Name: _____ Student No.: _____

Qualifying/Placement Exam
9:00 am, August 24, 2004

Put your NAME on every sheet of this 12 problem Exam -- NOW

You have 3 hours to complete the 12 problems on this exam. Show your work! Full credit will not be given for answers without justification. Some partial credit may be earned for the correct procedure, even if the correct answer is not achieved. Answers must be in the spaces provided. The **BACK** of the problem page may be used for lengthy calculations. Do not use the back of the previous page for this purpose!

You may need the following constants:

Speed of light in vacuum: $c = 3.00\times10^{8}$ m/s

Boltzmann constant: $k = 1.38\times10^{-23}$ J/K

Stefan-Boltzmann constant: $\sigma = 5.67\times10^{-8}$ W/(m^{2}K^4)

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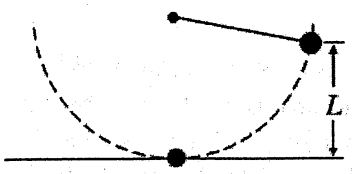
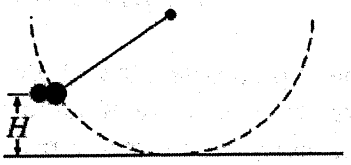
1. [10 pts] Tarzan, who has mass M_T, swings from a vine of negligible mass. He starts from rest on a branch that is height L above the ground. As he passes the lowest point in the arc, just above the ground, he picks up Jane, who has mass M_J. What is the height H of the highest branch that he and Jane could possibly come to rest on? (You can assume that the height of Tarzan and Jane can be neglected.)



Before



After

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2. [10 pts] Consider two springs, each with spring constant k, connected in series or in parallel. Determine the total energy stored, and the ratio of stored energies, when a series or a parallel combination of the two springs are stretched by the same force, F.

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3. [10 pts] A ramp of length d=1.5 m is set at an angle of θ=5 degrees to the horizontal. A solid cylinder of mass M=0.65 kg and radius R=0.047 m is initially at rest at the top of the ramp and starts rolling without slipping. How much time does it take for the cylinder to arrive at the bottom of the ramp?

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4. [10 pts] A metal rod 0.3 meters long rotates about an axis through one end and perpendicular to the rod, with an angular velocity of 7 rad/s. The plane of rotation is perpendicular to a uniform magnetic field of 0.5 T. What is the potential difference between the two ends of the rod induced by its motion in the magnetic field?

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5. [10 pts] The inner electrode of a spherical resistor is a spherical conducting shell of radius a, and the outer electrode is a concentric spherical conducting shell of radius b. The region $a<r<b$ is filled with a material whose electrical conductivity is σ (Siemens/meter). What is the resistance R between the electrodes?

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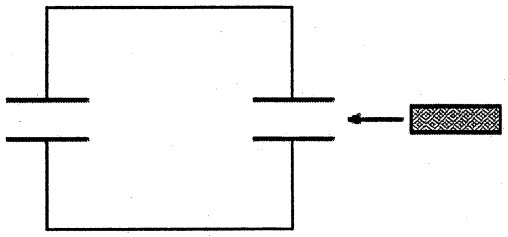
6. [10 pts] Two identical parallel plate capacitors of capacitance C are connected in parallel. The voltage on the capacitors is V. What is the total energy stored in the two capacitors. Now a slab of dielectric, with dielectric constant $\varepsilon = \kappa\varepsilon_0$, is slowly inserted (and fills the area) between the plates of one of the capacitors. What is the total energy stored in the two capacitors once the dielectric is in place? Does an external force have to do work to insert the dielectric?



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7. [10 pts] The half-life of Carbon 14 is $t_{1/2}$=5730 yr. Organisms accumulate this isotope from the atmosphere while they live, but cease doing so when they die. The skeleton of a mammoth is found to have a concentration of Carbon 14 that is 20 percent of the atmospheric value. When did the mammoth die? Assume that the atmospheric concentration of Carbon 14 is constant.

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8. [10 pts] Quasars are thought to be the active nuclei of galaxies in the early stages of their formation. A typical quasar radiates energy at the rate of 10^{41} W. At what rate is the mass of this quasar being reduced to supply this energy? Express your answer in solar mass units per year, where one solar mass unit (smu = 2×10^{30} kg) is the mass of our sun.

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9. [10 pts] The definition of the kinetic energy, K, of a particle of mass m moving with speed u, is the work done by an external force, F, in increasing the speed of the particle from zero to the value, u. Classically, this yields, $K=mu^2/2$. For motion in one dimension, show that this definition, using the relativistic momentum, $p=\gamma mu$, leads to the relativistic version of kinetic energy,

$$K=mc^2(\gamma-1),$$

where $\gamma = (1-u^2/c^2)^{-1/2}$.

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10. [10 pts] The maximum energy of photoelectrons from aluminum is E_1=2.3 eV for radiation of wavelength λ_1=200 nm, and it is E_2=0.9 eV for radiation of wavelength λ_2=261 nm. Use these data to estimate Planck's constant and the work function of aluminum.

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11. [10 pts] Consider a system in which a spin-1/2 fermion can be placed in one of four possible single-fermion states. Two of these single-fermion states (one with the fermionic spin up and the other with spin down) have energy $+E$ and likewise two of these single-fermion states have energy $-E$. Now, we put a total of three of these fermions into this system and put the system into contact with a heat reservoir at a temperature T. Assume there are no mutual interactions between the fermions.

a) You can completely characterize each three-fermion state by its occupancy (number of fermions) in each of the single-fermion states. What are the possible three-fermion states for this three-fermion system? What is the energy of each three-fermion state?

b) Find the average energy the three-fermion system when it is at temperature T.

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12. [10 pts] At the distance of the Earth's orbit about the Sun, the energy flux from the Sun is 1400 W/m^2. Suppose that the Earth absorbs 70% of the energy from the Sun that strikes its surface. Assuming this is the only source of energy, the Earth's surface is everywhere at the same temperature, and the surface radiates energy like a blackbody, calculate the equilibrium temperature of the Earth.