Student No.: _____

Put your **Student Number** on every sheet of this 6 problem Exam -- NOW

You have 2 hours to complete the 6 problems on Part-A of the exam. Show your work! Full credit will not be given for answers without justification. Some partial credit may be earned for the correct procedure, even if the correct answer is not achieved. Answers must be in the spaces provided. The **BACK** of the problem page may be used for lengthy calculations. Do not use the back of the previous page for this purpose!

You may need the following constants:

$$k_e = 8.99 \times 10^9 \, \text{Nm}^2/\text{C}^2 \quad \text{permittivity of free space}$$

$$\sigma = 5.7 \times 10^{-8} \, \text{Wm}^{-2}\text{K}^{-4} \quad \text{Stefan-Boltzmann constant}$$

$$k = 1.4 \times 10^{-23} \, \text{J/K} \quad \text{Boltzmann constant}$$

$$\hbar = 1.05 \times 10^{-34} \, \text{J} \cdot \text{s} \quad \text{Planck's constant}$$

$$c = 3.0 \times 10^8 \, \text{m/s} \quad \text{speed of light}$$

1. [10 pt] A cylindrical shell of radius a and length $2L$ is aligned around the z-axis from $z = -L$ to $z = +L$. A current I, is distributed uniformly on the cylinder and moves around the cylinder's z-axis. Find the magnitude of the magnetic field at the origin.
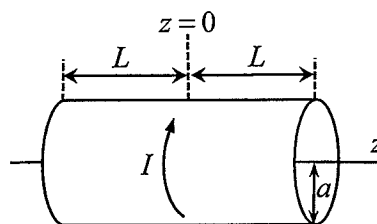
2. [10 pt] A perfectly conducting sphere of radius R is placed in an electric field $E_0 \hat{k}$. The presence of the sphere modifies the field. The sphere has a net charge, Q, and is placed at the origin of the coordinates.

(a) [8 pt] The potential function outside the sphere has the form:

$$V(r,\theta) = \frac{A}{r} + Br\cos\theta + \frac{C\cos\theta}{r^2}$$

in spherical polar coordinates (r,θ,ϕ) with respect to the $\hat{k}$ direction. Using the fact that the asymptotic field is $E_0\hat{k}$, and that the field is zero inside the sphere, determine the constants A, B, C in terms of E_0, Q, and R.

(b) [2 pt] Determine the charge density $\sigma(\theta)$ on the sphere.

3. [10 pt] An infinitely long cylinder of radius R has a uniform volume charge density, $\rho > 0$. Find the electric field, **E**, (magnitude and direction) at

    (a) [5 pt] a point inside the cylinder, and
    (b) [5 pt] a point outside the cylinder.
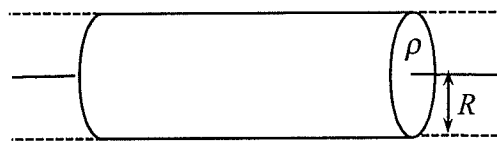
4. A mass, M_1, is constrained to move without friction on a horizontal bar. A second mass, M_2, hangs from the first mass by a rigid rod of negligible mass and length L, which can pivot without friction in the plane defined by the horizontal bar and the vertical.
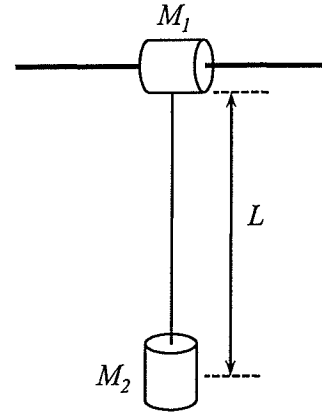
a) Letting x be the horizontal displacement of M_1 and θ be the angle of the rod with respect to the vertical, write expressions for the Kinetic Energy, T, and Potential energy, U, in terms of x and θ.

b) Write an expression for the horizontal component of the center of mass of the system x_{CM} in terms of x and θ.

c) Obtain the frequency for small oscillations of the system. [Hint: the answer to part b may be useful in diagonalizing the Kinetic Energy.

5. [10 pt] Consider the Lagrangian $L = \frac{1}{2} M\left(\dot{x}^2 + \dot{y}^2\right) + K\left(x\dot{y} - y\dot{x}\right)$, where M and K are constants.

    a) [5 pt] Find the Lagrange equations of motion.
    b) [5 pt] Solve these equations.

6. [10 pt] A particle of mass m moves in a one-dimensional harmonic oscillator with a spring constant k and damping constant Γ,

$$m\frac{d^2x}{dt^2} = -kx - \Gamma\frac{dx}{dt} + F_d\sin(\omega t).$$

Assuming the particle begins at rest at $x=0$, solve for $x(t)$ at large times and for any ω.

Qualifying/Placement Exam, Part-B
2:30 pm, August 21, 2012

Put your **Student Number** on every sheet of this 6 problem Exam -- NOW

You have 2 hours to complete the 6 problems on Part-B of the exam. Show your work! Full credit will not be given for answers without justification. Some partial credit may be earned for the correct procedure, even if the correct answer is not achieved. Answers must be in the spaces provided. The **BACK** of the problem page may be used for lengthy calculations. Do not use the back of the previous page for this purpose!

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1. [10 pt] In an infinite potential well of width a, where $V(x) = 0$ for $0 < x < a$, the wave function for a particle is

$$\psi(x) = Ax(a - x).$$

a) [3 pt] Normalize $\psi(x)$.

b) [3 pt] Using the Normalized $\psi(x)$, calculate $\langle x \rangle$.

c) [1 pt] What eigenstate of the infinite well system does $\psi(x)$ most nearly resemble?

d) [3 pt] Calculate $\langle H \rangle$.

2. [10 pt] The Hamiltonian H for a system is representative by the matrix

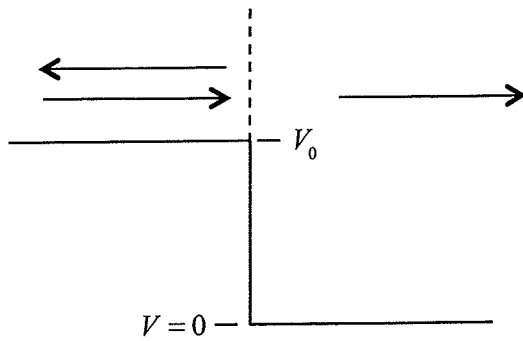$$H = \begin{pmatrix} E & -E \\ -E & E \end{pmatrix}$$

where $E \geq 0$.

  a) [2 pt] What are the possible energies of this system?
  b) [3 pt] Determine the (normalized) eigenvectors of H.
  c) [3 pt] The system described by H, is in the initial state $\psi(t=0)$ given by

$$\psi(t=0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

  What is $\psi(t)$?

  d) [2 pt] At what time will $\psi(t) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$?

$$V(x) = \begin{cases} V_0 & \text{for} \quad -\infty \le x \le 0 \\ 0 & \text{for} \quad 0 \le x \le \infty \end{cases}$$

3. [10 pt] A particle of mass m and energy $E > V_0$ approaches the 'step' at $x = 0$ from $x = -\infty$. At $x = 0$ it can be reflected or transmitted as indicated by the arrows in the figure above.

a) [4 pt] Assuming there is only a transmitted wave for $x > 0$, derive the general solution for the wave function in the regions $-\infty \le x \le 0$ and $0 \le x \le \infty$ starting from the Schrödinger equation. Define your notation.

b) [4 pt] Apply the boundary conditions at $x = 0$ and obtain the reflected and transmission amplitudes in terms of the amplitude of the incoming wave.

c) [2 pt] Derive the reflection probability.

4. [10 pts] The mean lifetime of a pion at rest is 26 ns. In the laboratory frame, a monoenergetic beam of pions is created with a mean lifetime of 200 ns. What is the kinetic energy of the pions, (Rest mass of a pion is 139.6 MeV/c^2.)

5. [10 pt] A particle of mass m in a harmonic oscillator whose potenial is

$$V(x) = \frac{1}{2}m\omega^2 x^2,$$

i.e., the eigen-energies are $\hbar\omega/2, 3\hbar\omega/2, \cdots$. Find the average energy of the particle if it interacts with a heat bath at temperature T.

6. [10 pt] A neutron is isolated in a vacuum until it decays.

    a) [5 pt] What are the products of this decay?

    b) [5 pt] Pick the most appropriate scale for the lifetime of the decay.

        * attoseconds (10^{-18} seconds)

        * minutes

        * millenia