

Student No.: _____

Qualifying/Placement Exam, Part-A
10:00 – 12:00, Aug. 20, 2018, 1400 BPS

Put your **Student Number** on every sheet of this
6 problem Exam -- NOW

You have 2 hours to complete the 6 problems on Part-A of the exam. Show your work! Full credit will not be given for answers without justification. Some partial credit may be earned for the correct procedure, even if the correct answer is not achieved. Answers must be in the spaces provided. The **BACK** of the problem page may be used for lengthy calculations. *Do not use the back of the previous page for this purpose!*

You may need the following constants:

| | |
|--|----------------------------|
| $k_e = 8.99 \times 10^9 \text{ Nm}^2/\text{C}^2$ | permittivity of free space |
| $\sigma = 5.7 \times 10^{-8} \text{ Wm}^{-2}\text{K}^{-4}$ | Stefan-Boltzmann constant |
| $k = 1.4 \times 10^{-23} \text{ J/K}$ | Boltzmann constant |
| $h = 1.05 \times 10^{-34} \text{ J} \cdot \text{s}$ | Planck's constant |
| $= 6.58 \times 10^{-16} \text{ eV} \cdot \text{s}$ | " |
| $c = 3.0 \times 10^8 \text{ m/s}$ | speed of light |
| $e = 1.602 \times 10^{-19} \text{ C}$ | charge of the electron |

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1. [10 pts] The vectors $\mathbf{v}_1 = (1,1,0)$, $\mathbf{v}_2 = (1,0,1)$, $\mathbf{v}_3 = (0,1,1)$, form a complete set of basis vectors, i.e., one can find a unique linear combination of these vectors to represent an arbitrary vector (x,y,z) :

$$(x, y, z) = c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + c_3\mathbf{v}_3 .$$

a) [5 pts] Prove that this linear combination is unique.

b) [5 pts] Determine the components, c_1, c_2, c_3 , of the general vector (x, y, z) .

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3. [10 pts] Find the eigenvalues and eigenvectors of the matrix:

$$M = \begin{pmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

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4. [10 pts] The National Ignition Facility (NIF) has the most powerful laser in the world. NIF uses 192 beams to aim 500 TW of power at a spherical target of diameter 2.00 mm with density 2000 kg/m³.

Suppose that *only one* of the 192 laser beams hits the target for 1.00 ns and the target *absorbs only 2.00%* of the laser light.

a) [7 pts] What is the radiation pressure (the absorbed power per unit area, divided by the speed of light) on the target.

b) [3 pts] What is the resulting acceleration of the target?

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5. [10 pts] Consider a long straight wire of radius a positioned at the center of a long conducting cylindrical shell of radius R . There is a current I in the \hat{z} direction in the wire that returns in the opposite direction in the cylindrical shell. Assume the current is uniformly distributed over the cross section of the wire and around the shell, and that the shell is arbitrarily thin.

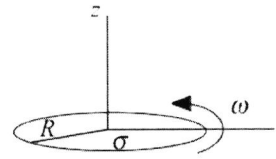
a) [6 pts] Find the magnetic field at all points, i.e., for $r < a$, $a < r < R$, and $r > R$.

b) [2 pts] What is the energy per unit length stored in the magnetic field?

c) [2 pts] If the length of the cylinder and wire is ℓ , what is the self-inductance of the circuit formed by the wire & shell?

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6. [10 pts] An insulating circular *disk* of radius R , carrying on its top surface a uniform surface charge density σ , rotates with an angular velocity ω , counterclockwise about the z-axis as shown.



a) [7 pts] On the disk, what is the *surface current density*, $\mathbf{K}(r) = K(r)\hat{\phi}$?

b) [3 pts] What is the magnetic moment of the disk, where an arbitrary current density, $\mathbf{J}(\mathbf{x})$,

has a magnetic moment, $\mathbf{m} = \frac{1}{2} \int \mathbf{x}' \times \mathbf{J}(\mathbf{x}') d^3 x'$? Hint: $\hat{r} \times \hat{\phi} = \hat{k}$.