Student No.: ______________________

Qualifying/Placement Exam, Part-A
10:00 – 12:00, August 18, 2016, 1400 BPS

Put your **Student Number** on every sheet of this 6 problem Exam -- NOW

You have 2 hours to complete the 6 problems on Part-A of the exam. Show your work! Full credit will not be given for answers without justification. Some partial credit may be earned for the correct procedure, even if the correct answer is not achieved. Answers must be in the spaces provided. The **BACK** of the problem page may be used for lengthy calculations. *Do not use the back of the previous page for this purpose*!

You may need the following constants:

$k_e = 8.99\times10^{9}\ \mathrm{Nm^2/C^2}$	permittivity of free space
$\sigma = 5.7\times10^{-8}\ \mathrm{Wm^{-2}K^{-4}}$	Stefan-Boltzmann constant
$k = 1.4\times10^{-23}\ \mathrm{J/K}$	Boltzmann constant
$\hbar = 1.05\times10^{-34}\ \mathrm{J\cdot s}$	Planck's constant
$= 6.58\times10^{-16}\ \mathrm{eV\cdot s}$	"
$c = 3.0\times10^{8}\ \mathrm{m/s}$	speed of light
$e = 1.602\times10^{-19}\mathrm{C}$	charge of the electron

Student No.: ________________________

1. [10 pts] Show that for a transformation matrix U,

$$\mathrm{Tr}\, U^{-1}AU = \mathrm{Tr}\, A$$

where $U^{-1}U = UU^{-1} = I$.

Student No.: ________________________

2. [10 pts] Consider the periodic function $y(x-2\pi) = y(x)$ for all x, and

$$y(x) = \begin{cases} 1, & -\pi/2 < x < \pi/2 \\ 0, & \pi/2 < x < 3\pi/2 \end{cases} .$$

For the expansion

$$y(x) = \sum_n a_n \cos nx + b_n \sin nx ,$$

list all the n for which $a_n \neq 0$, and all n for which $b_n \neq 0$. (Do NOT solve for the actual coefficients, just list the values of n)

Student No.: ________________________

3. [10 pts] Consider Schrödinger's equation for a particle of mass m moving in a two-dimensional harmonic oscillator potential with spring constant k,

$$\left[\frac{-\hbar^2}{2m}(\partial_x^2+\partial_y^2)+\tfrac{1}{2}k(x^2+y^2)\right]\psi = E\psi \,.$$

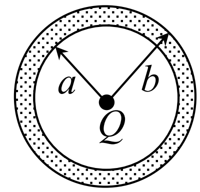
a) [5 pts] In the polar coordinates $r^2 = x^2 + y^2$, $\tan\phi = y/x$, find the radial differential equation for $\Psi_m(r)$, where the solution to Schrödinger's equation is

$$\psi(r,\phi) = e^{im\phi}\Psi_m(r)\,.$$

b) [5 pts] Find the lowest energy solution with $m = 0$, $\Psi_0(r)$.

Student No.: ________________________

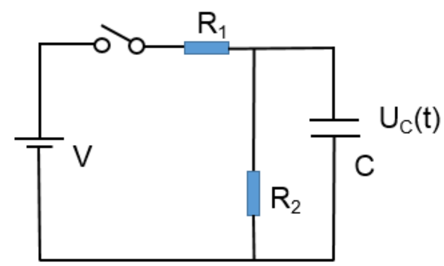
4. [10 pts] A point charge Q is placed at the center of an uncharged conducting sphere with inner radius a, and outer radius b, as shown in the figure.



a) [5 pts] Determine the field in the three regions: $r < a$, $a < r < b$, and $r > b$.
b) [3 pts] Determine the charge density at an arbitrary radius r, $b \geq r \geq a$.
c) [2 pts] Show that these charge densities are consistent with the change in the electric field when crossing each surface.

Student No.: ________________________

5. [10 pts] An electric circuit composed of a battery with potential V, two resistors R_1 and R_2, and a capacitor C is shown in the figure below. After being open for a long time, the switch is closed at t = 0. Calculate the potential drop across the capacitor as a function of time, $U_C(t)$.



Student No.: ________________________

6. [10 pts] Consider a grounded conducting surface in the *x*-*y* plane at $z = 0$. An infinite line of charge with charge density λ, is placed parallel to the *x*-axis at $z = a$ and $y = 0$.

   a) [8 pts] Find the electric field, $\mathbf{E}(x, y, z)$ for all $z > 0$.

   b) [2 pts] Find the charge density on the conducting surface.