Qualifying Exam

August 25, 1998

Exam number:

You have 3 hours to complete the 12 problems on this exam. Do all problems. Show your work! Full credit will not be given for answers without justification. Some partial credit may be earned for the correct procedure, even if the correct answer is not achieved. While waiting to begin, please enter your name and student number on the lines below.

Name: _____ Student No.: _____

List of subject areas

Subject area	Problem numbers
Mechanics	1, 2, 3
Electricity and Magnetism	4, 5, 6
Modern Physics	7, 8, 9, 10
Thermodynamics	11, 12

Do not turn this page and start the exam until you are told to begin.

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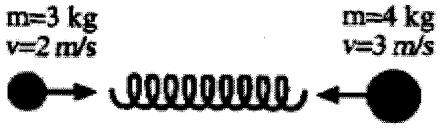
1. **Two particles with masses 3 kg and 4 kg are traveling toward each other with speeds of 2 m/s and 3 m/s, respectively. Between them is a relaxed massless spring with spring constant 50 N/m. When the two masses hit the spring they compress it so that their relative velocity is zero when the spring has its maximum compression, which is equal to x.**

(a) What is the velocity of the masses at the maximum spring compression?

(b) What is the maximum distance x by which the spring is compressed?



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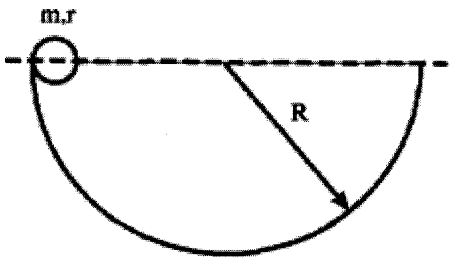
2. A sphere of radius r, mass m, and moment of inertia $I = 2mr^2/5$ is released from rest from the top of a semicircular track of radius R. It rolls down the track without slipping. Obtain an expression for the NORMAL force which the track exerts on the sphere when it is passing the bottom of the track.



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3. Two masses, m_1 and m_2, starting at rest, are suspended without friction by a massless string and pulley (Atwood's machine). The pulley is attached to the top of a doorway as shown in the figure.

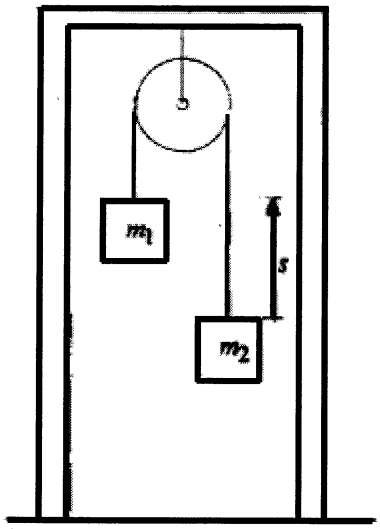
Use the work-energy theorem to derive the speed v after m_2 has risen a distance s, in terms of the masses and g.

$v =$ [ ]

Use this speed to determine the acceleration a of the masses, the tension T in the string, and the force F that the pulley applies to the top of the doorway.

$a =$ [ ]

$T =$ [ ] $F =$ [ ]



An Atwood's machine hung from a bar that is held off the earth by by two posts.

What is the net force F_E acting on the earth from this complete system – masses and doorway? What is the limit of F_E when $(m_1 - m_2) \to 0$?

$F_E =$ [ ] $\lim_{m_1 \to m_2} F_E =$ [ ]

Put your answers in the boxes above, but show your work for each answer below.

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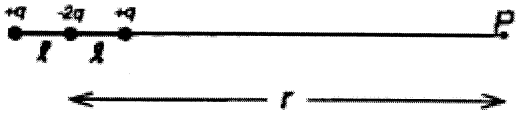
4. Shown in the figure is a linear quadrupole made of two identical dipoles lying on the same straight line, in opposite directions, with their negative charges at the same point. Find the electric potential and electric field at a point P on the extension of the line at distance r from the center (the negative charge). Assume $r \gg \ell$.



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5. The electric field in a linearly polarized electromagnetic plane wave is $\vec{E}(\vec{x},t) = E_0\hat{j}\cos(kx - \omega t)$ where $\hat{i}$, $\hat{j}$, $\hat{k}$ denote unit vectors along the x, y, z axes.

(a) (5 points) Determine the magnetic field.

(b) (5 points) Determine the Poynting vector.

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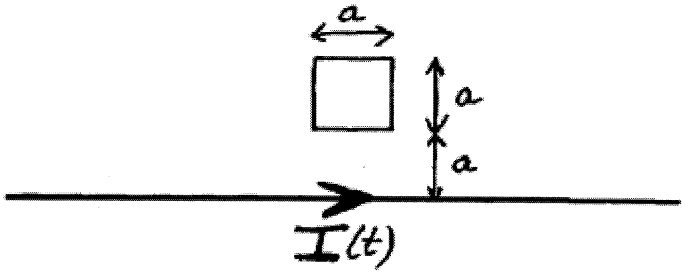
6. A square loop of wire, with dimensions $a \times a$, is placed near a long wire carrying time-dependent current $I(t)$, as shown in the Figure. The resistance of the loop is R.

(a) (4 points) Determine the magnetic flux through the loop if I is constant.

(b) (5 points) Determine the current $i(t)$ in the loop, due to electromagnetic induction, if $I(t)$ is changing.

(c) (1 point) What is the direction of $i(t)$ if $I(t)$ is increasing?



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7. A series of marbles, each with mass m, is dropped from a height H directly above a line on the ground. Although a high precision dropping device is used, each marble does not land on the line. Show that the typical distance from the line where a marble lands is

$$\Delta x \approx \left(\frac{\hbar}{m}\right)^{1/2}\left(\frac{2H}{g}\right)^{1/4}.$$

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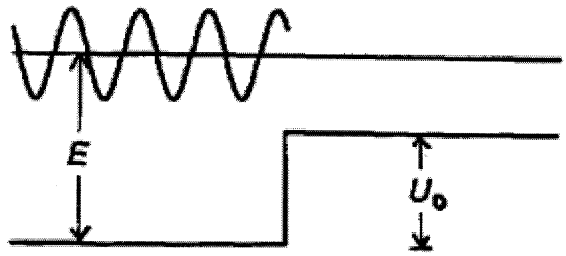
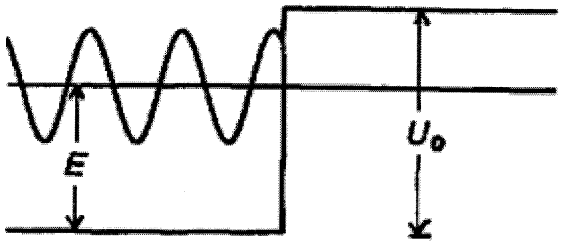
8. (a) (7 points) The mean lifetime of muons in their rest frame is 2×10^{-6} s. Suppose a muon is created in a cosmic ray event in the upper atmosphere, and its speed is $0.95\,c$, where $c = 3 \times 10^8$ m/s is the speed of light. What is the expected distance that the muon will travel before it decays? Give the result in km.

(b) (3 points) The muon rest mass is $106\,\mathrm{MeV}/c^2$. What is the total energy of the muon of (a)?

Exam number:

9. (a) (7 points) Below are two sketches of a step potential and the wave function of a particle in the region to the left of the step. In the first case the kinetic energy of the particle is greater than the step height U_0, and in the second case it is less than the step height. In each case the wave function is drawn only up to the position of the step. Draw on these figures the wave functions in the region to the right of the step. To be sure that your drawings show the added parts of the wave functions clearly, augment the drawings with a few words that tell of any differences between the wave functions on the two sides of the step.





(b) (3 points) The wave function of a particle bound by a harmonic oscillator potential is

$$\psi(x) = \left[\pi^{1/2} x_0\right]^{-1/2} \exp\left[-\frac{x^2}{2x_0^2}\right].$$

What is the probability of the particle being between $x = 0$ and $x = 0.01x_0$? Choose your answer from among these choices:

a. 0.14	f. 0.0056
b. 0.071	g. 0.0028
c. 0.033	h. 0.00062
d. 0.010	i. 0.00037
e. 0.0095	j. 0.00010

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10. (a) (7 points) In the radioactive decay chain from $^{235}_{92}U$ to $^{207}_{82}Pb$ a definite number of alpha particles and a definite number of beta rays are emitted. How many of each?

(b) (3 points) Energy has many forms. When $^{235}_{92}U$ captures a neutron and fissions, an energy of almost 200 MeV is released. In what form is most of the released energy?

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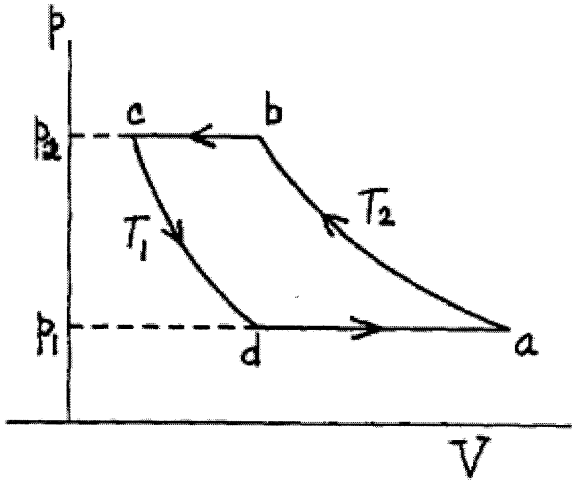
11. An ideal gas is used in the refrigeration cycle shown in the figure. *ab* and *cd* are isothermal processes at temperatures T_2 and T_1, respectively; *bc* and *da* are isobaric processes at pressures p_2 and p_1, respectively. The number of moles of gas is n. Express your answers in terms of p_1, p_2, T_1, T_2, and n.

(a) Determine the work done on the gas during processes *ab* and *cd*.

(b) Determine the work done on the gas during processes *bc* and *da*.

(c) Determine the heat Q_1 absorbed by the gas at temperature T_1, *i.e.*, during the process *cd*.

(d) The coefficient of performance K of this cycle is defined as Q_1/W_{net}, where W_{net} is the net work done on the gas in one cycle. Determine K in terms of T_1 and T_2.



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12. Find the change in entropy of the H_2O molecules when (a) 3 kg of ice melts into water at 273 K; and (b) 3 kg of water changes into steam at 373 K.

(c) On the basis of the answers to (a) and (b), discuss which change creates more disorder in the collection of H_2O molecules.

(Data: For the phase transitions of water, the heat of fusion is 3.33×10^5 J/kg, and the heat of vaporization is 2.256×10^6 J/kg.)