpendicularly penetrates a circular disk that lies in the $x-y$ plane. The disk has a radius a, thickness T, and resistivity ρ. You can assume the induced currents form circles in the disk which are concentric with it. You can assume that ρ is large enough that any magnetic fields produced by the induced current can be neglected.

a) Calculate the electric field inside the disk

b) Calculate the average total power dissipated as heat within the disk.

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7. [10 pts] One thousand ions of ^{44}V circulate in a storage ring with a velocity of 80% of the speed of light. Due to radioactive decays, after 20 ms only 340 ions are left. At rest, what is the half-life of ^{44}V?

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8. A particle of mass, m, is in the ground state of the one-dimensional "box" defined by the potential

$$V(x)=\begin{cases} 0, & 0<x<a \\ \infty, & \text{elsewhere} \end{cases}$$

The size of the box is *suddenly* increased by changing a to b, $b > a$. After the sudden expansion of the box, the energy of the particle is measured.

a) [5 pts] In the expanded box, what are the possible results of the energy measurement?

b) [5 pts] What are the probabilities associated with each of those possible results? (Well-defined integrals needn't be evaluated.)

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9. The proton – proton chain is the dominant mechanism for energy production in the center of the sun. The four reactions in the chain are:

$$p + p \rightarrow d + e^+ + \nu_e + 0.42 \text{ MeV} \quad (1)$$

$$e^+ + e^- \rightarrow \gamma + \gamma + 1.02 \text{ MeV} \quad (2)$$

$$p + d \rightarrow {}^3_2\text{He} + \gamma + 5.49 \text{ MeV} \quad (3)$$

$${}^3_2\text{He} + {}^3_2\text{He} \rightarrow p + p + \alpha + 12.86 \text{ MeV} \quad (4)$$

a) [4 pts] Write (next to the reaction) the primary force that is involved in each of the steps above. Choose among Electromagnetic, Gravitational, Weak Nuclear or Strong Nuclear.

b) [2 pts] Given the answers to part a) which of the steps do you expect to be the rate-limiting step? Explain why. (Assume the temperature is high enough that any coulomb repulsion in the collision does not change the relative rates.)

c) [2 pts] What is the total amount of energy released per α particle that is produced in the complete process?

d) [1 pt] Which of the final-state particles produced in the *pp* chain escape the sun with negligible scattering.

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10. [10 pts] The mass of the W^- boson (as produced in high energy collisions and measured using the decay products) is 80.4 GeV/c^2. Neutron beta decay is a weak interaction mediated by the W^- boson. Use the uncertainty principle to estimate the range of this weak interaction.

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11. [10 pts] Making reasonable assumptions, estimate the surface temperature of the planet Neptune. Neglect any possible internal sources of heat. What assumptions have you made about the planet's surface and/or atmosphere?

Relevant astronomical data: radius of the sun, $R_s = 7\times10^5$ km;

radius of Neptune, $R_N = 2.2\times10^4$ km;

mean sun-Neptune distance, $R_{sN} = 4.5\times10^9$ km; and the

surface temperature of the sun, $T_s = 6000$ K.

Also, see the front page of the exam for other perhaps useful constants.

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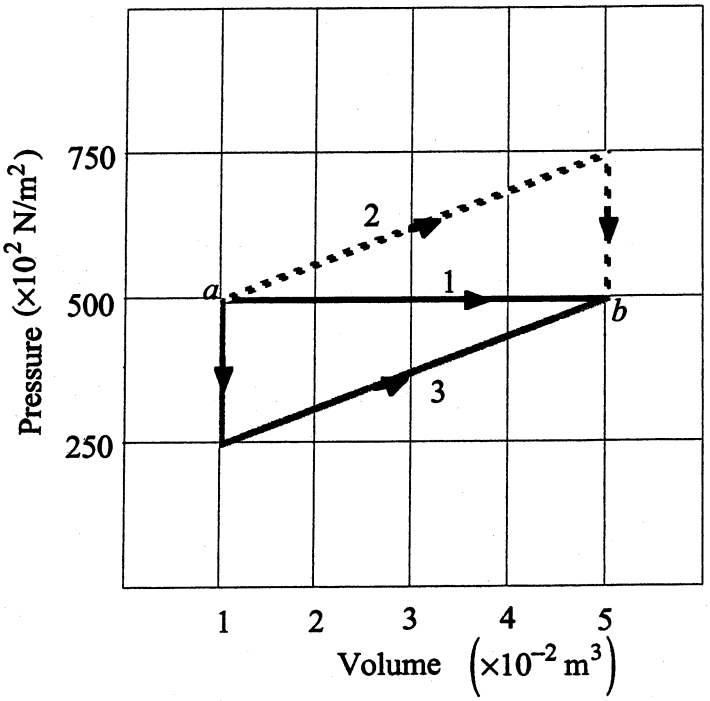
12. A sample of gas undergoes a transition from an initial state a to a final state b by three different paths (processes), as shown in the p–V diagram below. In process 1, the heat added to the gas is 5×10^3 J.



(a) [5 pts] What is the heat added to the gas in process 2?

(b) [5 pts] What is the change in internal energy that the gas undergoes in process 3?