Covariance Analysis of Symmetry Energy Observables from Heavy Ion Collision

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Abstract
Using covariance analysis, we quantify the correlations between the interaction parameters in a transport model and the experimental observables commonly used to extract information of the Equation of State of Asymmetric Nuclear Matter. Using the simulations of $^{124}\text{Sn}+^{124}\text{Sn}$, $^{124}\text{Sn}+^{112}\text{Sn}$ and $^{112}\text{Sn}+^{112}\text{Sn}$ reactions at beam energies of 50 and 120 MeV per nucleon, we have identified that the nucleon effective masses splitting are most strongly correlated to the neutrons and protons yield ratios from central collisions especially at high incident energy. The best observable to determine the slope of the symmetry energy, $L$, at saturation density is the isospin diffusion observable at low incident energy even though the correlation is not very strong ($\sim0.7$). Similar magnitude of correlation but opposite in sign exists for isospin diffusion and nucleon isoscalar effective masses. At 120 MeV/u, the effective mass splitting and the isoscalar effective mass also have opposite correlation for the double n/p and isoscaling p/p yield ratios. By combining the data and simulations at high and low energy, it should be possible to disentangle the effects the slope of symmetry energy ($L$), isoscalar effective mass ($m_s^*/m$) and effective mass splitting and place the constraints on $L$, $m_s^*/m$ and effective mass splitting with reasonable uncertainties.

Keywords: nucleon effective mass and effective mass splitting, symmetry energy, heavy ion collisions, covariance analysis

Knowledge about the isospin asymmetric nuclear equation of state is of

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\textit{Preprint submitted to Physics Letter B} \hspace{1cm} April 7, 2015
fundamental importance to our understanding of nature’s most asymmetric objects including neutron stars and heavy nuclei composed of very different numbers of neutrons and protons. In theory, there are two approaches to predict its properties. One is the microscopic approach which starts from a realistic two-body free NN and three-body NNN interaction[1, 2]. Based on many body approaches, different approximations are used, such as the relativistic Dirac-Bruckner-Hartree-Fock (DBHF) and its nonrelativistic counterpart Bruckner-Hartree-Fock (BHF)[3, 4, 5, 6, 7], Relativistic-Hartree-Fock (RHF)[8, 9] and chiral effective field[10] theories. Another approach is to use effective density dependent many body interactions such as the zero range Skyrme interaction [11, 12] and finite range Gogny interaction [13]. Among them, the effective Skyrme interactions are more commonly used in nuclear structure, reactions and astrophysics studies because they include translational invariance, Galilei invariance, rotational invariance, isospin invariance, parity invariance and time reversal invariance. The effective Skyrme force parameters allow a quantitative description of heavy nuclei properties while being sufficiently simple mathematically to make it computationally feasible[14]. They are usually obtained by fitting the properties of symmetric nuclear matter (such as the saturation density and the corresponding energy per nucleon and its incompressibility), asymmetric properties of nuclear matter (such as symmetry energy and isovector effective mass at saturation density), finite nuclei properties (such as binding energies and r.m.s radius of selected set of doubly magic nuclei) etc.[15]. In Ref [16], 240 Skyrme parameter sets [16] that fit the ground-state properties of stable nuclei and of symmetric and asymmetric nuclear matter are compiled. The large number of parameterization arises in part because there are strong correlations between some individual parameters or groups of parameters, with particular physical properties of the many-body nuclear system. These sets lead to very different neutron Equation of states[17, 18, 19] which may have different incompressibility $K_0 = 9\rho^2 \frac{\partial^2 \epsilon}{\partial \rho^2}$, symmetry energy coefficient $S_0 = S(\rho_0)$, slope of symmetry energy $L = 3\rho_0 \frac{\partial S(\rho)}{\partial \rho}|_{\rho_0}$, isoscalar effective mass $m_s^* = (1 + \frac{m}{k} \frac{\partial U(k,E)}{\partial k})^{-1}$, and isovector effective mass $m_v^* \frac{1}{1+\kappa}$, where $\kappa$ is the enhancement factor of the Thomas-Reich-Kuhn sum rule [20]. In this work, the magnitude of effective mass splitting, $(m_n^* - m_p^*)/m$ was not used as input variables since its form is much more complicated to be incorporated into the code. Instead we use $f_I = \frac{1}{2\delta} \frac{\partial (U_n - U_p)}{\partial E_k} = \frac{1}{2\delta} \left( \frac{m}{m_n^*} - \frac{m}{m_p^*} \right) = \frac{m}{m_s^*} - \frac{m}{m_v^*},$ where $\delta = (\rho_n - \rho_p)/(\rho_n + \rho_p)$, $\rho_n$ and $\rho_p$ are the neutron and proton den-
sity, $m_n^\ast$, $m_p^\ast$, and $m$ are the neutron, proton effective mass and free nucleon mass. In Skyrme Hartree-Fock approximation\[12, 15, 16\], $f_I$ increases with increasing density, but is independent of the momentum and kinetic energy which is similar to the behaviors in ImQMD-Sky. In the DBHF and RHF approximations \[9, 21, 22\], $f_I$ not only depends on the density but also on the kinetic energy of the in-medium nucleon.

To obtain the information of symmetry energy with heavy ion collision data, the symmetry potential used in transport models is changed by varying its input parameters, corresponding to the different values of $S_0$ and/or $L$ in the expression of density dependence of symmetry energy. The results of the calculations are then compared with data to find the best parameter sets. Recently, a consistent set of constraints on the symmetry energy near saturation density between $S_0$, and its slope, $L$, has been obtained from observables measured in both nuclear structure and nuclear reaction experiments\[19, 23, 24, 25\].

In Skyrme parametrizations, the symmetry energy parameters are expected to be correlated to other parameters such as those associated with the nucleon effective mass. Such correlations would affect the eventual symmetry energy constraints obtained from heavy ion collision data. In this work, we use the covariance analysis to quantitatively examine the correlations between model parameters and experimental observables. The correlation coefficient $C_{AB}$ between variable $A$ and observable $B$ is calculated as follows\[26\]:

$$C_{AB} = \frac{\text{cov}(A, B)}{\sigma(A)\sigma(B)}$$  

(1)

$$\text{cov}(A, B) = \frac{1}{N-1} \sum_i (A_i - <A>)(B_i - <B>)$$  

(2)

$$\sigma(X) = \sqrt{\frac{1}{N-1} \sum_i (X_i - <X>)^2}, X = A, B$$  

(3)

$$<X> = \frac{1}{N} \sum_i X_i, i = 1, N.$$  

(4)

$C_{AB} = 1$ means there is a linear dependence between $A$ and $B$, and $C_{AB} = 0$ means no correlations. In this work, we use the Improved Quantum Molecular Dynamic Model which incorporates the effective Skyrme interactions (ImQMD-Sky)\[27\] to simulate the collisions of heavy ions with parameter
set $i$ as in Table I. $A_i$ represents the $i$th parameter set of the transport variable $A = K_0, S_0, L, m_s^*, m_v^*$ or $f_I$ used as input to the ImQMD-Sky.

In ImQMD-Sky, the nucleonic potential energy density is $u_{\text{loc}} + u_{\text{md}}$, where

$$u_{\rho} = \frac{\alpha \rho^2}{2 \rho_0} + \frac{\beta \rho^{\eta+1}}{(\eta + 1) \rho_0^\eta} + \frac{g_{\text{sur}}}{2 \rho_0} (\nabla \rho)^2$$

$$+ \frac{g_{\text{sur},iso}}{\rho_0} [\nabla(\rho_n - \rho_p)]^2$$

$$+ A_{\text{sym}} \rho^2 \delta^2 + B_{\text{sym}} \rho^{\eta+1} \delta^2$$

and the energy density of Skyrme-type momentum dependent interaction are written based on its interaction form $\delta(r_1 - r_2)(p_1 - p_2)^2$ [11, 12],

$$u_{\text{md}} = u_{\text{md}}(\rho \tau) + u_{\text{md}}(\rho_n \tau_n) + u_{\text{md}}(\rho_p \tau_p)$$

$$= C_0 \int d^3p d^3p' f(\vec{r}, \vec{p}) f(\vec{r}, \vec{p}') (\vec{p} - \vec{p}')^2 +$$

$$D_0 \int d^3p d^3p' [f_n(\vec{r}, \vec{p}) f_n(\vec{r}, \vec{p}') (\vec{p} - \vec{p}')^2$$

$$+ f_p(\vec{r}, \vec{p}) f_p(\vec{r}, \vec{p}') (\vec{p} - \vec{p}')^2].$$

The 9 parameters $\alpha$, $\beta$, $\eta$, $A_{\text{sym}}$, $B_{\text{sym}}$, $C_0$, $D_0$, $g_{\text{sur}}$, $g_{\text{sur},iso}$ used in ImQMD-Sky can be derived from standard Skyrme parameter sets with 9 parameters \{t_0, t_1, t_2, t_3, x_0, x_1, x_2, x_3, \sigma\}[27, 28]. The coefficients of surface terms are set as $g_{\text{sur}} = 24.5 \text{MeV fm}^2$ and $g_{\text{sur},iso} = -4.99 \text{MeV fm}^2$ which are the same values derived from SLy4 parameter set[15], and varying of $g_{\text{sur}}$ and $g_{\text{sur},iso}$ for different Skyrme interactions have negligible effects on the experimental observables at intermediate energy. By using the relationship which was derived in reference[29, 30], the reduced 7 Skyrme parameter sets \{\alpha, \beta, \eta, A_{\text{sym}}, B_{\text{sym}}, C_0, D_0\} can be replaced by the parameter sets \{\rho_0, E_0, K_0, S_0, L, m_s^*, m_v^*\} which is directly related to the properties of nuclear matter at saturation density $\rho_0$. Choosing the experimental values of $\rho_0 = 0.16 \text{fm}^{-3}$, $E_0 = -16 \text{MeV}$, the parameter sets are further reduced to 5 variables, $A = K_0, S_0, L, m_s^*, m_v^*$. As explained above, to simplify the coding, the input variables used in ImQMD-Sky are $A = K_0, S_0, L, m_s^*, f_I(m_v^*)$.

For experimental observables, we adopt the ratios constructed from nucleon spectra. Most of the transport models over-predict the absolute yield of free nucleons due to inadequacies of the model in describing cluster production [31, 32, 33]. To minimize these effects, we construct the coalescence
invariant (CI) nucleon yield spectra and their ratios the same way as in Ref[34, 37, 38] by combining the nucleons in the light particles and free nucleons at given kinetic energy per nucleon as follows,

\[
dl{M_n,CI} \over dl{E_{c.m.}}/A = \sum N dM(N, Z) \over dl{E_{c.m.}}/A (7)
\]

\[
dl{M_p,CI} \over dl{E_{c.m.}}/A = \sum Z dM(N, Z) \over dl{E_{c.m.}}/A (8)
\]

The summation is up to \(Z \leq 6\) and \(A \leq 16\) particles in the calculations. Such CI neutron and proton yields obtained with ImQMD calculations with selected Skyrme parameter sets reproduce the Sn+Sn data[39] reasonably well especially at high beam energy. For simplicity, the CI neutron and proton yields from reaction \(i\) are represented as \(Y_i(n)\) and \(Y_i(p)\) respectively in the following. By convention, the more neutron-rich reaction is represented by the subscript "2" and in this work, \(i=1,2\) represent the reactions \(^{112}\text{Sn}+^{112}\text{Sn}\) and \(^{124}\text{Sn}+^{124}\text{Sn}\).

We simulate 10,000 events for each of the three systems, \(^{124}\text{Sn}+^{124}\text{Sn}, ^{124}\text{Sn}+^{112}\text{Sn}\) and \(^{112}\text{Sn}+^{124}\text{Sn}\). We construct four ratios from the coalescence invariant nucleon spectra: 1.) single n/p ratio CI-R\(_{2}(n/p)=Y_{2}(n)/Y_{2}(p)\), 2.) double n/p ratios CI-DR(n/p)=CI-R\(_{2}(n/p)/CI-R_{1}(n/p)=CI-R_{21}(n/n)/CI-R_{21}(p/p)\), 3.) isoscaling ratios CI-R\(_{21}(n/n)\) and 4.) CI-R\(_{21}(p/p)\) ratios. The experimental isoscaling ratios CI-R\(_{21}(n/n)=Y_{2}(n)/Y_{1}(n)\) and CI-R\(_{21}(p/p)=Y_{2}(p)/Y_{1}(p)\) and the double ratios CI-DR(n/p) have the advantage that they are minimally affected by detector systematic uncertainties and sequential decays[40, 41]. 5.) last isospin observable we analyze is the isospin transport ratios[42], \(R_{diff}\), that quantify diffusion of the nucleons in the neck region during the nuclear collisions. \(R_{diff}\) has been used to constrain the symmetry energy at subsaturation density. These constraints are consistent with constraints obtained in nuclear structure experiments[19].

In experiments, \(R_{diff}\) is defined as [42], \(R_{diff} = (2X - X_{aa} - X_{bb})/(X_{aa} - X_{bb})\) where \(X = X_{ab}\) is an isospin observable. In this work, we use the subscripts a and b to denote the projectile (first index) and target (second index) combination. By convention, \(aa\) and \(bb\) represent the neutron-rich and neutron-poor reactions, respectively. In theoretical calculations, \(X\) is the isospin asymmetry of the emitting source, which is constructed from all the emitted nucleons and fragments with certain velocity cuts and it is linearly related to \(X_{exp}\) as discussed in [42, 43], and have been used to
compare with data. One should note that, sensitivities of nucleon ratios to
effective mass splitting such as \( R_i(n/p), \) \( DR(n/p) \) rely mainly on high energy
nucleons\([27, 44, 45, 46, 47, 48]\), while \( R_{\text{diff}} \) are constructed with at least three
reaction systems to cancel the drift and mainly retain the information of the
diffusion, and carries the information of "nucleons" that have lower energy.
Thus complimentary information is obtained from the nucleon ratios and the
isospin transport ratios.

We first perform calculations by separately varying each variable of the
parameter set \( K_0, S_0, L, m^*_s, f_I \). In the following studies, 12 parameter sets
are constructed and listed in Table I. The average values of each variable in
the parameter space we used are, \( < K_0 >= 242.5 \text{MeV}, < S_0 >= 32 \text{MeV}, \)
\( < L >= 54.5 \text{MeV}, < m^*_s/m >= 0.7375 \) and \( < f_I >= -0.1835 \).

Table 1: List of twelve parameters used in the ImQMD calculations. \( \rho_0 = 0.16 \text{fm}^{-3}, \)
\( E_0 = -16 \text{MeV}, \) and \( g_{\text{sur}} = 24.5 \text{MeVfm}^2, g_{\text{sur,iso}} = -4.99 \text{MeVfm}^2 \)

<table>
<thead>
<tr>
<th>Para.</th>
<th>( K_0 ) (MeV)</th>
<th>( S_0 ) (MeV)</th>
<th>( L ) (MeV)</th>
<th>( m^*_s/m )</th>
<th>( f_I )</th>
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<td>46</td>
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<tr>
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<td>46</td>
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<td>0.7</td>
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<tr>
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<td>30</td>
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<tr>
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<td>1.00</td>
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<tr>
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<td>46</td>
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<td>0.0</td>
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<tr>
<td>12(SLy4)</td>
<td>230</td>
<td>32</td>
<td>46</td>
<td>0.7</td>
<td>0.178</td>
</tr>
</tbody>
</table>

To study the sensitivity of different force parameters in Skyme inter-
actions on isospin observables, we calculate the covariance coefficients \( C_{AB} \)
based on Eq.(1)-(4) using five force parameters: \( A=K_0, S_0, L, m^*_s \) and \( f_I \)
and five HIC observables: \( B=\text{CI}-R_2(n/p), \) CI-DR(n/p), CI-R_{21}(n/n), CI-
R_{21}(p/p) and \( R_{\text{diff}} \). All the nucleon yield observables in the simulations
are obtained for \( ^{124}\text{Sn}+^{124}\text{Sn} \) and \( ^{112}\text{Sn}+^{112}\text{Sn} \) collisions at incident energies
of 50 and 120 MeV per nucleon at impact parameter 2fm. The energy cut
\( E_{\text{c.m.}}/A > 40 \text{MeV} \) and angular cut with \( 70^\circ < \theta_{\text{c.m.}} < 110^\circ \) for CI-R_2(n/p),
CI-DR(n/p), CI-\(R_{21}(n/n)\) and CI-\(R_{21}(p/p)\) are imposed since nucleons with high kinetic energy are less influenced by sequential decay and are more sensitive to the effective mass splitting\([27, 44, 45, 46, 47, 48]\). For \(R_{\text{diff}}\), an additional mixed reaction \(^{124}\text{Sn} + ^{112}\text{Sn}\) is included and the calculations are performed with mid-peripheral impact parameter of 6 fm where a low density neck is formed and isospin transport between projectile and target occurs.

Figure 1 shows the covariance coefficient \(C_{AB}\) for Sn+Sn reactions at \(E_{\text{beam}}=50\) MeV/u (upper panels) and at \(E_{\text{beam}}=120\) MeV/u (lower panels). Red color bars represent positive correlations which mean observable B increases with parameter A, and blue shaded color bars show negative correlations which means observable B decreasing with increasing parameter A. To focus our search for the best experimental observables which are most sensitive to the model parameters, we will discuss only \(C_{AB}\) with values greater than 0.5. With this criterion, \(S_0\) and \(K_0\) are not very sensitive to any of the observables studied here.

![Figure 1](image)

Figure 1: (Color online) Correlations of five observables, CI-\(R_{2}(n/p)\) (a), CI-DR(n/p) (b), CI-\(R_{21}(n/n)\) (c), CI-\(R_{21}(p/p)\) (d), \(R_{\text{diff}}\) (e) with five force parameter, \(K_0\), \(S_0\), \(L\), \(m^*_s\) and \(f_1\). Up panels are the results for 50 MeV per nucleon, and bottom panels are for 120 MeV per nucleon.

In the case of \(L\), slope of the symmetry energy at saturation density,
$C_{LR, \text{diff}} \sim 0.7$ meets this criterion. Our analysis also shows similar influence from $m_s^*$, $C_{m_s^*, \text{diff}} \sim 0.64 - 0.70$. The observation that $R_{\text{diff}}$ is also sensitive to $m_s^*$ is consistent with the calculations from BUU approaches with and without momentum dependent interaction [35, 36, 38]. Interestingly, the sign of the correlation for $C_{LR, \text{diff}}$ and $C_{m_s^*, \text{diff}}$ are opposite suggesting that the data could determine both $L$ and $m_s^*$ values with reasonable uncertainties. Since isospin diffusion decreases with increasing incident energy due to the shorter contact time between projectile and target, one expects $R_{\text{diff}}$ values closer to $\pm 1$ (no diffusion) at 120 MeV per nucleon beam energy and it becomes more difficult to measure $R_{\text{diff}}$ experimentally. Thus it is not clear if the correlation value of $C_{LR, \text{diff}}$ at 120 MeV per nucleon is a robust correlation in practice.

$f_I$ shows the largest correlations to the experimental coalescence invariant nucleon observables, the single and double n/p yield ratios at both incident energies. As shown in panels (a), (f) and (b), (g) of Figure 1, the correlation between the single and double n/p yield ratios and $f_I$ reach above 0.8. However, strong correlation with $m_s^*$ is observed mainly at high energy, because the more violent nucleon-nucleon collisions at 120MeV/u causes larger momentum transfer and leads to the momentum dependent interaction (which can be characterized by the effective mass) to play more important roles. It can also be understood from the expression of the density dependence of symmetry energy in Skyrme interaction, where the density dependence of symmetry energy not only depends on isovector effective mass $m_v^*$, but also on the isoscalar effective mass $m_s^*$, i.e. $S(\rho) = \frac{1}{3} \epsilon_F \rho^{2/3} + A_{\text{sym}} \rho + B_{\text{sym}} \rho^{2/3} + C_{\text{sym}}(m_s^*, m_v^*) \rho^{5/3}$.

For isoscaling ratios, the sensitivity of CI-$R_{21}(n/n)$ to all variables $K_0$, $S_0$, $L$, $m_s^*$ and $f_I$ are borderline. On the other hand, CI-$R_{21}(p/p)$ are particularly sensitive to $f_I$ and $m_s^*$ at 120 MeV/u. Since the proton felt attractive from symmetry potential, it leads to negative correlation between CI-$R_{21}(p/p)$ and $f_I$. Due to the Coulomb repulsion, protons accelerate to higher kinetic energy than neutrons. At higher energy, the influence of effective mass splitting become more important, and it causes the very negative correlation between CI-$R_{21}(p/p)$ and $f_I$. The stronger positive correlation between CI-$R_{21}(p/p)$ and $m_s^*$ appears due to the violent nucleon-nucleon collisions at 120MeV/u. The correlation signs are opposite for them and the single and double n/p ratios. This opposite correlation signs can be exploited to extract constraints for both $f_I$ and $m_s^*$ from the single and double n/p ratios as well as the p/p ratios at 120MeV/u incident energy.
In summary, by separately varying the interaction parameter sets in the ImQMD-Sky code, we study the influence of \( K_0, S_0, L, m_s^*, \) and \( f_I \) on the high energy coalescence invariant neutron and proton yield ratios, for \(^{124}\text{Sn}+^{124}\text{Sn}\) and \(^{112}\text{Sn}+^{112}\text{Sn}\) at 50MeV/u and 120MeV/u incident energy. Sensitivities to \( S_0 \) and \( K_0 \) are relatively small, \( C_{AB} < 0.5 \). Instead, at incident energy of 120MeV/u, strong correlation are observed between observables constructed from coalescence invariant nucleon spectra at \( E_{c.m.}/A > 40 \) MeV, such as CI-\( R_2(n/p) \), CI-DR(n/p), and CI-\( R_{21}(p/p) \), and the effective mass splitting and the isoscalar effective mass, important input parameters to the transport models. The calculations also confirm the sensitivity of \( L \) to the isospin diffusion observable at low beam energy. However, the same observable is also sensitive to isoscalar effective mass but has opposite correlations—which should allow one to extract the constraints of \( m_s^* \) and \( L \) with reasonable uncertainties. Similarly the opposite correlations of the nucleon yield ratios, such as CI-DR(n/p) and CI-\( R_2(n/p) \) ratios to \( f_I \) and \( m_s^* \), at 120 MeV/u reactions would allow one to disentangle the effects of the effective nucleon mass splitting, isovector effective mass.

Acknowledgements This work has been supported by the National Natural Science Foundation of China under Grants No.(11475262, 11375062, 11275072), National Key Basic Research Development Program of China under Grant No. 2013CB834404. MBT acknowledges support from the USA National Science Foundation Grants No. PHY-1102511 and travel support from CUSTIPEN (China-US Theory Institute for Physics with Exotic Nuclei) under the US Department of Energy Grant No. DE-FG02-13ER42025.

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