Qualifying Exam

Aug 24, 1999

Exam number: 55

You have 3 hours to complete the 12 problems on this exam. Do all problems. Show your work! Full credit will not be given for answers without justification. Some partial credit may be earned for the correct procedure, even if the correct answer is not achieved. While waiting to begin, please enter your name and student number on the lines below.

Name: _____ Student No.: _____

List of subject areas

Subject area	Problem numbers
Mechanics	1, 2, 3
Electricity and Magnetism	4, 5, 6
Modern Physics	7, 8, 9, 10
Thermodynamics	11, 12

Do not turn this page and start the exam until you are told to begin.

1. Mechanics – I

A hollow cylinder is made out of steel (mass density ρ). It has inner radius r_1 and outer radius r_2. It has length ℓ.

[2 points] (a) What is the mass of the cylinder in terms of the symbols above?

[2 points] (b) What is the moment of inertia of the cylinder about its axis of greatest symmetry?

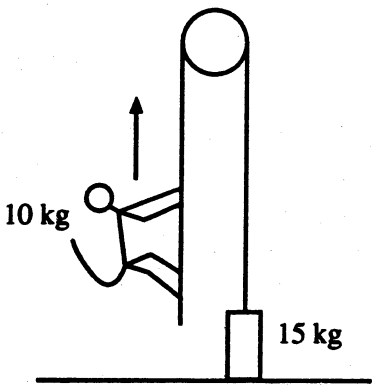
[6 points] (c) The cylinder rolls from rest without slipping down an inclined plane through a vertical distance h. What is the final velocity of the center of mass?

EXAM FF

2. (Classical Mechanics) A 10 kg monkey climbs up a massless rope that runs over a frictionless tree limb and back down to a 15 kg package on the ground.

A. What is the magnitude of the least acceleration the monkey must have if it is to lift the package off the ground?

B. If, after the package has been lifted, the monkey stops its climb and holds onto the rope, what is the monkey's acceleration?



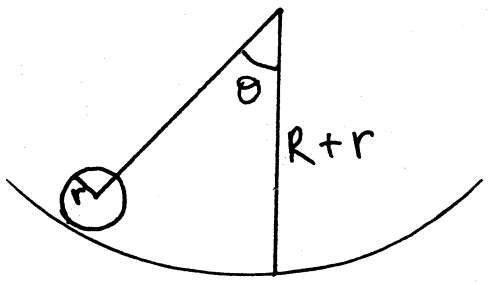
3. Mechanics – III

A solid ball of mass m and radius r rolls within a fixed spherical dish whose radius of curvature is $R + r$. A cross section is shown in the figure below. The elevation of the ball is constant so that the angle θ is fixed, *i.e.*, the ball rolls in a horizontal plane. Gravity operates in the downwards ($\theta = 0$) direction.

[5 points] (a) Find an expression for the velocity of the center of mass of the ball as a function of θ.

[5 points] (b) Find an expression for the total energy of the ball, assuming that the ball has zero potential energy when it rests at the bottom of the dish.

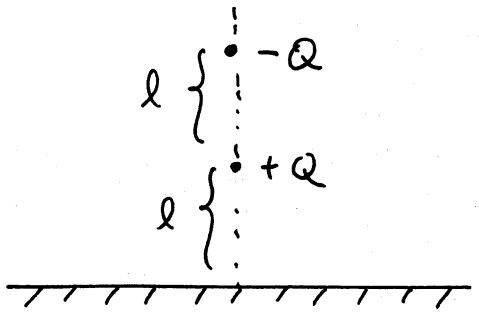
[Hints: The moment of inertia of the ball is $\frac{2}{5}mr^2$. The ball spins as fast as needed to roll without slipping.]



4. E & M – I

The region $z \leq 0$ is a conductor. On the z axis there are two point charges held at rest: $+Q$ at $z = \ell$ and $-Q$ at $z = 2\ell$.

[10 points] Calculate the electrostatic force on the charge $-Q$, i.e., the net force due to $+Q$ and the conductor.



5. E & M – II

A plane electromagnetic wave propagates through the vacuum. The electric field is

$$\vec{E}(x, y, z, t) = \hat{k} E_0 \sin\left[k(x - ct)\right].$$

($\hat{\imath}, \hat{\jmath}, \hat{k}$ denote unit vectors in the x, y, z directions.)

[10 points] Determine the magnetic field.

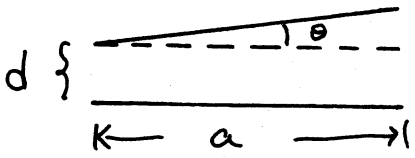
6. E & M – III

A capacitor is made of two square plates of length a on each side, separated by a small distance in the air. The plates are not quite parallel to each other, but rather make a small angle θ as shown end-on below.

[10 points] Find the capacitance *to first order in* θ.

[Hints: You can consider the capacitor to be a parallel combination of many strip capacitors, each with slightly different separation; or alternatively, you can approximate the charge density as a function of position.

You can use the approximation $ln(1+\epsilon) = \epsilon - \frac{1}{2}\epsilon^2 + ...$]



EXAM #

7. Modern - I

2 pts (i) The bound states of the Hydrogen atom are described by wave functions

$$\Psi_{nlm}(\mathbf{r}) .$$

Describe the quantum numbers n, l and m.

(ii) Consider a Hydrogen atom in a state described by the wave function

$$[\, \Psi_{100}(\mathbf{r}) + \Psi_{211}(\mathbf{r}) \,]/\sqrt{2} ,$$

2 pts a) what is the expectation value of the projection of the angular momentum on the z-axis L_z ?

2 pts b) what is the expectation value of the square of the angular momentum $\mathbf{L}^2$?

3 pts c) what is the expectation value of the energy in the units of E_o ? E_o is the energy of the ground state of the Hydrogen atom .

1 pt (iii) What is the numerical value of E_o in the units of eV ? A derivation is not required in this part.

Exam #

8. Modern - II

A photon with a wavelength of $\lambda = 500$ nm $= 5 \times 10^{-7}$ m collides with an electron at rest . The collision is a headon collision and therefore the momenta of the photon and of the electron after the collision are collinear with the initial momentum of the photon.

What is the initial energy of the photon (in eV) and what fraction of the photon energy is transferred to the electron during the collision ?

Assume that the motion of the electron is non-relativistic.

$h = 6.62 \times 10^{-34}$ J s
eV $= 1.602 \times 10^{-19}$ J
$\hbar c = 197$ eV nm
$m_{electron}\, c^2 = .511 \times 10^6$ eV

EXAM #F

9. (Modern Physics) – III

A particle moves in a one-dimensional square-well potential,

$$V(x) = \begin{cases} 0 & \text{for } -a/2 \le x \le a/2 \\ \infty & \text{for } |x| > a/2. \end{cases}$$

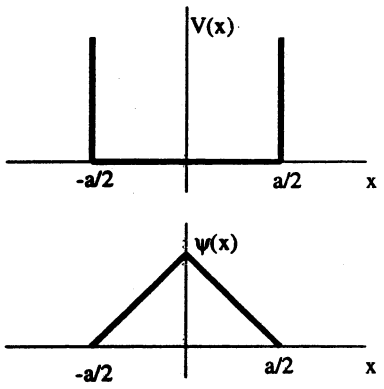
The particle's wave function is initially prepared with the triangle shape

$$\psi(x) = \begin{cases} C(a/2+x) & \text{for } -a/2 \le x \le 0 \\ C(a/2-x) & \text{for } 0 \le x \le a/2, \end{cases}$$

where $C = (12/a^3)^{1/2}$. The energy of the particle is then measured.

A. What is the probability that the particle is found to be in the lowest energy eigenstate?

B. What is the probability that the particle is found to be in the first excited state?



IV

10. (Modern Physics) Naturally-occuring potassium contains 0.0118 atomic percent of the isotope $^{40}_{19}\mathrm{K}$, which has two decay modes:

$$^{40}_{19}\mathrm{K} \rightarrow {}^{40}_{20}\mathrm{Ca} + e^- + \bar{\nu}_e \qquad (\beta \text{ decay}) \qquad (1)$$

$$^{40}_{19}\mathrm{K} + e^- \rightarrow {}^{40}_{18}\mathrm{Ar}^* + \nu_e \rightarrow {}^{40}_{18}\mathrm{Ar} + \gamma\,. \qquad (\text{electron capture}) \qquad (2)$$

The two decays are distinguished by the emitted e^- or γ, which occur on average in the ratio of 12 γ-rays to every 100 electrons. The total intensity of electrons emitted is $2.7 \times 10^4\ \mathrm{s}^{-1}$ per kilogram of natural potassium. Estimate the mean lifetime of $^{40}_{19}\mathrm{K}$.

(The atomic weight of natural potassium is 39.089. Avogadro's Number is $6.02 \times 10^{23}\ \mathrm{mol}^{-1}$.)

11. Thermodynamics – I

One mole of an ideal gas is initially at STP,

$$\begin{aligned} P_0 &= 1 \text{ atm} = 1.01 \times 10^5 \text{ N/m}^2 \\ V_0 &= 22.4 \times 10^{-3} \text{ m}^3 \\ T_0 &= 273 \text{ K} \end{aligned}$$

The volume is changed isothermally to $2V_0$.

[2 points] (a) Determine the final pressure.

[4 points] (b) Determine the work done by the gas.

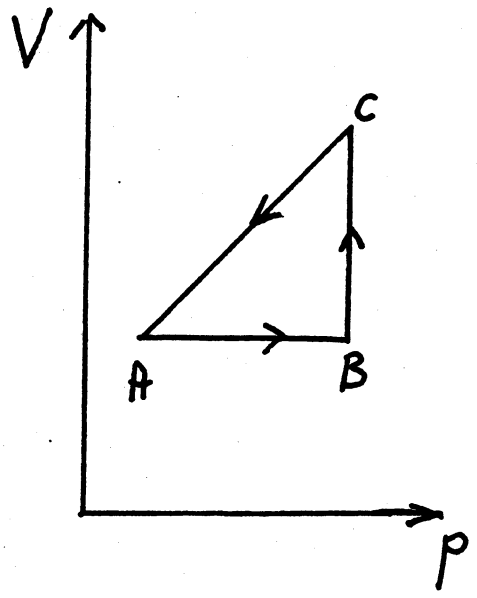
[4 points] (c) Determine the change in entropy of the gas.

Exam #

12. Thermodynamics -II

2 pts (i) State the first law of thermodynamics in the form of an equation.

8 pts (ii) A system is taken from an initial state A to another state B and back again to A, via state C, as shown by path ABCA in the p-V diagram below. Complete the table below by filling in +, − or 0 for the sign of each thermodynamic quantity associated with each process. In the table, Q is the heat absorbed by the system, W is the work done by the system and ΔU is the change in the internal energy of the system. Solve this problem for a general system and not necessarily for an ideal gas.



	Q	W	ΔU
A → B	+		
B → C			+
C → A			