Qualifying/Placement Exam, Part-B 2:00 – 4:00, August 18, 2016, 1400 BPS

## Put your **Student Number** on every sheet of this 6 problem Exam -- NOW

You have 2 hours to complete the 6 problems on Part-B of the exam. Show your work! Full credit will not be given for answers without justification. Some partial credit may be earned for the correct procedure, even if the correct answer is not achieved. Answers must be in the spaces provided. The **BACK** of the problem page may be used for lengthy calculations. *Do not use the back of the previous page for this purpose*!

You may need the following constants:

$k_e = 8.99 \times 10^9 \mathrm{Nm^2/C^2}$	permittivity of free space
$\sigma = 5.7 \times 10^{-8} \mathrm{Wm^{-2}K^{-4}}$	Stefan-Boltzmann constant
$k = 1.4 \times 10^{-23} \text{ J/K}$	Boltzmann constant
$\hbar = 1.05 \times 10^{-34} \mathrm{J} \cdot \mathrm{s}$	Planck's constant
$= 6.58 \times 10^{-16} \mathrm{eV} \cdot \mathrm{s}$	"
$c = 3.0 \times 10^8 \mathrm{m/s}$	speed of light
$e = 1.602 \times 10^{-19} \text{C}$	charge of the electron
$N_A = 6.022 \times 10^{23} \text{ mol}^{-1}$	Avogadro constant

Student No.:

1. [10 pts] An electron in a hydrogen atom is in the normalized state

$$\psi(\vec{r}) = \frac{1}{10} \left[ 5\psi_{10}^0(\vec{r}) + 3\psi_{21}^0(\vec{r}) - 7i\psi_{21}^1(\vec{r}) + 4\psi_{31}^{-1}(\vec{r}) + i\psi_{32}^2(\vec{r}) \right],$$

where  $\psi_{n\ell}^{m_{\ell}}(\vec{r})$  is denoted by principle quantum number *n*, angular momentum quantum number  $\ell$ , and magnetic quantum number  $m_{\ell}$ .

- a) [5 pts] What is the probability that a measurement of the electron's energy will yield this value:  $-1.51 \text{ eV} = -13.6/(3)^2 \text{ eV}$ ?
- b) [5 pts] What is the expectation value of the z-component of angular momentum,  $\langle L_z \rangle$ , in this state?

Student No.:

2. [10 pts] A wave in one dimension with energy  $E > V_0$  and amplitude A approaches a 'step' at x = 0 from  $x = -\infty$ . The reflected wave has amplitude B and the transmitted wave amplitude C.

$$V(x) = \begin{cases} V_0 & \text{for } -\infty < x < 0\\ 0 & \text{for } 0 < x < \infty \end{cases}$$



- a) [2 pts] Determine the solution for the wave function in region *I*. Let  $2m(E - V_0)/\hbar^2 = q^2$ .
- b) [2 pts] Determine the solution for the wave function in region *II*. Let  $2mE/\hbar^2 = k^2$ .
- c) [2 pts] Write down the two equations that match the solutions at x = 0.
- d) [4 pts] Solve for *B* and *C* in terms of *A*, and obtain the reflection and transmission coefficients.

- 3. [10 pts] The temperature of your skin is approximately 35.0 °C.
  - a) [4 pts] Assuming that your skin is a blackbody, what is the peak wavelength of the radiation it emits?
  - b) [4 pts] Assuming a total surface area of 2.00 m<sup>2</sup>, what is the total power emitted by your skin?
  - c) [2 pts] Given your answer to part (b), why don't you glow as brightly as a light bulb?

Note: Absolute zero temperature is -273.15 °C, Wein's displacement constant is  $2.90 \times 10^{-3}$  Km , and Stefan-Boltzmann constant is  $5.67 \times 10^{-8}$  Wm<sup>-2</sup>K<sup>-4</sup>.

Student No.:

4. [10 pts] An archaeologist found a 25.0 g piece of charcoal in the ruins of an ancient city. The sample shows <sup>14</sup>C activity of 250 decays/min. The charcoal came from a tree that died how long ago?

[Required information: The half-life of <sup>14</sup>C is 5730 yr, and the ratio of <sup>14</sup>C to <sup>12</sup>C in a living organism is  $1.3 \times 10^{-12}$ .]

- 5. [10 pts] A neutron beam with a selected speed of 0.40 m/s is directed through a double slit with a slit separation of 1.0 mm. An array of detectors is placed 10 m from the slit.
  - a) [4 pts] What is the de Broglie wavelength of the neutrons?
  - b) [6 pts] How far off axis is the first zero-intensity point on the detector array?

- 6. [10 pts] Consider an ideal mono-atomic gas of volume  $V_0$ , that expands adiabatically from a temperature of  $3.00 \times 10^3$  K to 2.75 K.
  - a) [4 pts] Calculate how much the volume of the gas has changed.
  - b) [5 pts] Assuming that the process is irreversible, calculate the change in the entropy of the gas based on the change in volume. Take the Boltzmann constant to be  $1.38 \times 10^{-23}$  J/K.
  - c) [1 pt] State why you think this is, or is not a good model for the temperature drop in the expansion of the Universe