PHY-852: QUANTUM MECHANICS

Final Subject Exam /Total: $5 \times 20 = 100$/

May 3, 2005

PERSONAL NUMBER.....

Additional information:

normalized $2p$-wave function for the hydrogen-like atom:

$$\psi_{21m}(\mathbf{r}) = \frac{1}{\sqrt{3}}\left(\frac{Z}{2a}\right)^{3/2}\frac{Zr}{a}e^{-Zr/2a}Y_{1m}(\mathbf{n}),$$

where a is the Bohr radius, Z nuclear charge, $\mathbf{n} = \mathbf{r}/r$.

Bohr magneton

$$1\mu_B = \frac{e\hbar}{2m_e c} = 0.927 \times 10^{-20}\,\mathrm{erg/Gs}$$

1. A hydrogen atom in the ground state is placed between capacitor plates. The electric field is switched on at $t = 0$ and then decreases with time at $t > 0$ as

$$\vec{\mathcal{E}}(t) = \vec{\mathcal{E}}_0 \exp(-t/\tau),$$

where $\mathcal{E}_0 = 10^4\,\mathrm{V/cm}$. Using perturbation theory, calculate the probability of finding the atom (after a long time interval $t \gg \tau$) in each of the three $2p$ states. Give a numerical answer for the case $\omega\tau \gg 1$, where ω is the transition frequency.

2. A potential acting on the particle of mass m can be modelled by a very narrow one-dimensional well, $U(x) = g\delta(x)$, where g is a negative constant.

   (a) Show that there exists a bound state of the particle and find its wave function and binding energy.

   (b) For this bound state find the expectation values of the coordinate x and momentum p, and their mean square fluctuations.

   (c) Check the uncertainty relation between the position and momentum of the particle for this bound state.

3. The wave function of a particle of mass m is

$$\psi(x, y, z) = A\, xy\, e^{-r^2/a^2},$$

where A and a are constants.

a. Find the probabilities of finding in this state various values of l, l_z and parity.

b. Find the expectation values of the orbital momentum projections l_x, l_y, l_z and their squares.

c. Show that this is a stationary state of an isotropic harmonic oscillator with a certain frequency, find this frequency and energy of the state; write down the given function in terms of the creation operators acting on the ground state.

4. A gas of atoms in the $^2D_{3/2}$ state is placed in the static magnetic field B_0. The resonant absorption of time-dependent electromagnetic field in the transition between the adjacent Zeeman components was observed at the frequency $\nu = 2.8\,\mathrm{GHz}$. Determine the value of B_0.

5. A particle is scattered from a molecule that has a form of a plane square with a side a and four identical heavy atoms in the corners. The interaction potential between the particle at a position $\mathbf{r}$ and each atom at a position $\mathbf{a}_i$, $i = 1, 2, 3, 4$, is $U(\mathbf{r} - \mathbf{a}_i)$. In the Born approximation, express the scattering amplitudes, differential and total cross sections for the scattering off the molecule in terms of the scattering amplitude from the individual atom. Calculate the approximate molecular cross section in terms of the atomic cross section for both low- and high-energy scattering.