

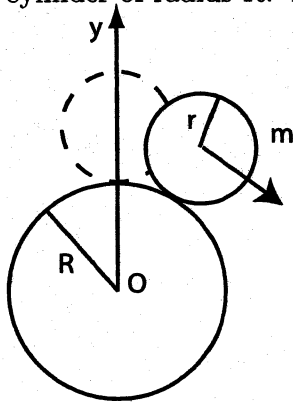
Classical Mechanics Graduate Exam

August 29, 2005

NOTE: Do NOT write your name on any paper you hand in. Only write the selected number for the subsequent identification, so that the problems may be graded anonymously.

Show all work. Justify your answers.

1. A uniform cylinder of mass m and radius r is placed on top of a rigidly held cylinder of radius R . The axes of the cylinders are parallel.



At time $t = 0$, the top cylinder is slightly touched and begins rolling, without slipping, down the bottom cylinder. (a) Find the moment of inertia of a cylinder of mass m and radius r about its axial symmetry axis. (b) What is the condition for the top cylinder detaching from the bottom cylinder, during its downward motion? (c) Find the elevation y , of the top cylinder's center above the bottom cylinder's center, when the top cylinder detaches.

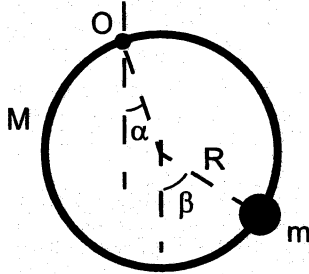
2. Particles 1 and 2, both of mass m , move along the x -axis. Particle 1 is affected by an electric trap potential characterized by a constant K . Particle 2 is neutral and not affected by the trap. The two particles interact with each other via potential characterized by constants D and d . The Lagrangian describing this situation is:

$$L = \frac{1}{2} m \dot{x}_1^2 + \frac{1}{2} m \dot{x}_2^2 - \frac{1}{2} K x_1^2 - \frac{1}{2} D \left((x_1 - x_2)^2 - d^2 \right)^2.$$

- (a) Find equilibrium points for the system of the two particles. What type of equilibrium, stable, unstable or neutral, do the respective points represent?
 (b) Choose a stable equilibrium point characterized by $x_1 \geq x_2$ and find angular frequencies of the free system vibrations around that point.

(Over)

3. A thin ring of mass M and radius R can swing around a nail, through point O , pinning the ring to a vertical wall. Threaded on and moving relative to the ring is a bead of mass m . Consider planar motion of the ring and bead, including the effects of gravity, but ignoring friction.



- (a) Find the kinetic energy T and Lagrangian L for the ring-bead system, employing the indicated angles α and β as generalized coordinates. (b) Find the kinetic energy T_{CM} associated with the motion of the center of mass of the system. Explain the origin of any differences compared to T in (a).
- (c) Obtain the generalized momenta conjugate to the angles α and β . Is any of those momenta conserved? Explain why. (d) Obtain the energy function h , referred also to as Jacobi integral, for the ring-bead system. Is that function conserved? Is the energy function identical to the energy for the system? Explain under what circumstances (i) the energy function is conserved and (ii) the energy function coincides with the energy.
4. The Hamilton's principle implies that the Lagrangian \tilde{L} , obtained by adding the time-derivative of an arbitrary differentiable function M of time and coordinates to the Lagrangian L ,

$$\tilde{L} = L + \frac{dM}{dt},$$

yields the same Lagrange equations of motion as L . Here, $L = L(q_1, \dots, q_n, \dot{q}_1, \dots, \dot{q}_n, t)$ and $M = M(q_1, \dots, q_n, t)$. (a) Let \tilde{p}_k and p_j represent the generalized momenta obtained, respectively, from \tilde{L} and L . Express \tilde{p}_k in terms of p , q and t . (b) Consider the transformation $(p, q) \rightarrow (\tilde{p}, \tilde{q})$, where $\tilde{q}_k = q_k$ and $\tilde{p}_k = \tilde{p}_k(q, p, t)$ is the relation from (a). For that transformation of the canonical variables, find how the fundamental Poisson brackets (of the canonical variables) transform. (c) Express the difference between the Hamiltonians \tilde{H} and H , obtained from \tilde{L} and L , respectively, in terms of p , q and t . (d) Construct a generating function $F(q, \tilde{p})$ for the transformation $(p, q) \rightarrow (\tilde{p}, \tilde{q})$. Verify that your function is consistent with the results in (a) and (c) and with $\tilde{q}_k = q_k$.