Student No.: ______________________

Qualifying/Placement Exam, Part-A
09:30 – 11:30, August 19, 2014, 1400 BPS

Put your **Student Number** on every sheet of this 6 problem Exam -- NOW

You have 2 hours to complete the 6 problems on Part-A of the exam. Show your work! Full credit will not be given for answers without justification. Some partial credit may be earned for the correct procedure, even if the correct answer is not achieved. Answers must be in the spaces provided. The **BACK** of the problem page may be used for lengthy calculations. *Do not use the back of the previous page for this purpose*!

You may need the following constants:

$k_e = 8.99\times10^{9}\ \mathrm{Nm^2/C^2}$	permittivity of free space
$\sigma = 5.7\times10^{-8}\ \mathrm{Wm^{-2}K^{-4}}$	Stefan-Boltzmann constant
$k = 1.4\times10^{-23}\ \mathrm{J/K}$	Boltzmann constant
$\hbar = 1.05\times10^{-34}\ \mathrm{J\cdot s}$	Planck's constant
$= 6.58\times10^{-16}\ \mathrm{eV\cdot s}$	"
$c = 3.0\times10^{8}\ \mathrm{m/s}$	speed of light
$e = 1.602\times10^{-19}\mathrm{C}$	charge of the electron

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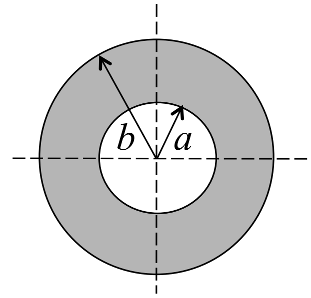

1. [10 pts] The space between two conducting coaxial cylinders of length L and radii, a and b, is completely filled with a material of resistivity ρ.

a) [8 pts] What is the resistance between the two cylinders?
b) [2 pts] Assuming a fill resistivity, $\rho = 30\ \Omega\text{m}$, and cylinders with dimensions, a = 1.5 cm, b = 2.5 cm, L = 50 cm, maintained at a potential difference of 10 V, find the current between them.

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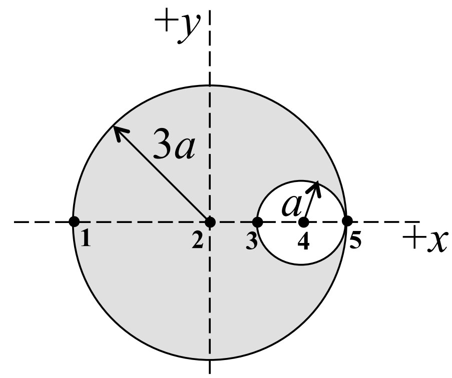
2. [10 pts] The figure at the right shows the cross-section of an infinitely long conducting cylinder of radius $3a$ with part of the area emptied by another cylinder of radius a. The current in the conductor, $\boldsymbol{I}$, is uniformly distributed with a direction pointing out of the plane.



Calculate the *magnitude H,* and *direction* of the magnetic field at the various positions labeled with 1, 2, 3, 4, 5. Note: for all of these points the value of y is zero.

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3. [10 pts] An infinite conducting plane is situated at $z = 0$ and kept at zero potential. Two point charges, $+Q$ and $-Q$, are placed at the coordinates $x = 0$, $y = 0$, $z = a$, and $x = 0$, $y = 0$, $z = -a$, respectively. Find the potential $V(x, y, \mathrm{z})$ for all points $z > 0$.

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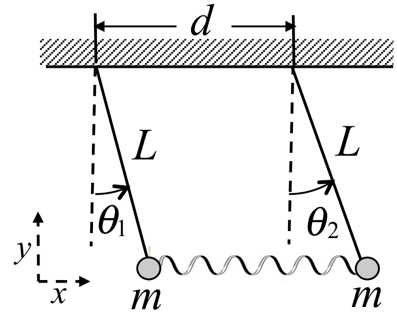
4. [10 pts] Two identical spheres of mass m, attached to the end of two massless rods at a distance L from a pivot, are connected together via a spring with a spring constant k, and are allowed to move only in the x-y plane, as shown in the figure on the right. The distance between pivots, d, is equal to the length of the non-stretched spring.



a) [3 pts] Express the Lagrangian in the coordinates, θ_1 and θ_2.

b) [7 pts] Find the normal modes of small oscillations and the associated frequencies.

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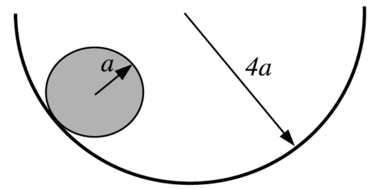

5. [10 pts] A uniform circular cylinder of radius a, and mass m, rolls without slipping on the surface of a fixed cylinder of radius $4a$. Find the frequency ω of small oscillations of the rolling cylinder.

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6. [10 pts] A particle of mass m is located at the center of a harmonic oscillator with spring constant k, and damping constant γ,

$$F = -kx - \gamma \frac{dx}{dt}.$$

An external force is applied,

$$F_{Ext}(t) = F_0 \sin \omega_0 t .$$

Find the behavior $x(t)$ for large times, i.e., your answer should not incorporate any transient behavior.

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Qualifying/Placement Exam, Part-B
13:30 – 15:30, August 19, 2014, 1400 BPS

Put your **Student Number** on every sheet of this 6 problem Exam -- NOW

You have 2 hours to complete the 6 problems on Part-B of the exam. Show your work! Full credit will not be given for answers without justification. Some partial credit may be earned for the correct procedure, even if the correct answer is not achieved. Answers must be in the spaces provided. The **BACK** of the problem page may be used for lengthy calculations. *Do not use the back of the previous page for this purpose*!

You may need the following constants:

$k_e = 8.99\times10^{9}\ \mathrm{Nm^2/C^2}$	permittivity of free space
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1. [10 pts] The time-independent wave function for a particle of mass m which moves in a one-dimensional potential, $V(x)$, has the form $\psi(x) = A\exp(-a^2x^2)$, where $a = \sqrt{m\omega / 2\hbar}$ and A is a normalization constant.

a) [8 pts] Using the time-independent Schrodinger equation, find $V(x)$ and the energy eigenvalue for $\psi(x)$.

b) [2 pts] Identify the system. Which one of the quantum states is described by $\psi(x)$?

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2. [10 pts] Consider the electron in a Hydrogen atom. At time $t = 0$ its wavefunction is a superposition of three eigenstates:

$$\psi(\vec{r}, t=0) = \tfrac{1}{\sqrt{2}}\psi_{2s}^{0}(\vec{r}) + \tfrac{i}{2}\psi_{2p}^{0}(\vec{r}) - \tfrac{i}{2}\psi_{2p}^{-1}(\vec{r}),$$

where, $\psi_{n\ell}^{m}(\vec{r})$ is the normalized wavefunction with principle quantum number n, orbital angular momentum quantum number ℓ, and magnetic quantum number m. The Rydberg ionization energy $E_R = \alpha^2 mc^2 / 2 = 13.6$ eV .

a) [5 pts] If the orbital angular momentum along the z-axis, L_z, is measured, what are the possible values that could be obtained?
b) [2 pts] What are the probabilities to obtain each possible value?
c) [3 pts] What is the wavefunction at a time $t \neq 0$ later (assuming no measurements have been made yet)?

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3. [10 pts] A particle of mass M is in the one-dimensional box $0 < x < a$, i.e.,

$$V(x) = \left\{ \begin{array}{cl} 0 & \text{if } 0 < x < a \\ +\infty & \text{otherwise} \end{array} \right\},$$

and its wavefunction is, $\psi(x) = Nx(a-x)$, where N is a normalization constant.

a) [5 pts] If you measure the energy of the particle, what is the smallest result you might find?
b) [5 pts] What is the probability that you would find that result? *You may leave clearly defined definite integrals in your answer - you do not need to evaluate them.*

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4. [10 pts] The mass of a muon is, $m_\mu = 106\ \mathrm{MeV/c^2}$, the mass of a charged pion is, $m_\pi = 140\ \mathrm{MeV/c^2}$ and $\hbar c = 197\,\mathrm{MeV \cdot fm}$ $(1\,\mathrm{fm} \equiv 10^{-15}\mathrm{m})$

a) [5 pts] How fast must a muon travel to have the same energy as a pion at rest?
b) [2 pts] What is the de Broglie wavelength of the muon?
c) [3 pts] If you could measure its momentum with 10% accuracy at time *t*, how well can you know the position of the muon at the same time?

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5. [10 pt] The first excited state of ^{57}Fe has an excitation energy of 14.4 keV and a mean lifetime of 141 ns.

a) [2 pts] What is the width ΔE of the excited state? Express it in eV if you can.
b) [5 pts] When the state decays to the ground state by emission of a photon, what is the recoil energy of the ^{57}Fe atom?
c) [3 pts] The photon cannot be reabsorbed by another ^{57}Fe atom in its ground state. Why not? (Assume the two atoms are initially at rest.)

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6. [10 pts] A massless, $E = pc$, particle moves in an infinite one-dimensional potential well. The energy states as determined by the boundary conditions are

$$E_n = n\hbar\omega_0,\ n = 1,2,\cdots$$

The system is heated to a temperature, T, with a single particle in the well.

a) [5 pts] What is the probability that the particle is in each of the n states?
b) [5 pts] What is the average energy of the particle?