Student No.: ______________________

Qualifying/Placement Exam, Part-B
2:00 – 4:00, August 19, 2019

Put your **Student Number** on every sheet of this
6 problem Exam -- NOW

You have 2 hours to complete the 6 problems on Part-B of the exam. Show your work! Full credit will not be given for answers without justification. Some partial credit may be earned for the correct procedure, even if the correct answer is not achieved. Answers must be in the spaces provided. The **BACK** of the problem page may be used for lengthy calculations. *Do not use the back of the previous page for this purpose*!

You may need the following constants:

$k_e = 8.99\times10^{9}\ \mathrm{Nm^2/C^2}$	electric force constant
$\sigma = 5.7\times10^{-8}\ \mathrm{Wm^{-2}K^{-4}}$	Stefan-Boltzmann constant
$k = 1.4\times10^{-23}\ \mathrm{J/K}$	Boltzmann constant
$\hbar = 1.05\times10^{-34}\ \mathrm{J\cdot s}$	Planck's constant
$= 6.58\times10^{-16}\ \mathrm{ev\cdot s}$	"
$c = 3.0\times10^{8}\ \mathrm{m/s}$	speed of light
$e = 1.602\times10^{-19}\ \mathrm{C}$	charge of the electron

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1. Consider the following three operators on the 2-dimensional Hilbert space of complex-valued column vectors:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

   a) [2 pts] What are the normalized eigenstates of σ_x?

   b) [6 pts] In the eigenstate of σ_x with an eigenvalue of 1, what are $\langle \sigma_z \rangle$, $\langle \sigma_z^2 \rangle$, and $\Delta\sigma_z$ (*i.e.,* the rms uncertainty of a measurement of σ_z in this eigenstate)?

   c) [2 pts] In the same eigenstate considered in part (b), what is the probability that a measurement of σ_z will return a result of 1?

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2. A particle is in the ground state of an infinite square well given by the potential

$$V(x)=\begin{cases} 0 & 0\le x\le a \\ \infty & \text{otherwise.} \end{cases}$$

a) [3 pts] What is the energy and the normalized wavefunction of the particle?
b) [2 pts] The right side of the well is instantaneously moved from $x=a$ to $x=2a$. What is the new ground-state energy of the particle after the change?
c) [5 pts] Assuming that the particle is still in its original wavefunction, what is the probability that a measurement of the particle's energy will give the new ground-state value after the change?

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3. Consider an electron in a hydrogen atom. At time $t = 0$ it is in a superposition of four eigenstates:

$$\Psi(\vec{r},0) = \frac{1}{\sqrt{15}}\left[\psi_{10}^{0}(\vec{r}) + 2i\psi_{21}^{0}(\vec{r}) - \psi_{21}^{1}(\vec{r}) - 3\psi_{31}^{1}(\vec{r})\right],$$

where $\psi_{n\ell}^{m}(\vec{r})$ is the normalized wavefunction with principle quantum number n, orbital angular momentum quantum number ℓ, and magnetic quantum number m. The Rydberg ionization energy is $E_R = 13.6$ eV.

a) [3 pts] If the energy is measured at $t = 0$, what is the expectation value for the energy?

b) [4 pts] If the orbital angular momentum along the z axis, L_z, is measured at $t = 0$, what are the possible values that could be obtained? What are the probabilities to obtain each value?

c) [3 pts] What is the wavefunction at a time $t \neq 0$ later (assuming that no measurements have been made yet)?

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4. A muon has the same electric charge as an electron, but has a larger mass of 105 $\mathrm{MeV/c^2}$, compared to the electron mass of 0.511 $\mathrm{MeV/c^2}$. In a muonic atom, a negatively-charged muon replaces an electron. Consider the muonic hydrogen atom, with one negative muon and a proton.
    a) Compare the ground state radius of a muonic hydrogen atom with the radius of a normal hydrogen atom.
    b) In a transition from the first excited state to the ground state, compare the energy of the photon emitted by a muonic hydrogen atom with that of the photon from a normal hydrogen atom.

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5. Here is a standard example that exhibits relativistic effects: Consider a barn of length L with doors on the front and back. A pole, which is 10% longer than the length of the barn (when both are at rest), is headed through the open barn doors at very high speed. Clearly, when the pole is traveling slowly, it cannot fit in the barn with both doors closed.
   a) [4 pts] In the rest frame of the barn, if the pole moves fast enough it can fit within the barn with both doors closed simultaneously. At what speed must the pole be moving (in the rest frame of the barn) to JUST fit inside, with both doors closed for an instant?
   b) [3 pts] In the pole's rest frame, the pole cannot fit within the barn with both doors closed simultaneously. Describe the sequence of door closings as observed from this moving frame.
   c) [3 pts] Assuming the barn to be 10 m long, calculate the time interval between the door closings in the frame of the moving pole.

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6. Consider a system with two single particle energy levels, 0 and ε. The levels are populated by two non-interacting electrons in equilibrium with a heat bath of temperature T.
   a) [2 pts] What is the average total energy of the two-electron system when $T = 0$?
   b) [2 pts] What is the average total energy when $T \to \infty$?
   c) [6 pts] What is the average total energy as a function of T?