Name: _____ Student No.: _____

Qualifying/Placement Exam
9:00 am, August 19, 2003

Put your NAME on every sheet of this 12 problem Exam -- NOW

You have 3 hours to complete the 12 problems on this exam. Show your work! Full credit will not be given for answers without justification. Some partial credit may be earned for the correct procedure, even if the correct answer is not achieved. Answers must be in the spaces provided. The **BACK** of the problem page may be used for lengthy calculations. Do not use the back of the previous page for this purpose!

You may need the following constants:

$k_e = 8.99\times10^{9}\ \mathrm{Nm^2/C^2}$	permittivity of free space
$\sigma = 5.7\times10^{-8}\ \mathrm{Wm^{-2}K^{-4}}$	Stefan - Boltzmann constant
$k = 1.4\times10^{-23}\ \mathrm{J/K}$	Boltzmann constant
$\hbar = 1.05\times10^{8}\ \mathrm{J\cdot s}$	Planck's constant
$c = 3.0\times10^{8}\ \mathrm{m/s}$	speed of light
$r_{univ} \approx 13\times10^{9}$ light - years	radius of universe (crude estimate)

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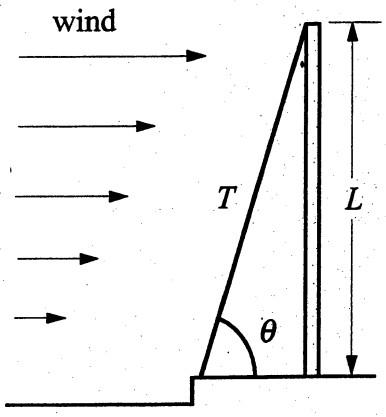
1. [10 pts] A seaside tower is supported by a tension cable. A horizontal wind that blows from the sea also depends on height according to force/unit length = αh, where h is the height above the bottom of the tower. If the height of the tower is L, and the angle of the tension cable to the base is θ, find an expression for the tension, T, in the cable in terms of the other variables in the problem.



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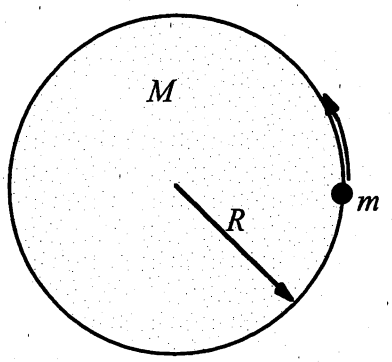
2. [10 pts] A turntable consists of a disk of radius, R, and mass, M, with a moment of inertia $I = \frac{1}{2}MR^2$, rotating on a frictionless bearing. Starting from rest, an ant of mass, m, crawls on the edge of the turntable, going around once, and returning to the exact spot on the turntable at which it started. In terms of m, M, and R, determine the angle through which the turntable has rotated.



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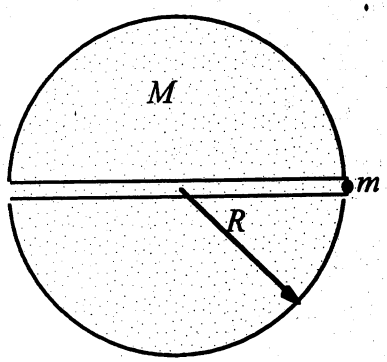
3. Assume the earth to be a constant density sphere of mass M, radius R. Imagine that it is possible to cut a *narrow* hole from one side of the earth, through the center, to the other side of the earth, as shown in the figure.



a) (5 pts) A ball of mass, m, is dropped down the hole from the surface. Show that as it drops the gravitational force acting on the ball has the form of a simple harmonic oscillator.

b) (5 pts) What is the period for the ball to return to its original position?

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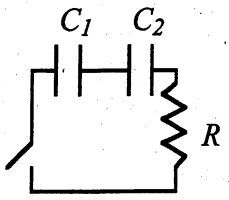
4. [10 pts] Consider two capacitors, C_1 and C_2, connected in series with a resistor, R, and a switch. At times, $t < 0$, the switch is open and capacitor C_1 holds charge, Q_0, and capacitor C_2 is uncharged. The switch is closed at $t = 0$. Find the charge on the capacitor C_1 as a function of time.



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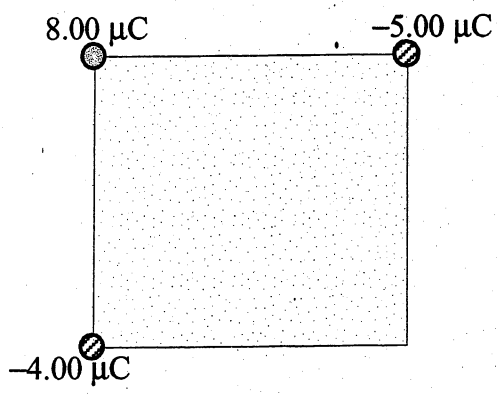
5. Three charges are placed on three corners of a square as shown. Each side of the square is 30.0 cm.



a) [5 pts] Compute the electric field **E** at the vacant corner. (2 significant figures)

b) [5 pts] Compute the force on a 6 μC charge placed at the vacant corner. Express this as a magnitude and angle wrt the horizontal axis.

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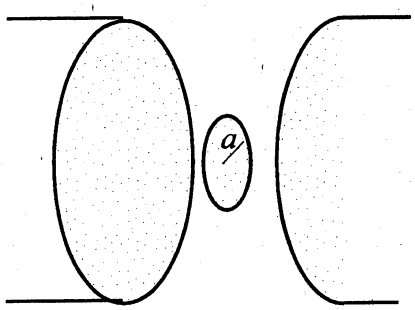
6. [10 pts] A circular loop of wire is placed between the pole faces of an electromagnet with its plane parallel to the pole faces. The loop has radius, a, and total resistance, R. If the magnet is turned on, producing a B field that is uniform across the area of the loop, what is the total charge q that flows past any point on the loop?



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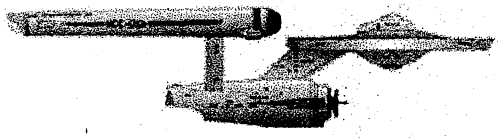
7. A spacecraft moving at a speed of 0.95c, travels from the Earth to the nearby star Alpha Centauri, which is 4.5 light years away.



a) How long will the trip take according to Earth's clocks?

b) How long will the trip take according to the spacecraft's clocks?

c) How far is it from Earth to Alpha Centauri according to the spacecraft's pilot?

d) What does the spacecraft's pilot compute for the speed of the spacecraft?

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8. [10 pts] Show that the kinetic energy of a mass, m, given by the total energy minus the rest energy, reduces to $KE = \frac{1}{2}mv^2$ for $v << c$.

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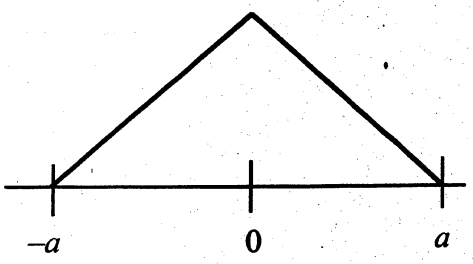
9. A particle is confined to a one-dimensional box with $|x| < a$. The initial spatial wave function is triangular.



a) [2 pt] An expansion of the wave function, $\psi(x)$, will require eigenfunctions that are

i) even parity.
ii) odd parity.
iii) both even and odd parity.
iv) neither even nor odd parity.

b) [2 pt] What is the expectation value of the momentum?
[Hint: first normalize the wavefunction)

c) [3 pt] What is the expectation value of the energy? (Hint: the discontinuity at $x = 0$ gives a non zero contribution to the energy, most easily obtained via integration by parts.)

d) [3 pt] If an energy measurement is made, what energy will be the most probable?

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10. A square sheet of metal with sides of length, L, is clamped on all four sides. It lies in the x-y plane and its displacement in the z-direction is described by the wave equation. Assume that the clamped sides lie at $x = 0$ and $x = L$, and $y = 0$ and $y = L$. Show that

$$z(x, y) = A\sin(kx)\sin(qy)\cos(\omega t)$$

is a solution of the wave equation that satisfies the boundary conditions. Also find the relation between k, q, and ω, which satisfies the wave equation.

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11. An ideal gas undergoes a reversible isothermal contraction. For each of the following quantities, indicate whether it is positive (>0), negative (<0), or zero (=0), and what leads you to make this choice.

(a) [2 pts] The work done by the gas (ΔW).

(b) [2 pts] The change in internal energy of the gas (ΔU).

(c) [2 pts] The heat absorbed by the gas (ΔQ).

(d) [2 pts] The change in entropy of the gas (ΔS_{gas}).

(e) [2 pts] The change in entropy of the universe ($\Delta S_{universe}$).

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12. Imagine the universe is a spherical cavity at a temperature of 3K. Using this assumption, estimate the total number of photons in the universe (useful constants on the front page of the exam).

Notes: integral: $\int_0^\infty \frac{x^2 dx}{e^x - 1} = 2.4$; black body energy density: $u(\omega, T) = \frac{\hbar}{\pi^2 c^3} \frac{\omega^3}{e^{\hbar\omega/kT} - 1}$.